Print ISSN: 2008-448X
Online ISSN: 2538-3310
International Journal of Business and Development Studies
University of Sistan and Baluchestan
Homepage: https://ijbds.usb.ac.ir

The Application of Game Theory in Determining Performance Tax Rates and Influential Factors

Arash Azami[1]

[1-] Ph.D Student in Economics, Departmen of Economics, Shahid Beheshti University, Tehran, Iran.
Email:arashesmy@gmail.com.

ARTICLE INFO

Article type:
Research

Article history
Received: 21.09.2024
Revised: 30.01.2025
Accepted: 23.03.2025
Published: 07.05.2025

Keywords:
Game Theory, Taxpayers, Optimal Tax Rate on Performance.

JEL Classification:
C70, O11, H21

Abstract:
The implementation of an effective tax system requires specific conditions, among the most critical being equity and efficiency. Taxation on income must align with the principle of the ability to pay; therefore, governments strive to set tax rates appropriately and impactfully. An disproportionate increase in income tax rates can lead to significant social repercussions on income distribution and public welfare. Consequently, calculating the optimal tax rate to maximize social welfare is deemed essential. This paper models the strategic interaction between taxpayers and the tax authority to achieve an optimal tax rate and identifies the factors influencing this rate. By designing a game among taxpayers, assumed to be uniformly distributed within the interval [0, 1], and evaluating the revenue generation of the tax authority, the paper delves into the game formulation and analyses the results. Findings indicate that the optimal tax rate exhibits an inverse relationship with the assessment rates set by investigating groups. Additionally, under specific conditions, the optimal performance tax rate has been determined to be 18%.

1. Introduction

Taxes represent one of the most significant and stable sources of revenue for governments, playing a crucial role as a tool in fiscal policies that contribute to economic growth and stability. On the one hand, citizens expect certain responsibilities from their governments, and the diversity of these responsibilities means that government expenditures hold different values for the public. For those who prioritize social welfare and equality, government spending can be perceived as more valuable, whereas those who place a higher importance on individual freedoms may view government expenditures as less significant. The expansion of government intervention in socio-economic realms, coupled with

Cite this article: A. Azami (2025). The Application of Game Theory in Determining Performance Tax Rates and Influential Factors. *International Journal Of Business and Development Studies*, 17 (1), 121-137. DOI: 10.22111/ijbds.2025.50578.2188.

increased commitments towards goals such as economic growth, price stability, job creation, and equitable income distribution, has led to a rising trend in government expenditures. Additionally, the government's role in addressing market failures and in redistributing economic benefits has further escalated these costs. Consequently, to finance these expenditures, governments employ various methods to collect diverse revenues. Among the most critical means of funding government expenses is tax collection (Maddah & et al, 2016).

The purpose of taxation extends beyond merely financing the government; taxes are also employed as instruments for promoting growth, stability, and reducing inequality within the economy. When the government increases taxes and allocates the revenue to infrastructure development, or enhances opportunities for private investment by lowering corporate taxes, it aims to pursue economic growth. An increase in consumption tax can serve as a tool to decrease aggregate demand, thereby helping to achieve price stability. Furthermore, the government imposes and collects taxes to finance public goods, ensuring the continuous provision of goods and services. Therefore, taxation can have both positive effects on utility and negative ramifications for overall welfare (Ghafari & et al, 2016).

The primary objective of governments, particularly in developing countries, is economic growth, job creation, equitable income distribution, and inflation reduction. Therefore, these countries must utilize their revenues to achieve these goals (Wawire, 2017). Government revenues can be classified into two categories: tax and non-tax revenues. Among these, tax revenue holds greater significance due to its controllable nature compared to other income sources. Conversely, studies such as those by Collier (2006) and Collier and Hoeffler (2005) emphasize that in countries that derive substantial income from natural resource sales, internal tax revenues are typically lower, leading to financial challenges. However, increasing taxes can also introduce difficulties. Thus, analyzing tax revenues and determining the factors influencing them in developing countries, particularly those rich in natural resources, is of special importance (Tamizi, 2018).

Based on the Laffer Curve[1], increasing taxes does not always lead to an increase in government revenue. As tax rates rise up to an optimal level, government revenue increases; however, further tax increases can result in a decline in government income. Therefore, an increase in government revenue is feasible only at this optimal level. When the government levies taxes, it influences the behavior, incentives, and choices of individuals. These effects can either lead society towards greater efficiency or push the economy further from an optimal point. Taxes, as one of the fiscal policy tools, impact the overall utility of the

[1] The hypothetical Laffer curve illustrates that when the tax rate is zero, tax revenue is also zero. As the tax rate increases, tax revenue initially rises until it reaches the optimal level t^*. Beyond this threshold, further increases in the tax rate result in a decline in government tax revenue.

economy in various ways: one through households and another through production firms, as taxes affect firms' investment decisions. Additionally, from the government's perspective, revenue is a function of tax rates. If tax rates are low, government revenue will be insufficient, leading to a budget deficit and hindering the government's ability to fulfill some of its responsibilities.

On the other hand, excessive taxation reduces the purchasing power and welfare of individuals, which in turn diminishes citizens' ability to pay taxes, ultimately leading to a decrease in government revenues and budget deficits. Lowering the income tax rate on capital gains may result in a short-term decline in tax revenues; however, it tends to increase the tax base (and tax revenues) over time. This occurs because a reduction in tax rates stimulates economic activities, including investments, which, following the tax cut, will eventually lead to increased production and, consequently, greater tax revenues (Auerbach, 2005; Mankiw & Weinzierl, 2006; Leeper & Yang, 2008; Strulik & Trimborn, 2012; Houndonougbo & Mohsin, 2016). The effective marginal tax rate is also suitable and applicable for analyzing research on the impact of tax incentives for new investments (Gupta and Newberry, 1997).

In the realm of economic theories, it is important to note that classical economists, such as Adam Smith, supported a proportional tax system. This means that a certain percentage of each individual's income is collected as tax. Smith considered a tax fair if it was applied uniformly across all income levels. Another type of tax is the progressive tax rate, where as income increases, a larger share of it is taxed in comparison to the previous income bracket. The opposite of a progressive rate is a regressive rate, which implies that as income rises, the rate of tax growth becomes milder. Arthur Laffer also emphasized the concept of the optimal tax rate, arguing that when the income tax rate rises excessively, overall tax revenue tends to diminish. When the income tax rate is at zero, no taxes are collected. Conversely, if the income tax rate reaches 100%, tax collection would also cease. Between these two extremes, individuals earn income and pay taxes; there exists a specific tax rate at which tax revenue is maximized (Mankiw, 2013). Given that taxation is a crucial tool in economic and social policy-making, it is necessary to establish principles and theories from various scholars to guide the implementation of tax policies. This ensures that tax systems are tailored to their specific economic and social conditions. Consequently, the discussion surrounding tax principles and theories is a fundamental aspect of tax studies, which can significantly contribute to the redistributive goals of taxation systems.

The theory of optimal tax finance has been developed in the tax literature with considerations for equity and redistribution in mind (Pourmoghimi & et al, 2005). The theories surrounding optimal tax rates are relatively recent and constitute a new area of study within economics. The initial effort in this field can be traced

back to Frank Ramsey (1927), followed by significant research conducted by Diamond & Mirrlees (1971). Subsequently, Saez & Stantcheva (2016) proposed the use of income elasticities to address the limitations of the Mirrlees model (societal constraints).
In his work on public finance, Pigou discusses how tax rates can be determined in markets while minimizing reductions in utility. However, it is now evident that Pigou's proposals are not widely regarded. In traditional public finance and public economics theories, the government was typically viewed as a benevolent entity that collects taxes for the welfare of the public. The assumption was that taxes would be allocated for the general benefit. Recent theories, however, have reexamined the nature of the government, moving towards the establishment of an optimal government through meritocratic paths combined with democratic tools. In an optimal government scenario, the private sector and taxpayers can readily observe the welfare effects of tax policies on their lives. The conceptualization of an optimal government and optimal taxation not only enhances economic efficiency but also contributes to greater political stability and social cohesion within the community. Thus, an optimal taxation system equates the marginal benefits gained from collecting one additional unit of tax with the marginal costs involved in that collection. In other words, if the government collects one thousand units of currency in taxes, the benefits derived from that revenue - such as improvements in social welfare and increased individual satisfaction - must outweigh the associated costs, which may include administrative and compliance expenses related to taxation (Dadgar & et al, 2013).
This article is organized into five sections. Following the introduction, the second section reviews the research background, while the third section presents game theory. The fourth section outlines the game modeling utilized, and the fifth and final section includes conclusions and recommendations.

2. Literature Review

Below, we will reference several studies conducted in the field related to the research topic:
In their research, Dadgar & et al (2013) conducted a strategic analysis of government structure and taxation by investigating optimal government size and optimal tax rates through two key metrics: the government consumption expenditure to GDP ratio and the tax revenue to GDP ratio. To assess the optimal size of the government, they applied the generalized Armey curve[1] as a utility function and employed the generalized method of moments (GMM) for estimation based on time series data. The Iranian government's dependence on

[1] The Armey curve demonstrates that, holding all other factors constant, the negative impact of sustained government size expansion, arising from the principle of diminishing marginal returns, will ultimately eliminate the beneficial effects of government expenditure on economic growth

crude oil export revenues during the analyzed period (2013-1973) has directly influenced the economic structure and the expansion of government size, creating a distinct Nash equilibrium in public goods allocation. The results reveal a significant disparity between the current government structure and tax regime in Iran, and the optimal government size and taxation models predicted by public economics theories, indicating inefficiencies in the current strategic framework.

Sameti & et al (2015) analyzes optimal tax rates using a Ramsey model in a multi-agent framework, applying the Samuelson-Bergson social welfare function. The study maximizes the social welfare function given a specified level of tax revenue for the government using the Lagrange method. The results indicate that at a zero rate of avoidance of social inequality, where the focus is solely on the efficiency of indirect taxes, the optimal tax rates are relatively close to each other. However, as the emphasis shifts towards social justice, leading to increased rates of avoidance of social inequality, the optimal tax rates begin to diverge. Some commodity groups may even qualify for subsidies under these conditions. Furthermore, as the rate of avoidance of social inequality increases, the marginal cost of social welfare related to changes in tax rates for goods and services diminishes. At the highest level of avoidance of social inequality, the decrease in welfare becomes negligible. Consequently, for commodity groups eligible for subsidies, a reduction in subsidies results in decreased social welfare, while for other groups, an increase in taxes leads to a further reduction in social welfare.

Farahnak and et al (2018) conducted a study examining the effects of changes in effective tax rates on public budgets, national production, and welfare using a general equilibrium model. They analyzed three main tax categories: value-added tax, import tax, and household income tax, along with simultaneous increases in all three types of taxes (three individual scenarios and one combined scenario). Comparing the results of the individual scenarios indicates that the most significant impacts arise from a 10% increase in the value-added tax, which raises public revenues and expenditures by 0.88% and 0.79%, respectively, while reducing the budget deficit and welfare by 14.04% and 1.58%. However, the overall best outcomes emerge from implementing the combined scenario, which results in a 19.74% reduction in the budget deficit, influenced by increases in public revenues and expenditures of 1.26% and 1.13%, alongside a 0.14% decline in welfare. This scenario is recommended as the preferred option for implementation. Additionally, due to the limited scope of the tax bases across all scenarios, only minimal negative changes in GNP were observed.

Aghaei and Maddah (2020) focused on determining the optimal tax rate in Iran with an emphasis on value-added tax. They calibrated their model using an endogenous neoclassical growth framework, utilizing parameters from the Iranian economy and data from the 2016 Social Accounting Matrix (SAM). Their analysis revealed that the implementation of a value-added tax at optimal rates occurs under two different scenarios: 5% and 10%.

Seadat Meher (2022) conducted a study on estimating the optimal rate of value-added tax (VAT) using the Laffer Curve approach. To this end, data related to VAT rates and the corresponding tax revenue from 24 provinces of the country was utilized for the period from 2008 to 2019, employing panel data methodology for the estimation of results. The findings of this research indicated that value-added tax has a Laffer effect in the Iranian economy. Additionally, the optimal VAT rate determined in this study was found to be 10.33%.

Altunoz (2017) estimated the Laffer Curve for various tax types in Turkey. To this end, fiscal policies from 1980 to 2014 were analyzed through four distinct models. The variables of total tax, direct tax, and income tax were tested in each model to determine whether the implemented tax policies in Turkey align with the principles of the Laffer Curve. The results indicated that all four models were consistent with the theoretical foundations of the Laffer Curve; however, the maximum and minimum tax rates did not conform to these theoretical underpinnings.

Abolade (2018) analyzes optimal taxation in Nigeria, Qatar, Saudi Arabia, Paraguay, Brunei, and Macedonia based on the Barnett-Wat method. The study identifies an optimal tax scheme through reduced tax rates within the existing tax frameworks. In the period under review, Nigeria's tax structure in its developing economy includes a 30% corporate tax rate for large companies, a 7% personal income tax rate, a 5% value-added tax, and sales tax. The findings suggest that to attract foreign investors into Qatar’s economy, it should implement a tax structure comprised of a 10% corporate income tax, coupled with no personal income tax. This combination enhances the productivity of smaller industries and strengthens the profitability of large corporations in Nigeria.

Kudrna & et al (2022) in their research on optimal capital taxation within the Overlapping Generations (OLG) model in Indonesia, consider the optimal linear capital coefficient in a model of overlapping generations where households face uncertain income risks across two life periods. They conclude that if the Ramsey tax rate, which maximizes sustainable utility, is positive, the implementation of this tax rate permanently enhances economic conditions.

Obara (2018) explores the quest to determine optimal taxation and necessary policy requirements using Ramsey models and optimization frameworks. The central thesis of this research posits that education possesses both consumptive and productive value, allowing individuals to choose their mode of education. While the government can set levels for labor income, capital income, and educational investment, it lacks the ability to discern the specific returns derived from education.

Eklund & Malmsten (2019) investigated optimal tax rates and their variations at the summit of the Laffer Curve through an empirical study using regression

methods. For this purpose, they considered 288 municipalities in Sweden from 2000 to 2017. Their findings indicated that the Laffer Curve does not possess a single maximum value; instead, the optimal rate differs depending on the specific conditions analyzed.
Senz (2022) presents an analytical model (developing a complete microeconomic framework) for the Laffer curve in the context of personal income tax. This model emphasizes that marginal tax rates on personal income not only affect the revenue collection from individual income taxes but also influence the aggregation of revenue from other tax sources. The findings indicate that exclusions (such as ignoring consumption taxes, etc.) create a false illusion in the Laffer curve, suggesting that the maximum revenue occurs at a narrower threshold than actually exists. If these exclusions are factored in, the optimal revenue is achieved by reducing the tax rate from 62.5% to 28.20%.
Salimian and Sobhanian (2024), in their research titled "Triangular Distribution Curve of Salah-Mohammad: A Substitute for the Laffer Curve," focused on determining the optimal tax rate. They employed a triangular probability density function for government tax revenues, which initially increase to a certain point and then decline thereafter, providing a suitable alternative to the Laffer Curve. Their findings indicate that under specific conditions, as the quality of the investigation group increases, the tax revenues collected by the Revenue Department rise. Ultimately, they concluded that the relationship between the tax rate and the variables of investigation quality and assessed tax is inversely correlated.
Based on the conducted analyses, extensive research has been done in the field of taxation; however, there are limited studies specifically addressing tax rates. Among these few studies, only a handful have focused on the determination of the optimal tax rate. These investigations primarily relate to value-added tax and often employ regression and econometric methods. Moreover, until now, no model has integrated utility functions, particularly utilizing game theory methodologies, to explore this area. Therefore, the application of these significant elements and the presentation of several key findings underscore the novelty and originality of this research.

3. Game Theory

Whenever an individual (such as a government or a group) engages in an action in relation to others, that action may provoke a response from the counterpart. This mutual interaction, particularly when both parties are aware of its implications, is referred to as a "game." The concept of game theory was first discussed by James Waldegrave in 1713 when he presented the Min-Max solution for a two-player game. It wasn't until Augustin Cournot, in his 1838 paper titled "Researches into the Mathematical Principles of Wealth," that game theory was pursued more broadly. In this work, Cournot examined bilateral monopolies and introduced a solution that aligns with the Nash equilibrium in a duopoly game. Game theory has evolved significantly through the sustained efforts of numerous scholars in the social sciences, particularly economics, as well as in pure sciences like mathematics and statistics. Today, it is recognized as one of humanity's crucial achievements, serving various fields including social sciences, natural sciences, engineering, and theoretical studies (Abdoli, 2013).

One of the most common types of games is static games with complete information. In a game involving multiple players, if each player has several possible choices (actions) but is unaware of the actions chosen by other players, it is categorized as a static game. In other words, players may select their actions at different times, but their choices remain unknown to one another; such games are also referred to as static games. Many real-world scenarios can be modeled as static games (Von Neumann & Morgenstern, 2007).

In these games, a fundamental assumption is that each player is unaware of the choices made by their opponents; it is as if all players simultaneously make their selections. Another essential assumption in these scenarios is that all outcomes of the game are known to every player (Mas-Colell, 2016). An outcome $\hat{a} = (\hat{a}^1, \hat{a}^2, \dots, \hat{a}^N)$,(where for each $\mathrm{i} = 1,2,\dots,\mathrm{N}$, $\hat{a}^i \epsilon A^i$) is referred to as a Nash Equilibrium (NE) if no player has an incentive to deviate from this outcome, assuming that the other players do not deviate from their strategies in the Nash outcome. In other words, for each player i ($\mathrm{i} = 1,2,\dots,\mathrm{N}$), and for all strategies $a^i \epsilon A^i$, the following condition holds:

$\pi^i(\hat{a}^i, \hat{a}^{-i}) \geq \pi^i(a^i, \hat{a}^{-i})$. If...

$$\begin{cases} \pi^i(\hat{a}^i, \hat{a}^{-i}) > \pi^i(a^i, \hat{a}^{-i}) & \text{for some } a^i \epsilon A^i \\ \pi^i(\hat{a}^i, \hat{a}^{-i}) = \pi^i(a^i, \hat{a}^{-i}) & \text{for some } a^i \epsilon A^i \end{cases}$$

(Shy, 1996).

Ultimately, it should be stated that if game theory seeks to provide a unique solution for a game, that solution must be a Nash equilibrium. A Nash equilibrium occurs when, firstly, each player chooses a strategy that maximizes their payoff based on their beliefs about the opponent's choice, and secondly, the player’s belief holds true, meaning that the opponent actually selects the strategy that the player believes they will. The strategies chosen by players in this manner constitute their Nash equilibrium strategies (Mas-Colell, 2016).

4. Game modeling

Assume that taxpayers are uniformly distributed over the interval [0, 1]. Following Salimian et al. (2023), a taxpayer located at point l on the mentioned interval after paying his diagnostic tax, has a surplus equal to:

$$u(l, i) = R - \frac{1}{1+r\theta}(l - T_i)^2 + q_i - E_i \qquad (1)$$

In this equation, R represents the reservation value of the goods or services, assumed to be sufficiently high so that all taxpayers opt to comply. The term $\frac{1}{1+r\theta}$ denotes the probability of non-detection of tax evasion. This suggests that the taxpayer may withhold some realities related to their payments. Here, r means the investigation by the tax investigation groups, that higher the r, lower the probability of not discovering violation ($r \geq 0$). Similarly, variable θ captures the likelihood of dishonesty from the taxpayer; an increase in variable θ leads to a lower probability of detection of evasion ($\theta \geq 0$). The variable l represents the taxpayer's position, while T_i denotes the tax assessment for taxpayer i. The term q_i reflects the quality of the investigating group for taxpayer i, with the understanding that differences in the quality of investigation groups are unknown to the taxpayers. E_i signifies the level of disclosure by taxpayer i. To examine the impact of uncertainty regarding the quality of investigation groups on taxpayers' disclosures, it is postulated that q_i is a random variable, which is unknown to the taxpayers at the time of disclosure.

Following the work of Salimian & Sobhanian (2024), the utility function (income) for the tax administration is defined as follows:

$$\begin{cases} te & \text{In case of non-processing} \\ t\big(e + q(T - e)\big) + f - a & \text{In case of processing} \end{cases} \quad (2)$$

In the aforementioned relationship, let variable t represent the tax rate ($0 \leq t \leq 1$), variable e denote the declared taxes by taxpayers, variable q indicate the quality of investigating groups ($0 \leq q \leq 1$), variable T symbolize the assessed taxes (post-investigation by the investigating group), variable f reflect the fines imposed on taxpayers (in cases of discrepancies between assessed and declared taxes, as well as delays in payment, etc.), and finally, variable a be the cost incurred for the investigating process by the tax authority. For simplicity, assume that the first investigating group is one that does not conduct thorough investigations but merely reviews and finalizes taxpayer cases based on their declarations. Conversely, the second investigating group represents those that conduct more rigorous reviews and assessments of taxpayer cases; thus, the term assessed taxes T in the second relationship pertains specifically to this group.

This function indicates that if the tax authority chooses not to investigation tax returns, it will derive a utility (income) equivalent to te, where t represents the tax rate and e denotes the declared taxable income of the taxpayer.

The second segment illustrates that if the tax authority aims to conduct investigations on tax files, it will derive a utility function represented as $t\big(e + q(T - e)\big) + f - a$. In this equation, the initial term captures the product of the tax rate and reported income, in addition to the differential between assessed tax and reported tax, which is multiplied by the value of q. Given that the detection of taxpayer misconduct (the discrepancy between assessed and reported tax) is contingent upon the efficacy of the investigating team, this variance is factored into the calculation. If the investigation team's capability in identifying violations is high, the multiplier approaches 1; conversely, if the team's effectiveness is subpar, this multiplier trends towards 0. Additionally, if q = 0, the utility function for the tax authority reverts to its original condition, incorporating solely the fines (added) and the expenses incurred during the investigation process (deducted).

The tax authority aims to maximize its expected revenue, while each taxpayer seeks to maximize their net surplus after tax payment. This concept indicates that a change in strategy for the tax authority, once the quality of the investigating groups for taxpayers is determined, can be highly costly (Salimian et al., 2023).

The expected revenue (receipts) of the tax authority, denoted as EI(t), is illustrated as being stochastically indifferent based on the positioning of the taxpayer within the probability landscape. Depending on whether the tax authority opts to investigation the taxpayer's financial records, and the specific location of the indifferent taxpayer in relation to the probability density function, the expected revenue will be segmented into two distinct areas. Here, $D(T)$ represents the cumulative distribution function associated with parameter t.

The questions that arise are: What is the optimal tax rate on income? What variables determine the optimal tax rate on income? What factors influence both declared tax and assessed tax by taxpayers, and is the relationship between these factors and the declared and assessed taxes direct or inverse? Additionally, what factors contribute to the revenue function of the tax authority, and how do they interrelate? These questions will be explored further below.

4.1. Equilibrium Declaration

Suppose variable z represent the number of taxpayers being investigationed by investigating group 1, while $1 - z$ denotes the number of taxpayers investigationed by group 2 ($z \in (0,1)$). According to the model proposed by Selimiān et al. (2023), the distribution of investigationed taxpayers between the two groups is determined by the indifferent taxpayer's position relative to both investigating groups 1 and 2. Thus, considering the relationship established in equation 1 and the fact that the indifferent taxpayer is positioned at z, we have:

$$E_1 + \frac{1}{1+r\theta}(z - T_1)^2 = -q + E_2 + \frac{1}{1+r\theta}(z - T_2)^2 \qquad (3)$$

in which $q = q_2 - q_1$. It is evident that the value of q can be positive, negative, or zero, depending on the quality of the investigating groups. The indifferent position of the taxpayer is contingent upon the tools, diagnostic measures, and the quality of the available investigating groups. Therefore, we express

$z = z(E_1, E_2, T_1, T_2, q)$. By solving above equation for $z \in (0,1)$, we obtain:

$$Z^* = \frac{T_1 + T_2}{2} + \frac{(r\theta+1)(E_2 - q - E_1)}{2(T_2 - T_1)} \quad (4)$$

Therefore, the revenue function for the stakeholder groups can be expressed as follows:

$$I_1 = E_1 z \quad , \quad I_2 = E_2(1 - z) \quad (5)$$

While:

$$z = \frac{T_1 + T_2}{2} + \frac{(r\theta+1)(E_2 - q - E_1)}{2(T_2 - T_1)} \text{ if } \frac{T_1 + T_2}{2} + \frac{E_2 - q - E_1}{2(T_2 - T_1)} \in (0,1) \quad (6)$$

(Salimian & et al, 2023). These relationships are derived from a uniform distribution of consumers. Now, we will extract the equilibrium equations. It is sufficient to merge the two equations above.

$$E_1^* = \frac{-q(r\theta + 1) - T_1^2 - 2T_1 + T_2^2 + 2T_2}{3(r\theta + 1)} \quad (7)$$

$$E_2^* = \frac{q(r\theta + 1) + T_1^2 - 4T_1 - T_2^2 + 4T_2}{3(r\theta + 1)} \quad (8)$$

The results demonstrate that the disclosure behavior of the second group of taxpayers (the investigationed cohort) is positively correlated with the quality of the investigating team. Specifically, as the quality of the investigating team enhances, the disclosure from the taxpayers increases correspondingly, and conversely.

In the subsequent section, the revenue function of the Tax Authority will be formulated, followed by an analysis and interpretation of the results.

4.2. Revenue Stream of the Tax Authority

In this section, we aim to find the maximum revenue potential for the tax administration using equilibrium tools. Conversely, taxpayers seek to maximize their expected utility. The expected revenue for the tax administration, denoted as EI, will be derived from whether the cases of taxpayers undergo examination or not. As previously mentioned, under these conditions, the expected revenue function for the tax administration can be expressed as follows:

$$\begin{cases} te & \text{In case of non-processing} \\ t\big(e + q(T - e)\big) + f - a & \text{In case of processing} \end{cases}$$

The Tax Administration's revenue is derived from two main avenues: non-examination (I_1) and examination (I_2) of tax filings by taxpayers:

$$= \begin{cases} I_1 = -\dfrac{l^2(T_1^{2} + 2T_1 - T_2^{2} - 2T_2 + q(r\theta + 1))}{6(r\theta + 1)} & \text{In case of non} - \text{processing} \\ I_2 = \dfrac{(l-1)(6a(r\theta+1) - 6f(r\theta+1) + (l+1)\left(T_1^2(q-1) + 4T_1(1-q) + T_2^2(1-q) - T_2(q(3r\theta-1)+4) + q(q-1)(r\theta+1)\right))}{6(r\theta+1)} & \text{In case of processing} \end{cases} \tag{9}$$

This is derived by integrating the probability density function of a uniform distribution over the interval [0, 1], where $D(T) = t$ represents the cumulative distribution function parameterized by t. Assume that α percent of taxpayer cases undergo further examination, thus leaving $1 - \alpha$ percent of tax cases unexamined (potentially settled through various means, such as Clause 100, etc.). Consequently, by aggregating these two relationships, we obtain:

$$I = \frac{6\alpha(l-1)(r\theta+1) + 6f(1-l)(r\theta+1)}{6(r\theta+1)} \tag{10}$$

$$+ \frac{l^2\left(T_1^2(q-2) + 2T_1(1-2q) + T_2^2(2-q) - T_2(q(3r\theta-1)+2) + q(q-2)(r\theta+1)\right)}{6(r\theta+1)}$$

$$+ \frac{T_1^2(1-q) + 4T_1(q-1) + T_2^2(q-1) + T_2(q(3r\theta-1)+4) - q(q-1)(r\theta+1)}{6(r\theta+1)}$$

As previously noted, it is posited that the revenue accrued by the tax authority originates from two segments: investigationed and non-investigationed cases. Consequently, the value of q can be assigned as zero (when no investigation occurs) or one (when an investigation is conducted, under the assumption that all taxpayer data is meticulously examined and tax liabilities are accurately determined). Therefore, considering these two values, we have:

$$I = \begin{cases} I_1 = -\dfrac{l^2(T_1^{2} + 2T_1 - T_2^{2} - 2T_2)}{6(r\theta + 1)} & \text{In case of non} - \text{processing} \\ I_2 = \dfrac{(l-1)(2a - 2f - T_2(l+1))}{2} & \text{In case of processing} \end{cases}$$

$$\Rightarrow I = \frac{6a(l-1)(r\theta+1) + 6f(1-l)(r\theta+1) - l^2(T_1^2 + 2T_1 - T_2(T_2 - 3r\theta - 1)) + 3T_2(r\theta+1)}{6(r\theta+1)} \tag{11}$$

Now, by taking the derivative of this function with respect to l, we arrive at:

$$\frac{\partial I}{\partial l} = \frac{3a(r\theta+1) - 3f(r\theta+1) - l(T_1^2 + 2T_1 - T_2(T_2 - 3r\theta - 1))}{3(r\theta+1)} \tag{12}$$

By resolving this equation, the resultant function is derived as follows:

$$l = \frac{3(a-f)(r\theta+1)}{{T_1}^2+2T_1-T_2(T_2-3r\theta-1)} \tag{13}$$

The tax rate on performance is a function of several variables: investigation costs, fine for non-compliance, diagnostic taxes imposed by investigating entities, the frequency of investigations, and the parameter representing taxpayer dishonesty. Considering that the diagnostic taxes levied by non-investigating groups - via mechanisms like Article 100 - are fixed, we can conceptualize the aggregate of these taxes as a constant, denoted as S. Therefore, we can express:

$$l = \frac{3(a-f)(r\theta+1)}{S-T_2(T_2-3r\theta-1)}$$

Also:

$$\frac{\partial l}{\partial T_1} = \frac{3(f-a)(r\theta+1)}{({T_2}^2-T_2(3r\theta+1)-S)^2}$$

$$\frac{\partial l}{\partial T_2} = \frac{3(a-f)(2T_2-3r\theta-1)(r\theta+1)}{({T_2}^2-T_2(3r\theta+1)-S)^2}$$

Assuming that the fines levied on taxpayers exceed the costs associated with enforcement, the findings suggest that the optimal tax rate exhibits an inverse relationship with the assessed tax liabilities of investigating entities. This conclusion is logically sound: an increase in assessed tax liabilities allows for a reduction in the tax rate. In other words, a lower tax rate can potentially generate greater tax revenue. These results are completely consistent with the results of Salimian & Sobhanian (2024).

We will now analyze the optimal tax rate on performance. This analysis will be based on the following relationship:

$$l = \frac{3(a-f)(r\theta+1)}{{T_1}^2+2T_1-T_2(T_2-3r\theta-1)}$$

Given that the Tax Administration determines its total tax revenue from both investigationed and non-investigationed cases, and that it possesses estimates for tax violations and investigating costs, it is straightforward to establish the tax rate for any given period. For instance, in the fiscal year 1402, the total tax revenue based on performance was estimated at approximately 400 trillion tomans (the sum of T_1 and T_2). Assuming that the total revenue from non-investigations is 50 trillion tomans and the revenue from investigations is 350 trillion tomans, and further assuming that a is one-fifth of variable f with the number of investigations being 1 and all taxpayers being truthful (indicating a noncompliance parameter of one), we can analyze the impact on the overall tax rate as follows:

$$\%18 = \frac{3(4)(1+1)}{(6.14)^2+2(6.14)-(35.8)((35.8)-3-1)}$$

The tax rate will accordingly be established at 18%. However, it is essential to highlight that this methodology is inherently dynamic. Annually, the optimal tax rate can be recalibrated in response to projected revenues from diverse sources, the strategic goals of the tax authorities concerning investigation frequency, the expenditure incurred in conducting these investigations, and the imposition of fiscal fines.

5. Conclusion and Recommendations

The determination of the optimal tax rate is among the most significant parameters influencing economic efficiency, production, income distribution, and the control of economic instability. This paper models the interaction between taxpayers and tax authorities in order to achieve an optimal performance tax rate and identify the factors that influence this rate.

To this end, we assume that taxpayers are uniformly distributed within the interval [0, 1]. Subsequently, we solve the proposed game and analyze the results. The findings indicate that the performance tax rate is a function of investigation costs, fines levied on taxpayers, the diagnostic tax rates of investigation groups, the frequency of investigations, and the parameter of taxpayer honesty. Furthermore, the reporting quality of investigationed taxpayers is positively correlated with the quality of the investigation team; that is, as the quality of the investigation team improves, taxpayers are more likely to disclose their true income, and vice versa.

Additionally, there is an inverse relationship between the optimal tax rate and the diagnostic tax rates of the investigation groups. Finally, based on approximate data and the presence of honest taxpayers who report their income accurately, the optimal performance tax rate was determined to be approximately 18 percent for the year 1402 (2023-2024). Therefore, considering that the Laffer curve indicates that increasing tax rates does not always lead to higher government revenue, it is recommended that the government selects an optimal performance tax rate that maximizes revenue while ensuring that taxpayers do not evade their tax responsibilities, thereby maximizing social welfare.

References

1. Abdoli, G. (2013). Game theory and its applications: Static and dynamic games with complete information (4th ed.). *Tehran: Jihad University Press*.
2. Abolade, F. (2018). Optimal Taxation In Africa: How Relevant Is Warren Buffet Model?.
3. Aghaei, A. M., & Maddah, M. (2020). Determining the optimal tax rate in Iran with an emphasis on value-added tax. *Tax Research Journal*, 28(47), 75-108.
4. Altunoz, U. (2017). The Application of the Laffer Curve in the Economy of Turkey. *Journal of International Social Research*, *10*(50).
5. Auerbach, A. J. (2005). Dynamic scoring: An introduction to the issues. *American Economic Review*, *95*(2), 421-425.
6. Collier, P., & Hoeffler, A. (2005). Resource rents, governance, and conflict. *Journal of conflict resolution*, *49*(4), 625-633.
7. Collier, P. (2006). Is aid oil? An analysis of whether Africa can absorb more aid. *World development*, *34*(9), 1482-1497.
8. Dadgar, Y., Nazari, R., & Siyami Iraqi, E. (2013). Government and optimal taxation in the public sector economy and the role of government and taxation in Iran. *Journal of Applied Economic Studies in Iran*, 5(2), 1-27.
9. Diamond, P. A., & Mirrlees, J. A. (1971). Optimal taxation and public production I: Production efficiency. *The American economic review*, *61*(1), 8-27.
10. Eklund, C. J., & Malmsten, M. (2019). On optimal tax rates and shifts in the peak of the Laffer curve-An empirical study of Swedish municipalities during the years from 2000 to 2017.
11. Farahnak, F., Maddah, M., & Shakeri, A. (2018). The effects of changes in the effective tax rate on public budget, national production, and welfare using a general equilibrium model. *Quarterly Journal of Applied Theories of Economics*, 5(2), 81-102.
12. Ghafari, H., Pourkazemi, M. H., Khodadadi Kashi, F., & Younesi, A. (2016). The optimal tax rate as a fiscal policy tool: A dynamic optimal control theory approach. *Tax Research Journal*, (29), 77.
13. Gupta, S., & Newberry, K. (1997). Determinants of the variability in corporate effective tax rates: Evidence from longitudinal data. *Journal of accounting and public policy*, *16*(1), 1-34.
14. Kudrna, G., Piggott, J., & Poonpolkul, P. (2022). *Extending pension policy in emerging asia: An overlapping-generations model analysis for indonesia*. Australian National University, Crawford School of Public Policy, Centre for Applied Macroeconomic Analysis.
15. Houndonougbo, A. N., & Mohsin, M. (2016). Macroeconomic effects of cost equivalent business fiscal incentives. *Economic Modelling*, *56*, 59-65.
16. Leeper, E. M., & Yang, S. C. S. (2008). Dynamic scoring: Alternative financing schemes. *Journal of Public Economics*, *92*(1-2), 159-182.
17. Maddah, M., Shafiei Nik Abadi, M., & Sami'i, N. (2016). Examining and determining optimal tax rates in line with the optimal demand level for public goods. *Tax Research Journal*, 30, 65-108.

18. Mankiw, N. G. (2013). Macroeconomics fifth edition.
19. Mankiw, N. G., & Weinzierl, M. (2006). Dynamic scoring: A back-of-the-envelope guide. *Journal of Public Economics*, *90*(8-9), 1415-1433.
20. Mas-Colell, A. (2016). *General equilibrium and game theory: ten papers*. Harvard University Press.
21. Obara, T. (2018). Essays on Optimal Taxation and Policy Implication. *(No Title)*.
22. Pourmoghimi, S. J., Nemat Pour, M., & Mousavi, M. H. (2005). Factors affecting tax revenue collection. *Economic Research Journal*, 17, 188.
23. Ramsy, F. P. (1927). A Contribution to the Theory of Optimal Taxation. *Economic Journal*, *37*, 47-61.
24. Saez, E., & Stantcheva, S. (2016). Generalized social marginal welfare weights for optimal tax theory. *American Economic Review*, *106*(01), 24-45.
25. Salimian, S., Beknazar, M. M., & Salimian, S. (2023). Modeling tax declaration behavior and quality of tax processing (game theory approach). *Journal of Tax reform*, *9*(1), 64-75.
26. Salimian, S., & Sobhanian, M. H. (2024). Game modeling of collusion and influential factors under uniform taxpayer distribution. *Journal of Dynamics and Games*, *11*(1), 63-75.
27. Salimian, S., & Sobhanian, M. H. (2024). Salah-Mohammad triangular distribution Curve is an Alternative to the Laffer curve. *Journal of Dynamics and Games*, *11*(3), 218-231.
28. Sameti, M., Amiri, H., & Izadi, S. (2015). The impact of optimal indirect tax rates on social welfare in Iran. *Quarterly Journal of Economic Research (Growth and Sustainable Development)*, 15(4), 51–73.
29. Sanz-Sanz, J. F. (2022). A full-fledged analytical model for the Laffer curve in personal income taxation. *Economic Analysis and Policy*, *73*, 795-811. Shy, O. (1996). *Industrial organization: theory and applications*. MIT press.
30. Seadat Meher, M. (2022). Estimating the optimal rate of value-added tax using the Laffer curve approach. *Quarterly Journal of Quantitative Economics*, 19(2), 93-110.
31. Shy, O. (1996). *Industrial organization: theory and applications*. MIT press.
32. Strulik, H., & Trimborn, T. (2012). Laffer strikes again: Dynamic scoring of capital taxes. *European Economic Review*, *56*(6), 1180-1199.
33. Tamizi, A. (2018). Examining the determinants of tax revenues in Iran: A Bayesian econometric approach. *Quantitative Economics Quarterly (Previous Economic Reviews)*, 15(1), 225-244.
34. Von Neumann, J., & Morgenstern, O. (2007). Theory of games and economic behavior: 60th anniversary commemorative edition. In *Theory of games and economic behavior*. Princeton university press.
35. Wawire, N. (2017). Determinants of value added tax revenue in Kenya. *Journal of Economics Library*, *4*(3), 322-344.

کاربرد نظریه بازی‌ها در تعیین نرخ مالیات بر عملکرد و عوامل مؤثر بر آن

چکیده:

به کارگیری یک سیستم مالیاتی مناسب دارای شرایطی است که از مهمترین آنها عدالت و کارایی است که بر اساس آن مالیات بر درآمد با اصل توانایی پرداخت تطبیق خواهد داشت، لذا سعی دولت ها بر این بوده که این نرخ ها را به طور مناسب و اثرگذار وضع کنند. چرا که افزایش نامتناسب نرخ های مالیات بر درآمد، اثرات اجتماعی زیادی را بر توزیع درآمد و رفاه عمومی در جامعه بر جای خواهد گذاشت؛ بنابراین محاسبه نرخ بهینه مالیات به صورتی که رفاه اجتماعی حداکثر شود امری ضروری به نظر می رسد. این مقاله به مدلسازی بازی بین مؤدیان و سازمان امور مالیاتی جهت دست‌یابی به نرخ بهینه مالیات و نیز عوامل مؤثر بر این نرخ بهینه است. در این مقاله با طراحی بازی بین مؤدیان مالیاتی که فرض شده است در بازه [0.1] به طور یکنواخت پخش شده و تعیین درآمدهای سازمان امور مالیاتی به طراحی بازی و تحلیل نتایج پرداخته شده است. نتایج نشان داد که نرخ بهینه مالیات رابطه معکوس با میزان مالیات تشخیصی گروه‌های رسیدگی کننده دارد. همچنین تحت شرایط خاصی، نرخ بهینه مالیات بر عملکرد برابر با 18٪ تعیین شده است.

طبقه‌بندی JEL: C70، O11، H21.

کلیدواژه‌ها: نظریه بازی‌ها، مؤدیان مالیاتی، نرخ بهینه مالیات بر عملکرد.