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Cryptocurrencies and Risk-based Strategies Portfolio Diversification

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ABSTRACT

Recently, many investors have become interested in investing in cryptocurrency market. Investing in an asset carries a lot of risk and may bankrupt the investor. The main way to control this risk is portfolio diversification. In this paper, we will investigate the effect of portfolio diversification by adding cryptocurrencies to the portfolio. We evaluate the performance of seven risk-based portfolio optimization strategies. these strategies are the minimum variance, inverse volatility, L2-norm constrained minimum variance, L2-norm constrained maximum decorrelation, risk parity portfolio and maximum diversification. Our portfolios consist of three markets stocks including, Tehran Stock Exchange, Commodities and Cryptocurrencies. Also, due to the fact that the cryptocurrency market has gained a significant attraction among investors, we will examine the positive and negative effects of adding the five selected currencies, simultaneously and separately to the base portfolio, which is Tehran Stock Exchange-Commodities portfolio. We investigate that whether adding cryptocurrencies to a stock portfolio can be considered as a tool to improve a risk-based portfolio. After analyzing portfolios, the best portfolio in each strategy and the best strategy in each portfolio are introduced from the aspects of risk, return and Sharpe ratio, and finally we have concluded that entering the cryptocurrency market in most of the strategies lead to an overall increase in the return, while the approach is to minimize the risk of the portfolio. So, it can be concluded that if the main goal is to build a more diversified portfolio, better outcome can be obtained for the investor considering the return gained.

Keywords— **cryptocurrencies, diversification, portfolio optimization, risk-based portfolio**

1. Introduction

The stock market, as one of the most significant areas of financial investment, has always attracted the attention of numerous investors. Portfolio selection is a key issue in investment. It deals with the allocation of limited capital to a number of potential assets in order to achieve profitable investment solutions [1]. The overwhelming majority of investments in Bitcoin compared to other cryptocurrencies in the market demonstrates that the investment portfolios of many investors lack diversification. It is crucial to acknowledge that by employing risk management strategies and diversifying investment portfolios through existing optimization solutions, severe financial losses can be prevented in all time periods.

The rapid proliferation of innovative activities, their alignment with the restructuring of the traditional global economic landscape, and the resulting transformations in all aspects of life, work, transportation, and even cognition and information processing, herald the advent of a new era for humanity.

Schwab terms this era the Fourth Industrial Revolution [3]. Schwab characterizes this revolution as a socio-technological process that pervades the digital, physical, and biological spheres.

This sweeping transformation hinges on the effective and innovative utilization of emerging digital technologies, facilitated by their efficient integration and interaction. The profound impact of this digital metamorphosis on all industries and economic sectors, manifesting in novel models and initiatives, is irrefutable [4].

Financial systems have already experienced the effects of these advancements in the form of cryptocurrencies and blockchain technologies [5].

Schwab believes that the concept of blockchain lies at the heart of the Fourth Industrial Revolution, and that Bitcoin, as the most popular and well-known

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cryptocurrency, should be acknowledged for its unique potential in providing individuals with access to a borderless payment protocol on a global scale.

The invention of cryptocurrencies in 2008 and the dramatic rise in the price of Bitcoin in 2017 sparked significant debate and controversy within global economic and financial circles. Individuals who invested in cryptocurrencies or acquired digital coins during this period experienced substantial gains. This phenomenon can be attributed to an inherent human behavioral trait: the instinct to invest for the future and allocate resources for potential exploitation and profit. This behavior, which can be seen as a form of self-preservation, stems from the individual's perceived sense of insecurity regarding potential future challenges.

The structure of this scientific research paper is as follows:

Section 2 provides a concise review of the scientific literature on risk-based asset allocation approaches and cryptocurrency portfolio optimization strategies. It then delves into the cryptocurrency market and its applications in the modern world and financial context. Section 3 precisely introduces and formulates the research problem. Section 4 presents the results and findings of the study. Section 5 discusses the obtained results in detail. Section 6 concludes the paper by offering intriguing directions for future research.

2. Related Work

In this section, we will briefly review the scientific research on risk-driven asset allocation approaches and cryptocurrency portfolio optimization strategies. We will then discuss the cryptocurrency market and its applications in the modern world and in the field of finance.

2.1. Risk-Driven Optimization

The pioneering study on portfolio selection employed the concept of the efficient set, introduced by Schwab [2]. In his study, which forms the basis of modern portfolio theory, Markowitz formulated portfolio selection as a mean-variance optimization problem with two fundamental objectives: maximizing return (expected return mean) and minimizing risk (variance of expected return) of the portfolio.

Notable research on portfolio optimization using the mean-variance model includes the works of [7-10].

Markowitz's mean-variance optimization approach has become known as the "error maximization" procedure [11]. However, due to the impracticality of this approach, the modern portfolio literature has proposed replacing the traditional Markowitz mean-variance optimization approach with modern portfolio theory through alternative capital allocation solutions.

Various portfolio construction schemes in the research literature are known as "risk-based asset allocation." One common feature of these "new paradigm" portfolio construction solutions is their focus on reducing risk while simultaneously increasing portfolio diversification. Instead of relying solely on expected return estimates, risk-based approaches are based on covariance matrices and the estimates derived from them. This approach generally neutralizes portfolio risk against possible errors in return estimates. Several previous studies have shown that these portfolios outperform and use the Markowitz optimization algorithm [12-15].

One popular approach for constructing mean-variance efficient portfolios is the equal-weighting strategy. In this strategy, all assets in a portfolio are assigned an equal weight. These portfolios have been widely used in practice [16-17].

According to the studies of DeMiguel et. al. and Plyakha et. al., the performance of the equally-weighted portfolio was significantly better than the value-weighted portfolio. However, the performance of the equally-weighted portfolio was worse than some portfolios that are not optimized based on selection models [15,18].

Another popular approach is the minimum-variance portfolio [6,13]. Behr et. al. and Clarke et. al. found that the minimum-variance portfolio outperforms the market-capitalization-weighted index, exhibiting higher returns, lower volatility, and hence better risk-adjusted performance [12-13].

The L2 maximum decorrelation strategy with soft constraints is based on the minimum-variance strategy [19]. However, instead of minimizing variance, its primary objective is to minimize the correlation between assets in the portfolio. This minimizes the number of input parameters required for optimization and thus the problem of return estimation errors.

Another approach is Inverse volatility. Although this approach is widely used in practice, it has received relatively little research attention, with the study by Maillard et. al., being the primary reference in this area [20].

Another, more recent approach is the Maximum Diversification (MD) portfolio, introduced by Choueifaty et. al. [21]. These researchers define diversification as "the ratio of the weighted average asset volatilities to the volatility of the overall portfolio of those same assets." The MD portfolio maximizes this diversification ratio. They found that the MD portfolio significantly outperforms the market-capitalization-weighted portfolio, as well as the minimum-variance and equally-weighted portfolios, exhibiting higher returns and lower volatility.

Navya et. al. empirically investigate the DR strategy in their paper using over 350 S&P 500 index stocks. In this analysis, it is assumed that stock losses are modeled using a flexible multivariate heavy-tailed model. This hypothesis is supported by empirical evidence [22].

The performance of the DR strategy is compared to four benchmark strategies: the equally weighted portfolio, the minimum variance portfolio, the extreme risk index portfolio, and the maximum diversification portfolio. The comparison metrics include annual portfolio return, modified Sharpe ratio, maximum drawdown, portfolio concentration, portfolio turnover, and degree of diversification. The results show that DR outperforms the other strategies. Specifically, DR delivers the highest return during the 2007-2009 global financial crisis while maintaining the highest level of diversification.

2.2. Cryptocurrency Portfolio Optimization

Cryptocurrency markets, as a new asset class and alternative to traditional investments [23-24], have attracted considerable interest from investors seeking alternative sources of investment returns. For example, Brière et. al. demonstrate that including Bitcoin in optimized portfolios improves portfolio diversification and risk-adjusted returns [25]. Eisl et. al. show that adding Bitcoin to diversified portfolios enhances diversification and risk-adjusted performance. More recent research has focused on the implications of adding cryptocurrencies to traditional asset portfolios [26]. Chuen et. al. are among the first to examine the inclusion of the cryptocurrency index CRIX-based cryptocurrencies in a portfolio of traditional assets [27]. Petukhina et. al. investigate the addition of cryptocurrencies to five traditional asset classes (equities, fixed income, fiat currencies, commodities, and real estate) and test whether optimizing such a portfolio improves real-world performance, concluding that cryptocurrencies are a valuable asset [28].

2.3. Integrating Cryptocurrencies into Investment Portfolios

Cryptocurrencies are units of complex computational bits or (depending on the type) blockchains designed to function as a means of payment or exchange of value. These blockchains, first introduced by Nakamoto in 2008, serve as the building blocks of a digital monetary system that typically lacks any central authority or oversight, where no single entity controls the wealth creation process, security mechanisms are unique, and the risk of even minimal competitor negligence is negligible. Cryptocurrencies have captured the attention of powerful members of large industries, individuals, and expert [29].

2.4. Perspectives on Cryptocurrency and Blockchain as a Financial Innovation

The financial landscape has undergone a remarkable transformation in recent years, transitioning from traditional payment models to the advent of blockchain technology and cryptocurrencies. While research prior to 2013 primarily focused on fintech models and mobile payments, a surge of investigations emerged post-2013, delving into the intricacies of blockchain and Bitcoin, Despite the skepticism surrounding blockchain, cryptocurrencies, and digital technologies, researchers have identified compelling investment opportunities within this domain [30-31]. Some scholars advocate for a more balanced perspective, emphasizing the societal benefits of blockchain and cryptocurrencies while acknowledging their potential for misuse [32] and Sidharth and his colleague also systematically examine the state-of-the-art associated with security concerns in cryptocurrencies from various perspectives in their paper This skepticism extends beyond the underlying technology to encompass prominent [33]. cryptocurrencies like Bitcoin and its associated blockchain technology, which fueled the extraordinary 30% returns experienced by firms incorporating these influential terms into their names [34].

3. Methodology

In this section, we will first elaborate on the relationship between risk and return. After becoming familiar with risk-based optimization models, we will introduce risk measures, variance, VAR, and CVAR.

3.1. Risk-Return Tradeoff

The risk-return tradeoff principle asserts that as expected returns increase, so does the associated risk. This principle implies that lower levels of uncertainty are accompanied by lower potential returns, while higher levels of uncertainty are associated with higher potential returns. Consequently, traders can potentially enhance their chances of achieving higher returns by investing in assets with a greater likelihood of loss [35].

3.2. Risk-Based Optimization Model

The optimization model in this study utilizes Equation (1) as the return equation, introduced by Markowitz [6]. The covariance and CVaR equations are used for the risk equation, and the constraints specified in this study which is Equations (1) is adopted from the study by Deb et. al. [36].

$$\begin{cases} \max\limits_{w_1,\dots,w_N} \sum_{i=1}^{N} w_i \mu_i \\ \min\limits_{w_1,\dots,w_N} \; risk\ measure\ (Covariance\ or\ CVaR) \end{cases} \quad (1)$$

$s.t.$

$\sum_{i=1}^{n} w_i = 1$

$$w_i = 0,\ \ or\ W_{min} \leq w_i \leq W_{max}$$

$d^{min} \leq d(x) \leq d^{max}$

In this equation, w_i represents the weight of asset i in the portfolio, μ_i represents the average return of asset i, $[W_{min}, W_{max}]$ represent the allowable range of asset weights, $[d^{min}, d^{max}]$ represent the minimum and maximum number of assets in the portfolio, $d(x)$ represents the number of stocks in the portfolio, and is obtained from this equation $\sum_{i=1}^{N} d_i$ where d_i represents the number of asset i. This study investigates variance as a measure of dispersion risk and VaR and CVaR as multi-quantile risk measures.

3.3. Risk Measures

Roman and Mittler in the year 2009 define a risk measure as an equation that assigns a numerical value as a measure of "riskiness" to any return function. According to them, risk models can be classified into two categories: dispersion measures and quantile measures [37].

Dispersion measures are based on a single return value and are further divided into two categories: symmetric and asymmetric measures. Symmetric measures consider the deviation of the return value from a predetermined target value (usually the expected value) as the measure of risk and account for both negative and positive deviations as risk. In contrast, asymmetric measures consider return values that are lower than the expected value as risk.

Quantile measures, also known as tail risk measures, delve into the extreme tails of the return distribution, capturing the probability and magnitude of potential losses or extreme events. Unlike dispersion measures that focus on the overall deviation of returns from the expected value, quantile measures concentrate on the worst-case scenarios, providing a more granular understanding of potential downside risks.

Variance

Variance as a Popular Measure in the Markowitz Model (1952) and Its Symmetry [6].

Variance has been widely criticized as a measure of risk due to its shortcomings in capturing investor behavior and its statistical limitations. One primary issue is that it treats both above-average and below-average returns as equally risky. Statistically, nce is limited in that it is only meaningful when ıs are symmetrically distributed, a condition often not met in empirical data [38].

The development and application of the Markowitz model, which utilizes variance as its risk measure, has fueled further criticisms and led to a plethora of alternative concepts, theories, and empirical studies on portfolio optimization and risk measurement.

Value at Risk(VaR)

Value at Risk (VaR) is a widely used statistical measure that quantifies the potential financial losses within a business unit, investment portfolio, or trading position over a specified time horizon. This metric is commonly employed by investors and commercial banks to assess the extent of possible financial losses in their institutional portfolios. It serves as a tool for measuring and controlling their overall risk exposure, both for individual positions and for entire portfolios subject to significant risk.

Conditional Value at Risk (CVaR)

This financial metric is a risk assessment statistic that quantifies the level of "tail risk" in an investment portfolio. It is calculated by taking the weighted average of the maximum losses in the tail of the distribution of possible returns. This metric is used to optimize portfolios based on effective risk management.

Conditional Value at Risk (CVaR) is defined based on Value at Risk (VaR). According to Rockafellar (2000), CVaR (with a confidence level of β%) is defined as the expected conditional loss of the portfolio given that the losses are greater than or equal to the VaR measure. Therefore, CVaR refers to the expected amount of losses when they exceed VaR. Rockafellar (2000) argues that this definition of CVaR ensures that the VaR measure at the β% confidence level will never be greater than the CVaR measure at the β% confidence level. As a result, portfolios with lower CVaR must also have lower VaR. Equation 2 shows how CVaR is calculated [39]:

$$F_a(\omega,\zeta) = \zeta + (1-a)^{-1} \cdot \sum_{j=1}^{J} \pi_j \left[f\left(w, r_j\right) - \zeta \right]^{+} \quad (2)$$

where ζ represents (VaR), α represents the confidence level, w represents the weights of the assets in the portfolio, r is the vector of average returns of the assets in the portfolio, $f(w,r)$ is the expected loss of the portfolio, and π is the probability of the scenarios that may occur.

3.4. Optimizing Risk Mitigation Solutions

This study employs seven risk-based portfolio construction strategies, which are:

1. Inverse volatility

2. Minimum variance
3. L2 Minimum variance
4. L2 Maximum decorrelation
5. Maximum diversification
6. Risk parity
7. Equally weighted crypto.

The performance of all these portfolios (based on the aforementioned strategies) is tested against the performance of an equally-weighted cryptocurrency portfolio. In equally-weighted portfolios, all assets are assigned equal weights in the investment. This is because this method is directly implementable and is known as the $\frac{1}{N}$ method, where the weight of each asset is determined by Equation (3):

$$W_i = \frac{1}{N}, \forall_i = 1,2,3, \dots, N \tag{3}$$

In this formula, w_i represents the weight assigned to asset i, and N represents the number of assets in the portfolio. Since the EW (Equally-weighted) portfolio invests equal amounts in each of the N assets, this type of portfolio is considered the most decentralized in terms of asset weighting. However, according to [40], the assumption that using this simple $\frac{1}{N}$ method guarantees portfolio diversification can be misleading.

The inverse volatility portfolio assigns weights to the N assets based on their inverse volatility, and then normalizes these weights so that they sum to one. Therefore, the weight of each asset is calculated using Equation (4):

$$W_i = \frac{1/\sigma_i}{\sum_{i=1}^{N} 1/\sigma_i}, \forall i = 1,2,3, \dots, N \tag{4}$$

In this equation, w_i represents the weight assigned to asset i, and σ_i represents the volatility of asset i. One of the main advantages of the IV (inverse volatility) portfolio is its ease of calculation and practical appeal.

The minimum variance portfolio allocates the weights of the N assets in such a way that the variance of the portfolio, σ_p^2, is minimized. According to Markowitz's mean-variance framework, this variance reduction occurs while ignoring expected return forecasts. This point lies at the farthest left point on the efficient frontier and is the only portfolio on the efficient frontier that does not require return forecasts. It can be determined by optimization methods using only the covariance matrix forecast.

The minimum variance portfolio can be calculated by solving the following optimization problem, which includes Equation (5):

$$W_i = \arg min\, W' \sum W\, t$$

s.t.

$$W_i \geq 0$$

$$\sum_{i=1}^{N} W_i = 1 \tag{5}$$

W is an $N \times 1$ vector of asset weights and $\sum$ is $N \times N$ Variance-covariance matrix.

In the portfolio weight allocation problem, the portfolio weights are obtained by solving the minimization problem (6) subject to soft normal constraints.

$$W_i = \arg min\, W' \sum W\, t$$

s.t.:

$$W_i \geq 0$$

$$\sum_{i=1}^{N} W_i = 1$$

$$\|W\|_2^2 = \sum_{i=1}^{N} W_i \leq \frac{3}{N} \tag{6}$$

W is a $N \times 1$ vector of asset weights and $\sum$ is $N \times N$ Variance-covariance matrix and $\|.\|_2$ are flexible, focused constraints that are applied to 2-norm. The portfolio with the maximum uncorrelation coefficient is closely related to the minimum variance portfolio, but its aim is to reduce the number of input parameters. Therefore, the optimal portfolio weights are obtained through the optimization problem (7).

$$W_i = \arg min\, W' \sum W\, t$$

s.t:

$$W_i \geq 0$$

$$\sum_{i=1}^{N} W_i = 1$$

$$\|W\|_2^2 = \sum_{i=1}^{N} W_i^2 \leq \frac{3}{N} \tag{7}$$

W is a $N \times 1$ vector of asset weights and $\sum$ is $N \times N$ Variance-covariance matrix and $\|.\|_2$ are flexible, focused constraints that are applied to L2-norm. Therefore, unlike the MV and MVN portfolio approaches, which aim to reduce risk by concentrating on low-volatility components, the MDN portfolio seeks to exploit the risk reduction effects that arise from investing in assets with minimal correlation. While this approach prevents concentration on specific assets by ignoring differences in the volatilities of each weight, it can still lead to high asset concentration because it focuses on assets with lower correlations with other assets. Therefore, we again use L2-norm constraints in the optimization problem to improve the issue of high concentration on specific weights.

The goal of the maximum diversification portfolio is to construct a portfolio that is as diversified as possible. Mathematically, the maximum diversification portfolio is created by solving the optimization problem (8):

$$W_i = \arg max \frac{W'\sigma}{\sqrt{W'\Sigma W}} \quad (8)$$

s.t.:

$$W_i \geq 0$$

$$\sum_{i=1}^{N} W_i = 1$$

W is a $N \times 1$ vector of asset weights and $\sum$ is $N \times N$ Variance-covariance matrix and σ is an $N \times 1$ vector of Asset volatility.

The equally risky portfolio (ERP) or risk parity portfolio is a portfolio in which the risk contributions of all assets are equal. This portfolio mimics the diversification effect of the equally weighted (EW) portfolio while considering the individual and common risk contributions of all assets. In simpler terms, the goal of RP portfolios is to ensure that no asset has a greater impact on the overall portfolio risk compared to another asset (no asset has a greater impact on the overall risk than another). The theoretical foundations of the RP approach have been extensively studied in the portfolio literature [41-42].

We solve the optimization problem using sequential quadratic programming (9):

$$W^* = argmin\, f(x)$$

s.t.:

$$W_i \geq 0$$

$$\sum_{i=1}^{N} W_i = 1$$

$$f(x) = \sum_{i=1}^{N} \sum_{i=1}^{N} (w_i (\sum w)_i - w_j (\sum w)_j)^2 \quad (9)$$

From the above equation, it can be seen that the existence of an RP portfolio is guaranteed only if the condition $W_i(\sum W)_i = W_j(\sum W)_j$ holds for all assets in the portfolio. If all assets have equal volatility and if all pairwise correlation coefficients are equal, the risk parity portfolio becomes identical to the $\frac{1}{N}$ portfolio.

4. Results

Daily closing prices of 15 assets were considered, including 5 stocks from Tehran Stock Exchange (TSE), 5 commodities, and 5 cryptocurrencies. The stocks were selected based on the highest market capitalization, the commodities were selected based on the highest trading volume, and the cryptocurrencies were selected based on the highest market capitalization at the beginning of the study period.

In this study, five stocks from the Tehran Stock Exchange (TSE) with the highest market capitalization were selected, in the following order of ticker symbols: Fars, Hmkho, Marun, Khabar, and Folad. For the commodities market, the five commodities with the highest trading volume were selected, in the following order: coffee, gold, wheat, cotton, and corn. Finally, the five cryptocurrencies with the highest market capitalization were selected, in the following order: Bitcoin, Ethereum, Litecoin, Monero, and Ripple.

Data for the Tehran Stock Exchange was obtained from the Tehran Stock Exchange Technology Management Company website. Data for commodities was obtained from Yahoo Finance, and data for cryptocurrencies was obtained from CoinMarketCap. The time period considered for the data for all three groups was from January 1, 2017, to June 30, 2022 (a period of 5 years and 6 months).

Data for TSE stocks were obtained from the TSE website, data for commodities were obtained from Yahoo Finance, and data for cryptocurrencies were obtained from CoinMarketCap. Logarithmic returns were used in all portfolios.

Table 1 to 3 presents the descriptive statistics data .Descriptive statistics show that while all selected assets provide positive returns to their investors over the considered period, their average daily returns are highly heterogeneous, ranging from 3.2% for gold to 33% for Mobarakeh Steel Company. The TSE stock group had the highest average return of 24.7%.

The daily standard deviation ranges from 0.01 for gold to 3.89 for Bitcoin, with the cryptocurrency group having the highest standard deviation. The Sharpe ratio is highest for Mobarakeh Steel Company with a value of 2.1 and lowest for Bitcoin with a value of 0.11.

The skewness of the TSE stock group is not normal for the symbols Akhabr and Maroon, and is normal for the symbols Fars, Hamrah, and Foolad. In the case of Akhaber, the return on one day was 63.57% (which is a high return compared to the average return of this stock over the considered period), which caused a very high skewness in its distribution. If this return is removed, the skewness becomes 0.6 and falls within the normal distribution range.

The same is true for the skewness of the symbol Panbeh in the commodity group. With the removal of the return with a value of 23.88, the skewness of this

Table1. Descriptive Statistics

	Fars	Hamrah	Maroon	Akhaber	Fulad
Mean	29.0%	16.0%	27.0%	18.6%	**33.0%**
Annual mean	66%	37%	63%	40%	76%
Median	0.09%	-0.01%	0.16%	0.00%	0.15%
Standard Deviation	0.02	0.02	0.02	0.03	0.02
Volatility/SD	0.16	0.15	0.15	0.17	0.16
SR	1.87	1.09	1.79	1.07	**2.11**
Kurtosis	1.77	1.23	3.25	155.50	0.62
Skewness	0.31	0.34	0.45	**7.57**	0.28
Minimum	-10.81	-7.03	-7.88	-6.97	-6.67
Maximum	12.25	8.66	17.80	**63.57**	11.02
Observations	1219	1219	1219	1219	1219

Table2. Descriptive Statistics

	Bitcoin	Ethereum	Litecoin	Monero	Ripple
Mean	21.1%	29.1%	15.4%	20.9%	18.4%
Annual mean	77%	**106%**	56%	76%	67%
Median	0.18%	0.24%	0.05%	**0.28%**	0.00%
Standard Deviation	**3.89**	0.05	0.05	0.05	0.06
Volatility/SD	**1.97**	0.22	0.23	0.23	0.25
SR	**0.11**	1.32	0.67	0.91	0.75
Kurtosis	9.26	7.48	5.66	10.51	16.28
Skewness	-0.45	-0.47	-0.30	-0.23	1.37
Minimum	-37.17	-42.35	-36.17	-41.38	-42.33
Maximum	18.75	25.95	28.20	41.19	56.01
Observations	1219	1219	1219	1219	1219

Table3. Descriptive Statistics

	Coffee	Gold	Wheat	Cotton	Corn
Mean	6.9%	**3.2%**	9.3%	4.7%	4.2%
Annual mean	17%	**8%**	23%	12%	10%
Median	-0.04%	0.06%	**-0.04%**	0.02%	0.00%
Standard Deviation	0.02	**0.01**	0.02	0.02	0.01
Volatility/SD	0.14	**0.10**	0.14	0.13	0.11
SR	0.48	0.33	0.66	0.36	0.37
Kurtosis	1.63	5.41	1.33	31.25	3.97
Skewness	0.42	-0.18	0.34	**-2.20**	0.18
Minimum	-8.63	**-4.98**	-7.92	-23.88	-6.25
Maximum	10.03	5.95	8.07	**5.57**	7.02
Observations	1219	1219	1219	1219	1219

commodity decreases to 0.85, which is within the normal distribution range. Based on the skewness values, the distribution of returns for the two commodities coffee and wheat are also within the normal distribution range.

The skewness of gold and corn is not normal. Due to the high volatility of the cryptocurrency group, the distribution of this group is far from normal, which causes the skewness values to be far from those of the TSE stock and commodity groups.

The kurtosis of the selected TSE stocks is positive and to the right. The kurtosis of gold and cotton is negative and the kurtosis of the rest is positive. The kurtosis of cotton is very high. Among the selected cryptocurrencies, only the kurtosis of Ripple is positive and the kurtosis of the rest is negative. Among the selected cryptocurrencies, Ripple has the highest kurtosis. The minimum returns of the 15 selected symbols are -42.35% for Ethereum and the maximum is 63.57% for Akhaber. The number of observations is 1219 rows for each of the 15 selected symbols.

The correlation coefficient between all selected assets is calculated and reported as follows:

The highest correlation in the TSE stock group is between the symbols Akhaber-Fars, Foolad-Maroon, and Foolad-Akhaber, and the lowest correlation is between Fars-Foolad and Akhaber-Maroon. The correlation between the symbols Maroon-Hamrah and Akhaber-Hamrah is zero.

The highest correlation in the commodity group is between coffee and cotton and the lowest is between corn and cotton. In the cryptocurrency group, the highest correlation is between Litecoin and Ethereum and the lowest correlation is between Ripple and Monero. In the correlation between the two groups of TSE stocks and commodities, the highest value is between Maroon and coffee and the lowest value is between Maroon and gold.

In the correlation between the two groups of TSE stocks and cryptocurrencies, the highest value is between Akhaber and Monero and the lowest value is between Fars and Ripple. In the correlation between the two groups of commodities and cryptocurrencies, the highest correlation is between gold and Bitcoin and corn and Litecoin, and the lowest correlation is between cotton and Ripple.

The highest correlation between all three groups is between the two cryptocurrencies Ethereum and Litecoin with a value of 0.82 and the lowest correlation is between cotton and Ripple with a value of -0.04. We begin our analysis by comparing the performance of risk-based portfolio optimization.

A comparative analysis between a cryptocurrency portfolio, Tehran Stock Exchange (TSE) shares, global commodities, and five portfolios where each cryptocurrency was added to the base portfolio showed that the highest return of 19.84% was achieved in the Ethereum + TSE shares + global commodities portfolio with the equal-weight strategy. The lowest return of 9.59% was observed in the Monero + TSE shares + global commodities portfolio with the minimum variance under normality constraints strategy (Table 4).

In all six created portfolios, the minimum variance under normality constraints strategy had the lowest return. The minimum variance strategy had the highest frequency of creating higher returns in the portfolios, achieving the highest return in four portfolios. The equal-weight strategy achieved the highest return in the other two portfolios (Table 1).

Table 5 shows the standard deviation of the portfolios under 7 selected strategies. As can be seen, the cryptocurrency portfolio has a much higher standard deviation in all strategies except the maximum Sharpe ratio with soft constraints strategy. In this portfolio, the minimum standard deviation is 0.85 for the maximum Sharpe ratio with soft constraints strategy and the maximum standard deviation is 4.38 for the equal-weight strategy. The standard deviation of the base portfolio is 0.75 in the equal-weight strategy, which is the lowest value among all portfolios in this strategy. This indicates that adding the cryptocurrency portfolio and other assets did not improve risk control.

After examining the average return and standard deviation of the portfolios created in the selected strategies, we now turn to the Sharpe ratio.

Table4. Performance of the Studied Portfolios with Selected Strategies

Portfolio name	RPP	MDP	L2MDP	L2MVP	MVP	IVP	EWP
Tehran Stock Exchange	24.72%	24.65%	24.00%	24.64%	24.64%	24.78%	24.72%
Cryptocurrency	21.12%	20.67%	20.47%	22.65%	20.99%	21.19%	20.99%
Commodity	5.04%	4.92%	5.46%	3.77%	3.99%	5.12%	5.67%
Tehran Stock Exchange+commodity	12.26%	12.68%	15.03%	9.56%	9.71%	11.39%	15.20%
Tehran Stock Exchange+Commodity+Cryptocurrency	10.85%	13.33%	16.63%	9.79%	9.91%	11.54%	17.13%
Tehran Stock Exchange+Commodity+Bitcoin	10.85%	13.53%	16.90%	9.81%	19.01%	11.58%	17.18%
Ethereum+Tehran Stock Exchange+Commodity	13.90%	14.13%	19.69%	9.70%	19.09%	11.68%	19.84%
Litecoin+Tehran Stock Exchange+Commodity	12.75%	12.84%	14.92%	9.73%	18.91%	11.39%	15.26%
Monero+Tehran Stock Exchange+Commodity	13.11%	13.28%	16.78%	9.59%	18.98%	11.47%	17.08%
Ripple+Tehran Stock Exchange+Commodity	10.85%	12.99%	15.97%	9.68%	18.95%	11.43%	16.28%

Table5. Portfolio Standard Deviation Across Different Strategies

Portfolio name	RPP	MDP	L2MDP	L2MVP	MVP	IVP	EWP
Tehran Stock Exchange	1.13	1.13	0.47	1.12	1.12	1.13	1.16
Cryptocurrency	4.32	4.37	0.85	3.90	3.87	4.31	4.38
Commodity	0.78	0.77	0.53	0.71	0.71	0.79	0.89
Tehran Stock Exchange+commodity	0.66	0.67	0.73	0.61	0.61	0.67	0.75
Tehran Stock Exchange+Commodity+Cryptocurrency	0.66	0.74	0.59	0.61	0.61	0.66	1.55
Tehran Stock Exchange+Commodity+Bitcoin	0.66	0.73	0.59	0.61	0.61	0.66	1.40
Ethereum+Tehran Stock Exchange+Commodity	0.75	0.74	0.59	0.61	0.61	0.66	1.70
Litecoin+Tehran Stock Exchange+Commodity	0.78	0.76	0.60	0.61	0.61	0.66	1.84
Monero+Tehran Stock Exchange+Commodity	0.76	0.75	0.59	0.61	0.61	0.66	1.83
Ripple+Tehran Stock Exchange+Commodity	0.66	0.75	0.58	0.61	0.61	0.66	2.09

Table 6 shows the Sharpe ratios. In the base portfolio of Tehran Stock Exchange (TSE) shares + commodities, the highest Sharpe ratio is 20.7 in the L2 Maximum decorrelation

and the lowest is 15.6 in the minimum variance under normality constraints strategy. The average Sharpe ratio for this portfolio across different strategies is 18.1. Four of the seven selected strategies have a Sharpe ratio higher than the average and three have a Sharpe ratio lower than the average.

Then we add the cryptocurrency portfolio to the base portfolio and examine the new portfolio. In this portfolio, the highest Sharpe ratio is 28.2 in the maximum decorrelation coefficient with soft constraints strategy and the lowest is 11 in the equal-weight strategy. The average Sharpe ratio for this portfolio across different strategies is 17.7. Two of the seven selected strategies have a Sharpe ratio higher than the average and five have a Sharpe ratio lower than the average.

After adding and examining the cryptocurrency portfolio to the base portfolio, we add each cryptocurrency individually to the base portfolio. We start with Bitcoin. The highest Sharpe ratio is 31.3 in the minimum variance strategy and the lowest is 12.3 in the equal-weight strategy. The average Sharpe ratio for this portfolio across different strategies is 20.1. Two of the seven selected strategies have a Sharpe ratio higher than the average and five have a Sharpe ratio lower than the average.

Then we add Ethereum to the base portfolio. The highest Sharpe ratio is 33.6 in the maximum decorrelation coefficient with soft constraints strategy and the lowest is 11.6 in the equal-weight strategy. The average Sharpe ratio for this portfolio across different strategies is 21.1. Two of the seven selected strategies have a Sharpe ratio higher than the average and five have a Sharpe ratio lower than the average.

The next cryptocurrency is Litecoin. After adding it to the portfolio, the highest Sharpe ratio is 31 in the minimum variance strategy and the lowest is 8.3 in the equal-weight strategy. The average Sharpe ratio for this portfolio across different strategies is 18.7. Two of the seven selected strategies have a Sharpe

Table6. Portfolio Sharpe Ratio Across Different Strategies

Portfolio name	RPP	MDP	L2MDP	L2MVP	MVP	IVP	EWP
Tehran Stock Exchange	21.8%	21.8%	52.7%	21.9%	21.9%	21.8%	21.3%
Cryptocurrency	4.9%	4.7%	24.0%	5.8%	5.4%	4.9%	4.8%
Commodity	6.4%	6.4%	10.2%	5.3%	5.6%	6.4%	6.4%
Tehran Stock Exchange+commodity	18.5%	18.9%	20.7%	15.6%	15.9%	17.1%	20.2%
Tehran Stock Exchange+Commodity+Cryptocurrency	16.4%	18.1%	28.2%	16.1%	16.3%	17.5%	11.0%
Tehran Stock Exchange+Commodity+Bitcoin	16.4%	18.5%	28.6%	16.2%	31.3%	17.6%	12.3%
Ethereum+Tehran Stock Exchange+Commodity	18.5%	19.0%	33.6%	16.0%	31.4%	17.7%	11.6%
Litecoin+Tehran Stock Exchange+Commodity	16.4%	17.0%	25.0%	15.9%	31.0%	17.1%	8.3%
Monero+Tehran Stock Exchange+Commodity	17.2%	17.6%	28.3%	15.8%	31.2%	17.3%	8.3%
Ripple+Tehran Stock Exchange+Commodity	16.4%	17.2%	27.4%	15.9%	31.1%	17.3%	7.8%

ratio higher than the average and five have a Sharpe ratio lower than the average.

Adding Monero to the base portfolio results in a highest Sharpe ratio of 31.2 in the minimum variance strategy and a lowest Sharpe ratio of 9.3 in the equal-weight strategy. The average Sharpe ratio for this portfolio across different strategies is 19.5. Two of the seven selected strategies have a Sharpe ratio higher than the average and five have a Sharpe ratio lower than the average.

Finally, we add Ripple to the base portfolio. The highest Sharpe ratio is 31.1 in the minimum variance strategy and the lowest is 7.8 in the equal-weight strategy. The average Sharpe ratio for this portfolio across different strategies is 19. Two of the seven selected strategies have a Sharpe ratio higher than the average and five have a Sharpe ratio lower than the average.

Based on the above values, the highest Sharpe ratio of 33.6 is achieved in the maximum decorrelation coefficient with soft constraints strategy and in the Ethereum + TSE shares + commodities portfolio. This indicates the good performance of this portfolio across all portfolio-strategies. The lowest Sharpe ratio of 7.8 is in the equal-weight strategy and in the Ripple + TSE shares + commodities portfolio, which indicates the weak performance of this portfolio across all portfolio-strategies.

After evaluating the portfolios in the selected strategies, the analysis of the strategies in the created portfolios is reviewed. As can be observed in Figure1 in the Mean-Variance strategy in the base portfolio, the return value is 15.20%, and by adding the cryptocurrency portfolio and individual cryptocurrencies, the Mean-Variance return in all portfolios is greater than the base portfolio. In the Reverse Oscillation strategy in the base portfolio, the return value is 11.39%, and by adding the cryptocurrency portfolio and individual cryptocurrencies, the Mean-Variance return in all portfolios is at least equal to or greater than the base portfolio. In this strategy, all Mean-Variance returns are within the range of the base portfolio. In the Minimum Variance strategy in the base portfolio, the return value is 9.71%, and by adding the cryptocurrency portfolio and individual cryptocurrencies, the Mean-Variance return in all portfolios is greater than the base portfolio. Only the cryptocurrency portfolio is within the range of the base portfolio, and in the remaining portfolios, the return has increased to nearly 100%. In the Minimum Variance strategy under normal constraints in the base portfolio, the return value is 9.56% And by adding the cryptocurrency portfolio and individual cryptocurrencies, the Mean-Variance return in all portfolios is greater than the base portfolio. The

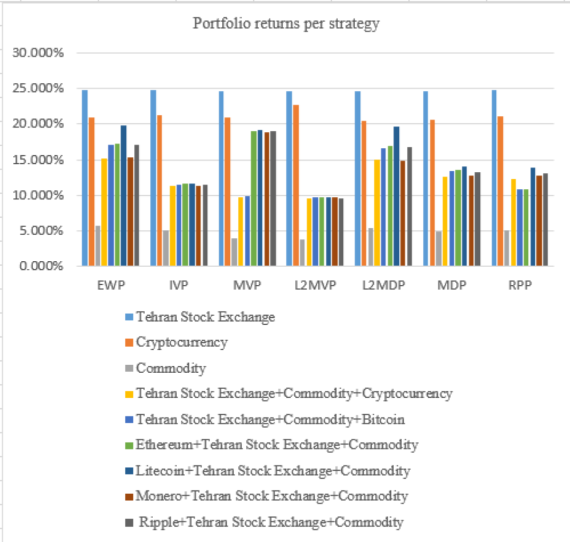

Figure. 1. Return of Each Strategy in 10 Portfolios

return in all portfolios is within the range of the base portfolio.

In the Maximum Uncorrelated Weight strategy with normal constraints in the base portfolio, the return value is 15.03%. By adding the cryptocurrency portfolio and individual cryptocurrencies, the Mean-Variance return in all portfolios is greater than the base portfolio, except for the Litecoin+Tehran Stock Exchange+Commodity portfolio, which has a value of 14.92%.

In the Maximum Diversification strategy in the base portfolio, the return value is 12.68%. By adding the cryptocurrency portfolio and individual cryptocurrencies, the Mean-Variance return in all portfolios is greater than the base portfolio.

In the Risk Equalization strategy in the base portfolio, the return value is 12.26%. By adding only, the Ethereum, Litecoin, and Monero cryptocurrencies, the Mean-Variance return is greater than the base portfolio. However, by adding the cryptocurrency portfolio and the Bitcoin and Ripple cryptocurrencies, the return is less than the base portfolio.

Among the 6 portfolios created after adding the cryptocurrency portfolio and individual cryptocurrencies to the base portfolio, and in the 7 selected strategies, the Mean-Variance return is 14.01%. The highest is 19.84% and the lowest is 9.59%. Of the 7 selected strategies, the Mean-Variance return was calculated in 6 portfolios, which are, from highest to lowest return, based on the strategies of Minimum Variance Purchase, Mean-Variance Weighting, Maximum Uncorrelated Weight with Normal Constraints, Maximum Diversification, Risk Equalization, Reverse Oscillation, and Minimum Variance under Normal Constraints.

As can be observed in Figure 2 in the base portfolio, the minimum standard deviation is 0.61 for the Minimum Variance strategy and the maximum is 0.75 for the Mean-Variance strategy. After adding the cryptocurrency portfolios to the base portfolio, the minimum decreases to 0.59 for the Maximum Uncorrelated Weight strategy with normal constraints and the maximum increases to 1.55 for the Mean-Variance strategy. As can be seen in the table, the Mean-Variance strategy has at least more than double the value of the other strategies.

We then examine the standard deviation before and after the base portfolio by adding individual cryptocurrencies to the base portfolio. After adding Bitcoin to the base portfolio, the minimum standard deviation is 0.59 for the Maximum Uncorrelated Weight strategy with normal constraints and the maximum standard deviation is 1.40 for the Mean-Variance strategy. After adding Ethereum to the base portfolio, the minimum standard deviation is 0.59 for the Maximum Uncorrelated Weight strategy with normal constraints and the maximum standard deviation is 1.70 for the Mean-Variance strategy. With the addition of Litecoin to the base portfolio, the minimum standard deviation is 0.6 for the Maximum Uncorrelated Weight strategy with normal constraints and the maximum standard deviation is 1.84 for the Mean-Variance strategy. Adding Monero to the base portfolio and examining its standard deviation, the minimum value is 0.58 for the Maximum Uncorrelated Weight strategy with normal constraints and the maximum standard deviation is for the Mean-Variance strategy. Finally, adding Ripple to the base portfolio and examining its standard deviation, the minimum value is 0.58 for the Maximum Uncorrelated Weight strategy with normal constraints and the maximum value is 2.09 for the Mean-Variance strategy.

As reported in the portfolio-strategy level analysis, it was found that in all portfolios, the Maximum Uncorrelated Weight strategy with normal constraints has the minimum standard deviation and the Mean-Variance strategy has the maximum standard deviation for the portfolios.

Figure 3 show the Sharpe ratios of the selected strategies for the portfolios.

In the Mean-Variance strategy, the highest Sharpe ratio is 12.3% for the Bitcoin+Tehran Stock Exchange+Commodity portfolio and the lowest is 7.8% for the Ripple+Tehran Stock Exchange+Commodity portfolio. The average Sharpe ratio in this strategy is 10.1%, which is higher in 3 portfolios and lower in 3 portfolios than the average Sharpe ratio in this strategy.

In the Reverse Oscillation strategy, the highest Sharpe ratio is 17.7% for the Ethereum+Tehran Stock Exchange+Commodity portfolio and the lowest is 17.1% for the Litecoin+Tehran Stock Exchange+Commodity portfolio. The average Sharpe ratio in this strategy is 17.4%, which is higher in 3 portfolios and lower in 3 portfolios than the average Sharpe ratio in this strategy.

In the Minimum Variance strategy, the highest Sharpe ratio is 31.4% for the Ethereum+Tehran Stock Exchange+Commodity portfolio and the lowest is 16.3% for the Cryptocurrency+Tehran Stock Exchange+Commodity portfolio. The average Sharpe ratio in this strategy is 28.7%, which is higher in 5 portfolios and lower in 1 portfolio than the average Sharpe ratio in this strategy.

In the Minimum Variance under Normal Constraints strategy, the highest Sharpe ratio is 16.2% for the Bitcoin+Tehran Stock Exchange+Commodity portfolio and the lowest is 15.8% for the Monero+Tehran Stock Exchange+Commodity portfolio. The average Sharpe ratio in this strategy is 16%, which is higher in 3 portfolios and lower in 3 portfolios than the average Sharpe ratio in this strategy.

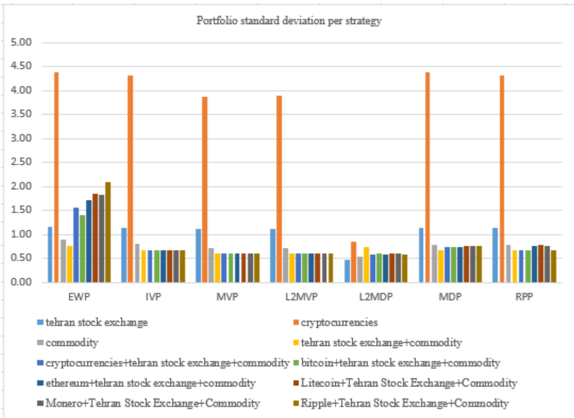

Figure. 2. Standard Deviation of Each Strategy Across 10 Portfolios

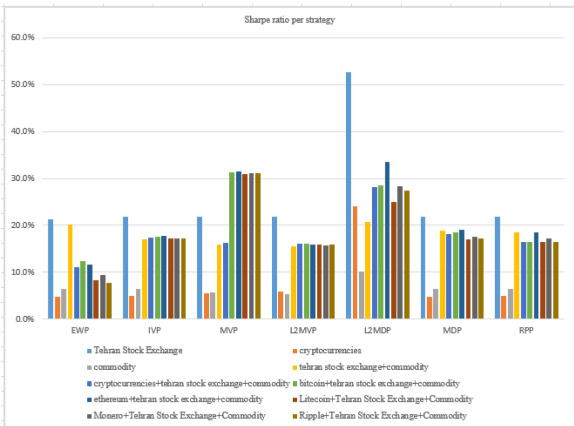

Figure. 3. Sharpe Ratio of Each Strategy Across 10 Portfolios

In the Maximum Uncorrelated Weight with Normal Constraints strategy, the highest Sharpe ratio is 33.6% for the Ethereum+Tehran Stock Exchange+Commodity portfolio and the lowest is 25% for the Litecoin+Tehran Stock Exchange+Commodity portfolio. The average Sharpe ratio in this strategy is 28.5%, which is higher in 2 portfolios and lower in 4 portfolios than the average Sharpe ratio in this strategy.

In the Maximum Diversification strategy, the highest Sharpe ratio is 19% for the Ethereum+Tehran Stock Exchange+Commodity portfolio and the lowest is 17% for the Litecoin+Tehran Stock Exchange+Commodity portfolio. The average Sharpe ratio in this strategy is 17.9%, which is higher in 3 portfolios and lower in 3 portfolios than the average Sharpe ratio in this strategy.

In the Risk Equalization strategy, the highest Sharpe ratio is 18.5% for the Ethereum+Tehran Stock Exchange+Commodity portfolio and the lowest is 16.4% for the Litecoin+Tehran Stock Exchange+Commodity portfolio. The average Sharpe ratio in this strategy is 16.9%, which is higher in 2 portfolios and lower in 4 portfolios than the average Sharpe ratio in this strategy.

Based on the above, it can be concluded that among the 6 portfolios created, the highest Sharpe ratio of 33.6% is for the Ethereum+Tehran Stock Exchange+Commodity portfolio in the Maximum Uncorrelated Weight with Normal Constraints strategy, and the lowest Sharpe ratio of 7.8% is for the Ripple+Tehran Stock Exchange+Commodity portfolio in the Mean-Variance strategy.

In this part of the analysis, all portfolios in each strategy are considered. The average Sharpe ratio across the 7 selected strategies is 19.4%, with 2 strategies having a higher Sharpe ratio and 5 strategies having a lower Sharpe ratio than the average. This indicates the weak performance of these 5 strategies. If we consider all portfolios as a unit and only want to compare the strategies, we can see that the Minimum Variance strategy has the best performance and the Maximum Uncorrelated Weight with Normal Constraints strategy ranks second.

4.1. Risk Analysis from the Perspective of VaR and CVaR

The main goal of risk-based strategies is to protect investors' capital in times of crisis. Therefore, in this section, we analyze the tail risk.

Tail risk is the risk that the price of an asset or portfolio will exceed a certain threshold, typically three standard deviations from the mean. In a less strict definition, it also refers to the risk or probability of unlikely events occurring.

The results of the tail risk analysis (shadow risk) for different portfolio-strategies show that the Value at Risk (VaR) and Conditional Value at Risk (CVaR) of all portfolios under the base strategy are higher than the six strategies on average, at confidence levels of 1% and 5%. Therefore, from the perspective of tail risk analysis, all two strategies performed better than the base strategy.

Table 7 and 8 shows the results of the sequential risk analysis (shadow risk) for different portfolio strategies.

The lowest VaR and CVaR values in the base + all cryptocurrencies, base + Bitcoin, and base + Ripple portfolios were achieved by the RP strategy. For the remaining portfolios, from a tail risk analysis perspective, the L2MVP strategy had the best performance.

Considering the VaR measure at a 95% confidence level, the worst performance in all cryptocurrency portfolios was achieved by the maximum decorrelation coefficient with soft constraints strategy. For the cryptocurrency + TSE + commodities, Bitcoin + TSE + commodities, and Ripple + TSE + commodities portfolios, the lowest VaR was achieved using the risk equalization strategy. For the remaining cryptocurrency portfolios, the best performance was with the minimum variance with soft constraints strategy.

Considering the CVaR measure at a 95% confidence level, the worst and best performances among the strategies are similar to the VaR measure. Considering the VaR measure at a 99% confidence level, the worst performance in all cryptocurrency portfolios is with the maximum decorrelation coefficient with soft constraints strategy, except for the cryptocurrency + TSE + commodities portfolio,

Table7. Sequential risk analysis for portfolio EWP strategy

strategy	*portfolio*	*VaR*		*CVaR*	
		95	99	95	99
EWP	Tehran Stock Exchange + commodity.	-1.42	-2.43	-1.96	-2.76
	Tehran Stock Exchange + commodities + cryptocurrencies	-3.13	-6.00	-4.77	-7.66
	Tehran Stock Exchange + commodity + Bitcoin	-2.88	-5.08	-4.21	-6.65
	Tehran Stock Exchange + commodity + Ethereum	-3.33	-6.09	-4.98	-7.98
	Tehran Stock Exchange + commodity + Litecoin	-3.57	-6.66	-5.34	-8.62
	Tehran Stock Exchange + commodity + Monero	-3.44	-6.24	-5.17	-8.58
	Tehran Stock Exchange + commodity + Ripple	-3.60	-6.64	-5.49	-9.30

Table 8. Sequential risk analysis for portfolio IVP strategy

strategy	portfolio	VaR	CVaR		
		95	99	95	99
IVP	Tehran Stock Exchange + commodity.	-1.30	-2.22	-1.82	-2.57
	Tehran Stock Exchange + commodities + cryptocurrencies	-1.42	-2.46	-2.01	-2.90
	Tehran Stock Exchange + commodity + Bitcoin	-1.42	-2.44	-2.01	-2.90
	Tehran Stock Exchange + commodity + Ethereum	-1.41	-2.44	-2.00	-2.88
	Tehran Stock Exchange + commodity + Litecoin	-1.38	-2.38	-1.95	-2.79
	Tehran Stock Exchange + commodity + Monero	-1.39	-2.39	-1.96	-2.83
	Tehran Stock Exchange + commodity + Ripple	-1.39	-2.39	-1.97	-2.84

where the highest VaR was achieved by the equal-weight strategy.

For the cryptocurrency + TSE + commodities, Bitcoin + TSE + commodities, and Ripple + TSE + commodities portfolios, the lowest VaR was achieved using the risk equalization strategy. For the remaining cryptocurrency portfolios, the best performance was with the minimum variance with soft constraints strategy.

Considering the CVaR measure at a 99% confidence level, the worst and best performances among the strategies are similar to the VaR measure. The difference is that for all cryptocurrency portfolios without exception, the maximum decorrelation coefficient with soft constraints strategy had the worst performance.

An analysis of the weights allocated in the portfolios shows that the maximum decorrelation coefficient with soft constraints and equal weight strategies rank first and second in terms of the highest share of allocated weights, and the lowest weight in the risk equalization strategy is allocated to the Ripple + TSE + commodities portfolio.

5. Discussion and Conclusion

In the Weighted Average Strategy, the highest return was 72.24% and the lowest return was 26.15%, respectively for the Ethereum+Tehran Stock Exchange+Commodity and Litecoin+Tehran Stock Exchange+Commodity portfolios.

The lowest standard deviation in this strategy was 0.04 and the highest standard deviation was 0.09, respectively for the Ethereum+Tehran Stock Exchange+Commodity and Ripple+Tehran Stock Exchange+Commodity portfolios.

The highest Sharpe ratio in this strategy was 2.92 and the lowest was 0.78, respectively for the Bitcoin+Tehran Stock Exchange+Commodity and Ripple+Tehran Stock Exchange+Commodity portfolios.

In the Inverse Oscillation Strategy, the highest return was 11.68% and the lowest return was 11.39%, respectively for the Ethereum+Tehran Stock Exchange+Commodity and Litecoin+Tehran Stock Exchange+Commodity portfolios.

All portfolios in this strategy have the same standard deviation of 0.66.

The highest Sharpe ratio in this strategy was 17.68 and the lowest was 15.17, respectively for the Ethereum+Tehran Stock Exchange+Commodity and Litecoin+Tehran Stock Exchange+Commodity portfolios.

In the minimum variance strategy, the highest return is 19.09% and the lowest return is -9.91%, for the portfolios Ethereum+Tehran Stock Exchange+Commodities and Cryptocurrency+Tehran Stock Exchange+Commodities, respectively. In this strategy, all portfolios have the same standard deviation of 0.61. The Sharpe ratio in this strategy is the highest 16.34 and the lowest 31.41, for the portfolios Ethereum+Tehran Stock Exchange+Commodities and Cryptocurrency+Tehran Stock Exchange+Commodities, respectively.

In the minimum variance strategy with soft constraints, the highest return is 9.81% and the lowest return is -5.59%, for the portfolios Bitcoin+Tehran Stock Exchange+Commodities and Monero+Tehran Stock Exchange+Commodities, respectively. In this strategy, all portfolios have the same standard deviation of 0.61. The Sharpe ratio in this strategy is the highest 16.17 and the lowest 15.76, for the portfolios Bitcoin+Tehran Stock Exchange+Commodities and Monero+Tehran Stock Exchange+Commodities, respectively.

In terms of the Sharpe ratio from a portfolio perspective in the selected strategies, for the Cryptocurrency+Tehran Stock Exchange+Commodities portfolio, the highest Sharpe ratio is in the Minimum Variance strategy with soft constraints (0.92) and the lowest is in the Mean-Variance strategy (0.32). For the Bitcoin+Tehran Stock Exchange+Commodities portfolio, the highest Sharpe ratio is in the Minimum Variance strategy (1.14) and the lowest is in the Mean-Variance strategy (0.42). For the Ethereum+Tehran Stock Exchange+Commodities portfolio, the highest Sharpe ratio is in the Maximum Non-correlation strategy with soft constraints (1.27) and the lowest is in the Mean-Variance strategy (0.47). For the Litecoin+Tehran Stock Exchange+Commodities portfolio, the highest Sharpe ratio is in the Minimum Variance strategy (1.19) and the lowest is in the Mean-Variance strategy (0.43). For the Monero+Tehran Stock Exchange+Commodities portfolio, the highest Sharpe ratio is in the Minimum Variance strategy (1.16) and the lowest is in the Mean-Variance strategy (0.41). For the Ripple+Tehran Stock Exchange+Commodities portfolio, the highest Sharpe ratio is in the Minimum Variance strategy (1.13) and the lowest is in the Mean-Variance strategy (0.40).

As mentioned above, the Mean-Variance strategy has underperformed in all portfolios. The strategy with the highest number of occurrences in terms of the most frequent strategy is Minimum Variance (4 portfolios), followed by Maximum Non-correlation strategy with soft constraints (2 portfolios).

In the Maximum Non-correlation strategy with soft constraints, the highest return is 19.69% and the lowest return is 14.92%, for the Ethereum+Tehran Stock Exchange+Commodities and Litecoin+Tehran Stock Exchange+Commodities portfolios, respectively. In this strategy, the lowest standard deviation is 0.58 and the highest standard deviation is 0.60, for the Ripple+Tehran Stock Exchange+Commodities and Litecoin+Tehran Stock Exchange+Commodities portfolios, respectively. These values are very close to each other. The Sharpe ratio in this strategy is the highest 33.59 and the lowest 25, for the Ethereum+Tehran Stock Exchange+Commodities and Litecoin+Tehran Stock Exchange+Commodities portfolios, respectively.

In the Maximum Diversification strategy, the highest return is 14.13% and the lowest return is 12.84%, for the Ethereum+Tehran Stock Exchange+Commodities and Litecoin+Tehran Stock Exchange+Commodities portfolios, respectively. In this strategy, the lowest standard deviation is 0.73 and the highest standard deviation is 0.76, for the Bitcoin+Tehran Stock Exchange+Commodities and Litecoin+Tehran Stock Exchange+Commodities portfolios, respectively. These values are very close to each other. The Sharpe ratio in this strategy is the highest 18.97 and the lowest 16.96, for the Ethereum+Tehran Stock Exchange+Commodities and Litecoin+Tehran Stock Exchange+Commodities portfolios, respectively.

5.1. Risk Equalization Strategy

In the Risk Equalization strategy, the highest return is 13.90% and the lowest return is 10.85%, for the Ethereum+Tehran Stock Exchange+Commodities and Ripple+Tehran Stock Exchange+Commodities portfolios, respectively. In this strategy, the lowest standard deviation is 0.66 for the two portfolios Bitcoin+Tehran Stock Exchange+Commodities and Cryptocurrency+Tehran Stock Exchange+Commodities, and the highest standard deviation is 0.78 for the Litecoin+Tehran Stock Exchange+Commodities portfolio. The Sharpe ratio in this strategy is the highest 18.05 and the lowest 16.37, for the Ethereum+Tehran Stock Exchange+Commodities and Litecoin+Tehran Stock Exchange+Commodities portfolios, respectively.

Overall, among the created portfolios and selected strategies, the highest return of 19.84% and the lowest return of 59.9% are for the Ethereum + TSE + commodities - equal weight and Monero + TSE + commodities - minimum variance under normality constraints portfolio-strategies, respectively. The lowest standard deviation of 0.58 is for the Ripple + TSE + commodities portfolio in the maximum decorrelation coefficient with soft constraints strategy, and the highest standard deviation of 2.09 is for the Ripple + TSE + commodities portfolio in the equal-weight strategy. The highest Sharpe ratio for all portfolio-strategies is 33.59 in the Ethereum + TSE + commodities portfolio and the maximum decorrelation coefficient with soft constraints strategy, and the lowest is 7.8 for the Ripple + TSE + commodities portfolio in the equal-weight strategy.

Overall, the best portfolio-strategy based on the Sharpe ratio (which is a combination of both risk and return measures) is the Ethereum + TSE + commodities portfolio.

5.2. Limitations of the Study

One of the limitations of this study is the lack of a consistent data period for all selected assets. The study was conducted with the lowest number of observations available. Another limitation is that the portfolio selection approach in this study was based on risk minimization. If the portfolio selection criteria were to change, the results could be different.

Also, due to the non-normality of cryptocurrency returns, the use of portfolio management and optimization methods such as mean-variance is not recommended in most studies.

5.3. Suggestions for Future Research

It is suggested that in the initial study, the two base assets be combined into a single portfolio, and then the selected portfolio or cryptocurrencies be added to the portfolio separately and analyzed. This analysis would then involve two portfolios in the second stage. For example, the base portfolio with two assets would be combined into a single portfolio, and when the cryptocurrency portfolio is added, there would again be two portfolios for solving the problems. The weight, return, and standard deviation would be considered for the two assets, and the results would be reported.

In the next suggestion, all fifteen assets would be combined and the results reported for the seven risk-based strategies.

It is suggested that in both of the above suggestions and in the present study, several return-based strategies be added to the set of strategies and then the performance of the strategies be analyzed.

Declarations

Authors' contributions

MKh: Study design, Interpretation of the results, Revision the manuscript and Supervision;

HHM: Acquisition of data, Modeling and statistical analysis;

MAA: Interpretation of the results, statistical analysis, drafting the manuscript.

Conflict of interest

The authors declare that no conflicts of interest exist.

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