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\* Corresponding author. Tel.: +98-86-33400661; Fax: +98-86-33670020. E-mail address: *n.hamta@arakut.ac.ir, nima.hamta@gmail.com*  Applied-Research Paper

# An Optimization Model for Designing a Supply Chain Network with a Value-Based Management Approach

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#### Abstract

Traditional approaches applied in supply chain management consider only the physical logistic operations and ignore the financial aspects of the chain. In this study, a mathematical model has been developed to address the supply chain network design problem with a value-based management approach. This model integrates both operations and financial aspects to maximize the value created and measured by shareholder value analysis (SVA) as an objective function. The results attributed to the developed model and the basic model are compared. The results indicate that creating more value for the company and its shareholders is achievable with appropriate financial decisions. To validate and show the applicability of the proposed model, it was solved by GAMS software with data provided by literature. Finally, sensitivity analyses on financial parameters were performed to evaluate the results. The results clearly reveal the improvement of using the new approach and convince managers to take advantage of the proposed approach.

# **1** Introduction

One of the main goals of supply chain management (SCM) is to maximize the profitability and competitiveness of a company since it provides an opportunity to enhance synergy [1-3]. The overall financial performance of a company can be affected by its strategic decisions and operational actions. The design of the supply chain network, which leads to the identification of the overall structure of the chain, requires the adoption of decisions that limit the decisions at lower levels, and therefore have a direct impact on the performance of the supply chain, particularly its competitiveness in the market. Financial decisions in supply chain management also can affect future tactical and operational decisions [4]. Therefore, they should be simultaneously considered for optimizing the supply chain network. Many previous studies have mentioned the importance of financial decisions in supply chain management and suggested considering them when modeling a supply chain [5]. However, a limited number of these studies have an optimization model that merges supply chain planning with financial decisions such as investment, financing and dividend decisions. Based on the previous studies, there are two different approaches in this field of research. In the first approach, financial considerations are considered as endogenous variables and optimized with other variables. In the second approach, financial aspectsare applied in objective functions and constraints as known parameters [6-11]

- ☑ This study aims to enrich the literature on supply chain network design by using mathematical programming techniques and financial considerations to address the problem of designing a supply chain network. The objective function of the model is to maximize the company value, measured by Shareholder Value Analysis (SVA), which is one of the most prominent metrics being used in business today. In order to integrate financial aspects in supply chain network design, a mixed-integer nonlinear programming (MINLP) model has been developed that considers operational and financial decisions simultaneously for designing a deterministic multi-echelon, multi-product, and multi-period supply chain network. To show the model applicability, the data of a case study were employed and solved using BARON solver in GAMS software. The major contributions of this study can be summarized as follow:
- ☑ This study presents a mathematical model to solve a supply chain network design problem that considers tactical, strategic and financial decisions at the same time.
- ☑ Maximizing the creation of economic value for shareholders measured by shareholder value analysis (SVA) as a new objective function instead of traditional approaches such as maximizing profits or minimizing costs. It has not been yet used in the general model in supply chain network design problems.
- ☑ Providing the possibility of opening or closing facilities in order to deal with market fluctuations at any time period of the planning horizon.
- ☑ The proposed model considers the amount of loan, bank repayment and new capital from shareholders as decision variables, therefore, it provides an accounts payable policy for the company managers instead of considering that all payments should be paid in cash. This is a contribution to the literature because previous studies consider them as parameters.
- At the strategic level, the model specifies the number and location of each facility. At the tactical level, it determines the products quantities to be produced and stored to satisfy customers demand. Regarding to financial decisions, the model specifies the amount of investment and their sources such as cash, bank debt or shareholders' capital as decision variables and it provides a repayment policy for managers.
- ☑ Regarding the constraints, in addition to common operational constraints, we also consider lower limit and/or upper limit values for financial ratios (performance, efficiency, liquidity and leverage), in order to support the financial health of the corporation. To retain a better financial performance, the proposed model provides a balance between new capital entries, loans and repayment. With consideration of large cost of new capital entries, the model imposes an upper bound on it and to avoid an ever-increasing debt, it considers a lower bound for bank repayments. Besides, these benefits, the proposed model provides an accounts payable guideline for managers.
- ☑ In contrast with basic models in previous studies which have too many assumptions, the presented model uses accounting principles with less assumptions that makes it more realistic. For example, we use the net liabilities in the analysis of financial statements that balances bank loans and payments, determines the exact value of deprecation by knowing the lifetime of each asset in each time period, and applies real cash value instead of pre-determined proportion of profit.

The remaining sections of this paper are as follows: In section 2, the relevant studies are reviewed. Section 3 describes the problem and presents a mathematical model for designing a supply chain with financial considerations. Section 4 explains a numerical example and discusses the results. Finally, in Section 5 the conclusions and some suggestions for future studies are given.

# **2 Literature Review**

As mentioned before, the available published studies on supply chain network design which simultaneously take operations and financial dimensions into account are still rare. The available published studies on supply chain network design that simultaneously take operations and financial dimensions into account are still rare [13-34]. In these studies, Longinidis et al. [5] introduced an MINLP SCN design model that considers the sale and leaseback (SLB) technique model to find the optimal configuration of an SCN, under uncertainty in product demand. Their model's financial objectives are maximizing net operating profits after taxes (NOPAT) and unearned profit on SLB (UPSLB).

Ramezani et al. [12] presented a financial approach that considers financial and physical flows to model a supply chain network design for long-term and mid-term decisions. They applied the change in a company equity as the objective function instead of traditional approaches such as minimizing cost or maximizing profit. Mussawi and Jaber [13] formulated a nonlinear program to find the optimal order amounts and the payment time of the supplier by using cash management integration. In their model, maximizing cash level and loan amount are financial decisions that need to be made to minimize inventory and financial costs.

Badri et al. [14] proposed a stochastic MILP programming model for a value-based supply chain network design. In their model, to maximize the company value (EVA), decisions on financial flow and physical flow (raw materials and finished products) are integrated.

Mohammadi et al. [15] developed a MILP model to consider financial and physical flows in mid-term and long-term decisions. The objective functions of their study are maximizing the economic value added (EVA), shareholders' equity, and corporate value. Saberi et al. [16] considered a trade-off between funding and its effect on environment in order to optimize NPV in a forward supply chain. Brahm et al. [25] addressed the planning problem of which considers physical and financial flows at the same time. In their research, supply chain contracts were combined and supply chain tactical planning was also considered within an uncertain condition; budgetary, environmental, and contractual constraints were also incorporated. They also developed and implemented a planning model on a rolling horizon basis to minimize the impact of uncertainties.

Yazdimoghaddam [26] presented a mathematical model that integrated strategic and tactical aspects of a supply chain as well as financial flows. His study compared the traditional approach (maximize profit) with a new approach (maximize the change in equity). Goli et al. [27] addressed a supply chain network design with uncertain parameters. They presented a model to incorporate the financial flow, constraints of debts, and employment under fuzzy uncertainty with three objective functions: maximize the cash flow, maximize the reliability of raw materials, and maximize the total jobs created.

Izadikhah [35] proposed a new variant of two stage DEA models and further evaluates the banks and financial institutes in Tehran stock exchange by considering the financial ratios.

Saeedi Aghdam et al. [36] developed a mathematical model and prediction of ensemble learning in order to evaluate crowdfunding projects. Their model determines the cost of funding for the entrepreneur and the return investors will receive per period. The results show the designed model improved performance in predicting the evaluation of success or failure of Crowdfunding projects.

Rezaei et al. [37] proposed a supply chain network design model focusing on the interactions between logistic and financial considerations. From the logistic point of view, their model determines the optimal location of production facilities and the assignment of these facilities to customers. From the financial

point of view, it plans logistics decisions such that a financial indicator is maximized and Adjusted Present Value (APV) has been applied as the objective function.

Goli and Kianfar [38] developed a bi-objective mathematical model and Fuzzy  $\varepsilon$ -constraint method for a closed-loop mask supply chain design with the objectives of increasing the total profit and reducing the total environmental impact is presented. In their problem, there are some potential locations for collection, recycling and disposal centers and the model should decide about location of the established centers as well as the amount of produced masks and raw materials. Izadikhah [39] applied the modified ERM model to evaluate 15 private bank branches in Markazi province. For this purpose, His study followed the primary goal that was maximizing the shareholders' satisfaction level and chose two financial bank efficiency measurement approaches, i.e. the production approach and the user cost approach. The approaches led to finding four regions for all branch performances.

Based on the above-mentioned works, this study suggests a mathematical model that simultaneously considers physical and financial aspects in a supply chain planning problem. A deterministic Mixed Integer Nonlinear Programming (MINLP) model is developed to specify the number and location of facilities and the links between them. The model also determines the quantities to be produced, stored and transported in order to meet customers' demands as well as maximize shareholder value analysis (SVA). As financial decisions, we consider the amount to invest, the source of the money needed (cash, bank loan, or new capital from shareholders), and repayments to the bank.

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### **3** Problem Definition and Assumptions

In this study, a multi-echelon, multi-period, and multi-product supply chain was discussed. Its semantic configuration is shown in Figure 1. The supply chain consists of plants, warehouses, distribution centers and customer zones. The problem incorporates operational and financial decisions simultaneously, therefore, the mathematical formulation needs proper variables and parameters.



----- Product or Material Flow if Plant/Warehouse/Distribution Center is Established

Advances in Mathematical Finance and Applications [5]

Figure 1: The Schematic of the Proposed Supply Chain Network

The objective function and financial constraints are calculated based on the studies by Jin et al. [40], Brealey et al. [45] and Borges et al. [46]. The goals of the proposed model are to determine:

- ☑ Strategic decisions about the facilities (plants, warehouses and distribution centers) to be established (opening or closing) in given locations and the supply routes among them for each time period.
- ☑ Tactical operation decisions regarding the quantity produced for each product at each factory, the materials flow between facilities and the levels of inventory that consist of maximum inventory at plants, products safety stock and max and min inventory of products at warehouses and distribution centers.
- ☑ Financial decisions for determining the amount of bank loans, new capital entries and total investments to establish the network and the quantity of repayments to the bank for each time period.

These three kinds of decisions were made for maximizing the value of the company at the end of planning horizon that was measured by SVA as an indicator of the corporation's profitability. As presented in the previous sections, supply chain strategic decisions and their operation impact corporate finances and consequently financial value created for shareholders [43]. SVA is a method that values the whole equity in a company. This method assumes that the value of a business is the net present value of its future cash flows, discounted at the appropriate cost of capital. Once the value of a business is calculated, the next step is to calculate the shareholder value by the equation:

shareholder value = value of business - debt

This method was first presented by Alfred Rappaport in the 1980s. That shows how well the company utilizes its properties in order to create value. This method is one of the most accepted lines of thought on how the corporate performance relates to the shareholder value [44].

Moreover, the assumptions of the proposed model can be summarized as follows: In each duration, the demand of each customer zone is clear. To satisfy customers' demands, the company can decide what kind of facilities to be involved at a particular time. Products can be kept at the company as inventory or distributed among warehouses. There is not any back-order. Transportation of products among different facilities has capacity limitations. Cost and revenue are derived from the operation of firm. Fixed and variable expenses are related to transportation and production. The establishment of facilities has fixed costs. Financial considerations are defined regarding capital cost, financial ratios, tax and depreciation rates and long-term borrowing.

# **3.1 Mathematical Model**

The proposed model considers both supply chain operation and financial decisions in the supply chain. mixed-integer nonlinear programming (MINLP) was used for designing a deterministic multi-echelon, multi-product, and multi-period supply chain network. The indices, parameters and decision variables applied in the mathematical model of this study are defined in Table 1:

Table 1: No	Sets and Indices
E	Resources of production indexed by e
E I	Products indexed by i
J	Locations of plant, indexed by j
J K	Locations of DC, indexed by $\mathcal{K}$
L	Locations of CZ, indexed by I
M	Locations of WH, indexed by m
$\mathcal{T}$	Planning periods indexed by s and t
	Parameters
A <sup>P</sup> <sub>jt</sub>	Plant market price j during the TP t, with $j \in J$ and $t \in T$
A <sup>W</sup> <sub>mt</sub>	Warehouse market price <i>m</i> during the TP t, with $m \in M$ and $t \in T$
$A_{kt}^{D}$	Distribution center market price $\mathcal{K}$ at TP t, with $\mathcal{K} \in K$ and $t \in \mathcal{T}$
$C_{jt}^{P+}$	Cost for establishing a plant at location j during the TP t, with $j \in J$ and $t \in T$
cW+	Cost for establishing a WH at location <i>m</i> during the TP t, with $m \in M$ and $t \in T$
$C_{mt}^{W+}$ $C_{kt}^{D+}$	Cost for establishing a W11 at location $\mathcal{K}$ at TP t, with $\mathcal{K} \in K$ and $t \in \mathcal{T}$
$C_{kt}^{P-}$	Cost for closing a plant at location j during the TP t, with $j \in J$ and $t \in T$
$C_{mt}^{W-}$	Cost for closing a WH at location $m$ during the TP t, with $m \in M$ and t $\in \mathcal{T}$
$C_{kt}^{D-}$	Cost for closing a DC at location $\mathcal{K}$ during the TP t, with $\mathcal{K} \in K$ and $t \in \mathcal{T}$
C <sub>ijt</sub>	Fixed production cost for product i at plant j at TP t, with $i \in J$ , $j \in J$ , and $t \in \mathcal{T}$
C <sub>ijt</sub>	Unit production cost for product i at plant j at TP t, with i $\in$ I, j $\in$ J, and t $\in \mathcal{T}$
$C_{ijmt}^{FTPW}$	Fixed transportation cost of product i from plant j to WH m at TP t, with $i \in I$ , $j \in J$ , $m \in M$ , and $t \in T$
C <sup>VTPW</sup> <sub>ijmt</sub>	Unit transportation cost of product i from plant j to WH <i>m</i> at TP t, with $i \in I$ , $j \in J$ , $m \in M$ , and $t \in T$
C <sub>imkt</sub>	Fixed transportation cost of product i from WH <i>m</i> to D.C $\mathcal{K}$ at TP t, with $i \in I, m \in M, \mathcal{K} \in K$ and $t \in \mathcal{T}$
C <sup>VTWD</sup>	Unit transportation cost of product i from W H m to D, C $\mathcal{K}$ at TP t, with $i \in I, m \in M, \mathcal{K} \in K$ and $t \in \mathcal{T}$
$C_{iklt}^{FTDC}$	Fixed transportation cost of product i from DC $\mathcal{K}$ to CZ l at TP t, with $i \in I, \mathcal{K} \in K, l \in L$ and $t \in \mathcal{T}$
C <sub>iklt</sub>	Unit transportation cost of product <i>i</i> from D.C $\mathcal{K}$ to CZ l at TP t, with $i \in I, \mathcal{K} \in K, l \in L$ and $t \in \mathcal{T}$
C <sub>ijt</sub>	Unit inventory cost of product i at plant j at TP t, with $i \in I, j \in J$ and $t \in T$
C <sup>IW</sup> <sub>imt</sub>	Unit inventory cost of product i at WH <i>m</i> at TP t, with $i \in I$ . $m \in M$ . and $t \in T$
C <sub>ikt</sub>	Unit inventory cost of product i at DC $\mathcal K$ at TP t, with i $\in$ I. $\mathcal K \in$ K. and t $\in \mathcal T$
$D_k^{max}$	Maximum capacity of DC $\mathcal{R}$ , with $\mathcal{K} \in K$
$D_k^{min}$	Minimum capacity of DC $\mathcal{K}$ , with $\mathcal{K} \in K$
I <sup>max</sup>	Maximum inventory level of product i being held at plant j at the end of TP t, with $i \in I$ . $j \in J$ . and $t \in T$
0 <sub>ilt</sub>	Demand of product i from customer zone l at TP t, with $i \in I, l \in L$ , and $t \in T$
P <sub>ij</sub> <sup>max</sup>	Maximum production capacity of product i at plant j with $i \in I$ end $j \in J$
P <sub>ij</sub>	Minimum production capacity of product i at plant j with $i \in I$ end $j \in J$
PR <sub>ilt</sub>	Unit selling price of product i at CZ l at TP t, with $i \in I, l \in L$ , and $t \in T$
Q <sup>PW</sup> <sub>im</sub>	Maximum limit of products that can be transferred from plant j to WH $m$ , with $j \in J$ end $m \in M$
$Q_{mk}^{WD}$	Maximum limit of products that can be transferred from WH $m$ to D.C $\mathcal{K}$ , with $m \in M$ end $\mathcal{K} \in K$
$Q_{kl}^{DC}$	Maximum limit of products that can be transferred from DC $\mathcal{K}$ to C.Z l, with $\mathcal{K} \in K$ end $l \in L$
R <sub>je</sub>	Available quantity of resource e at plant j, with $e \in E$ and $j \in J$
Wmax	Maximum capacity of WH $m$ , with $m \in M$
$W_{m}^{min}$	Minimum capacity of WH $m$ , with $m \in M$
$SS_{ikt}^{D}$	Safety stock of product i at DC $\mathcal{K}$ , during the TP t with $j \in J, \mathcal{K} \in K$ , and $t \in \mathcal{T}$
$SS_{imt}^W$	Safety stock of product i at WH $m$ , during the TP t with $i \in I. m \in M$ , and $t \in T$
CRt	Lower bound for cash ratio during the TP t, with $t \in \mathcal{T}$
CURt	Lower bound for current ratio during the TP t, with $t \in T$
CCRt	Lower bound for cash coverage ratio during the TP t, with $t \in \mathcal{T}$

# Table 1: Notations

$ \begin{array}{llllllllllllllllllllllllllllllllllll$		
$ \begin{aligned} &  TDR_t \\ Upper bound for long-term debt ratio during the TP t, with t \in \mathcal{T} \\ TDR_t \\ Upper bound for rotum on equity ratio during the TP t, with t \in \mathcal{T} \\ &  PM_t \\ Lower bound for rotum on equity ratio during the TP t, with t \in \mathcal{T} \\ &  PM_t \\ Lower bound for quick ratio during the TP t, with t \in \mathcal{T} \\ &  PR_t \\   Lower bound for quick ratio during the TP t, with t \in \mathcal{T} \\ &  PR_t \\   Lower bound for quick ratio during the TP t, with t \in \mathcal{T} \\ &  PR_t \\   Lower bound for quick ratio during the TP t, with t \in \mathcal{T} \\ &  PR_t \\   Ret of lang-treatmont assets ratio during the TP t, with t \in \mathcal{T} \\ &  PR_t \\   Ret a class of depreciation of a facility opened at TP and closed during the TP t, with s and t \in \mathcal{T} \\ &  PR_t \\   Ret a class of depreciation of a facility at the end of TP t, with s and t \in \mathcal{T} \\ &  PR_t \\   Ret of depreciation of a facility at the end of TP t, with t e \mathcal{T} \\ &  PR_t \\   Coefficient relating resource utilization rate of e to produce product i in plant j, with e \in E, F \in I, and j \in J \\ &  PR_t \\   Coefficient relating loss during the TP t, with t e T \\ &  Pt \\ & Coefficient relating payables outstanding at TP t, with t \in T \\ &  Pt \\ &  Pt \\ & Coefficient relating now during the held at Plant j at TP t, with i \in I, let   Pt   Pt   Pt   Pt   Pt   Pt   Pt   $	ATR <sub>t</sub>	Lower bound for assets turnover ratio during the TP t, with $t \in T$
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$\begin{array}{lll} \mu_t & \mbox{Coefficient relating payables outstanding at TP t, with t \in \mathcal{T} \\ \mbox{Coefficient relating revenues outstanding at TP t, with t \in \mathcal{T} \\ \hline \mbox{Decisions and Auxiliary Variables} \\ \hline \mbox{G}^{\dagger}_{It} & \mbox{Inventory level of product i being held at plant j at TP t, with i \in L, n \in M, and t \in \mathcal{T} \\ \mbox{M}^{\dagger}_{It} & \mbox{Inventory level of product i being held at DC $\mathcal{H}$ at TP t, with i \in L, m \in M, and t \in \mathcal{T} \\ \mbox{M}^{\dagger}_{It} & \mbox{Inventory level of product i being held at DC $\mathcal{H}$ at TP t, with i \in I, j \in J, and t \in \mathcal{T} \\ \mbox{Pitt} & \mbox{Product quantity i transferred from plant j at TP t, with i \in I, j \in J, and t \in \mathcal{T} \\ \mbox{Pitw} & \mbox{Product quantity i transferred from WH $m$ to DC $\mathcal{H}$ to TP t, with i \in I, $m \in M, and t \in \mathcal{T} \\ \mbox{Pitw} & \mbox{Product quantity i transferred from DC $\mathcal{H}$ to C2 loturing TP t, with i \in I, $m \in M, and t \in \mathcal{T} \\ \mbox{Visite} & \mbox{Product quantity of product i transferred from DC $\mathcal{H}$ to C2 loturing TP t, with i \in I, $m \in M, and t \in \mathcal{T} \\ \mbox{Visite} & \mbox{Visite} $		
$ \begin{aligned} \alpha_t & \text{Coefficient relating revenues outstanding at TP t, with t \in \mathcal{T} \\ \hline \textbf{Decisions and Auxiliary Variables} \\ \hline \textbf{q}_{1jt}^{II} & \text{Inventory level of product i being held at plant j at TP t, with i \in I, i \in J, \text{ and } t \in \mathcal{T} \\ \hline \textbf{q}_{1kt}^{III} & \text{Inventory level of product i being held at WH m at TP t, with i \in I, m \in M, \text{ and } V \in \mathcal{T} \\ \hline \textbf{q}_{1kt}^{III} & \text{Inventory level of product i being held at DC \mathcal{H} at TP t, with i \in I, m \in M, \text{ and } V \in \mathcal{T} \\ \hline \textbf{q}_{1kt}^{III} & \text{Inventory level of product i being held at DC \mathcal{H} at TP t, with i \in I, m \in M, \text{ and } V \in \mathcal{T} \\ \hline \textbf{q}_{1kt}^{IIII} & \text{Product quantity i produced at plant j at TP t, with i \in I, j \in J, m \in M, \text{ and } t \in \mathcal{T} \\ \hline \textbf{x}_{1kt}^{IIII} & \textbf{Product quantity i transferred from VH m to DC \mathcal{H} in TP t, with i \in I, m \in M, \mathcal{H} \in K \text{ and } t \in \mathcal{T} \\ \hline \textbf{x}_{1kt}^{IIII} & \textbf{Q}_{1kt}^{IIIII} & \textbf{f a plant at location } j \text{ is opened at TP t;} \\ \hline \textbf{0} & \text{otherwise.} \\ \hline \textbf{with } j \in J \text{ and } t \in \mathcal{T} \\ \hline \textbf{y}_{1t}^{W^*} & \left\{ \begin{array}{c} 1 & \text{if a plant at location } j \text{ is obsend at TP t;} \\ \hline \textbf{0} & \text{otherwise.} \\ \hline \textbf{with } j \in J \text{ and } t \in \mathcal{T} \\ \hline \textbf{y}_{1t}^{W^*} & \left\{ \begin{array}{c} 1 & \text{if a W. H at location } m \text{ is opened at TP t;} \\ \hline \textbf{0} & \text{otherwise.} \\ \hline \textbf{with } m \in M \text{ and } t \in \mathcal{T} \\ \hline \textbf{y}_{1t}^{W^*} & \left\{ \begin{array}{c} 1 & \text{if a W. H at location } m \text{ is opened at TP t;} \\ \hline \textbf{0} & \text{otherwise.} \\ \hline \textbf{with } m \in M \text{ and } t \in \mathcal{T} \\ \hline \textbf{y}_{1t}^{W^*} & \left\{ \begin{array}{c} 1 & \text{if a W. H at location } \mathcal{H} \text{ is opened at TP t;} \\ \hline \textbf{0} & \text{otherwise.} \\ \hline \textbf{with } m \in M \text{ and } t \in \mathcal{T} \\ \hline \textbf{y}_{1t}^{W^*} & \left\{ \begin{array}{c} 1 & \text{If a D. C at location } \mathcal{H} \text{ is opened at TP t;} \\ \hline \textbf{0} & \text{otherwise.} \\ \hline \textbf{with } \mathcal{H} \in K \text{ and } t \in \mathcal{T} \\ \hline \textbf{y}_{1t}^{W^*} & \left\{ \begin{array}{c} 1 & \text{If a D. C at location } \mathcal{H} \text{ is closed at TP t;} \\ \hline \textbf{0} & \text{otherwise.} \\ \hline \textbf{with } \mathcal{H} \in K \text{ and } t \in \mathcal{T} \\ \hline \textbf{y}_{1t}^{W^*} & \left\{ \begin{array}{c} 1 & \text{If a D. C at location } \mathcal{H} \text{ is closed at TP t;} \\ \hline \textbf{0} & \text{otherwise.} \\ \hline with$	γt	
Decisions and Auxiliary Variables $q_{ijt}^{\dagger}$ Inventory level of product i being held at plant j at TP t, with $i \in l, i \in J$ , and $t \in T$ $q_{int}^{\dagger}$ Inventory level of product i being held at DC $\mathcal{K}$ at TP t, with $i \in I$ . $\mathcal{M} \in M$ , and $t \in T$ $q_{int}^{\dagger}$ Inventory level of product i being held at DC $\mathcal{K}$ at TP t, with $i \in I$ . $\mathcal{M} \in M$ , and $t \in T$ $p_{ijt}$ Product quantity i produced at plant j at TP t, with $i \in I$ . $\mathcal{J} \in \mathcal{K}$ , and $t \in T$ $p_{ijt}$ Product quantity i transferred from plant j to WH $m$ in TP t, with $i \in I$ . $\mathcal{M} \in \mathcal{K}$ , and $t \in T$ $\chi_{imt}^{WD}$ Product quantity i transferred from DC $\mathcal{K}$ to $\mathcal{C}$ . $\mathcal{D}$ fund TP t, with $i \in I$ . $\mathcal{M} \in \mathcal{K}$ , $l \in L$ and $t \in T$ $\chi_{imt}^{WD}$ Quantity of product i transferred from DC $\mathcal{K}$ to $\mathcal{C}$ . $\mathcal{D}$ fund TP t, with $i \in I$ . $\mathcal{K} \in \mathcal{K}$ . $l \in L$ and $t \in T$ $\chi_{ipt}^{WP}$ $\left\{ \begin{array}{c} 1 & \text{if a plant at location } j \text{ is opened at TP } t; \\ 0 & \text{otherwise.} \\ \text{with } j \in J$ and $t \in \mathcal{T}$ $y_{it}^{W^*}$ $\left\{ \begin{array}{c} 1 & \text{if a vent to \mathcal{I} opened at TP t; \\ 0 & \text{otherwise.} \\ \text{with } j \in J and t \in \mathcal{T}y_{imt}^{W^*}\left\{ \begin{array}{c} 1 & \text{if a W}. Hat location m is opened at TP t; \\ 0 & \text{otherwise.} \\ \text{with } m \in M and t \in \mathcal{T}y_{imt}^{W^*}\left\{ \begin{array}{c} 1 & \text{if a W}. Hat location \mathcal{K} is opened at TP t; \\ 0 & \text{otherwise.} \\ \text{with } m \in M and t \in \mathcal{T}y_{imt}^{W^*}\left\{ \begin{array}{c} 1 & \text{if a W}. Hat location \mathcal{K} is opened at TP t; \\ 0 & \text{otherwise.} \\ \text{with } \mathcal{M} \in K and t \in \mathcal{T}y_{imt}^{W^*}\left\{ \begin{array}{c} 1 & \text{if a D. C at location \mathcal{K} is closed at TP t; \\ 0 & \text{otherwise.} \\ \text{with } \mathcal{K} \in K and t \in \mathcal{T}y_{imt}^{W^*}$		
$ \begin{array}{ll} \mathbf{q}_{  t }^{\mathbf{p}} &  \text{Inventory level of product i being held at plant j at TP t, with i \in \mathbf{l}, i \in \mathbf{j}, \text{ and } t \in \mathcal{T} \\ \mathbf{q}_{  t }^{W} &  \text{Inventory level of product i being held at WH m at TP t, with i \in \mathbf{l}, m \in \mathbf{M}, \text{ and } t \in \mathcal{T} \\ \mathbf{q}_{  t }^{W} &  \text{Inventory level of product i being held at DC & at TP t, with i \in \mathbf{l}, m \in \mathbf{M}, and t \in \mathcal{T} \\ \mathbf{p}_{  t } &  \text{Product quantity i produced at plant j at TP t, with i \in \mathbf{I}, j \in \mathbf{j}, and t \in \mathcal{T} \\ \mathbf{p}_{  t }^{W} &  \text{Product quantity i transferred from plant j to WH m in TP t, with i \in \mathbf{I}, j \in \mathbf{j}, m \in \mathbf{M}, and t \in \mathcal{T} \\ \mathbf{x}_{  t }^{WD} &  \text{Product quantity i transferred from DC & no C & no TP t, with i \in \mathbf{I}, m \in \mathbf{M}, \mathcal{K} \in \mathbf{K} \text{ and } t \in \mathcal{T} \\ \mathbf{x}_{  t }^{WD} &  \text{Quantity of product i transferred from DC & no C & no TP t, with i \in \mathbf{I}, m \in \mathbf{M}, \mathcal{K} \in \mathbf{K} \text{ and } t \in \mathcal{T} \\ \mathbf{x}_{  t }^{WD} &  \text{Quantity of product i transferred from DC & no C & no TP t, with i \in \mathbf{I}, m \in \mathbf{M}, \mathcal{K} \in \mathbf{K} \text{ l } \in \mathcal{T} \\ \mathbf{y}_{  t }^{P^-} &  \left\{ \begin{array}{l} & \text{if a plant at location } j \text{ is opened at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } j \in \mathbf{J} \text{ and } t \in \mathcal{T} \\ \mathbf{y}_{  t }^{P^-} &  \left\{ \begin{array}{l} & \text{if a plant at location } j \text{ is opened at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } j \in \mathbf{J} \text{ and } t \in \mathcal{T} \\ \mathbf{y}_{  t }^{W^+} &  \left\{ \begin{array}{l} & \text{if a W. H at location } m \text{ is opened at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } m \in \mathbf{M} \text{ and } t \in \mathcal{T} \\ \mathbf{y}_{  t }^{W^+} &  \left\{ \begin{array}{l} & \text{if a W. H at location } m \text{ is closed at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } \mathcal{K} \in \mathbf{K} \text{ and } t \in \mathcal{T} \\ \mathbf{y}_{  t }^{P^-} &  \left\{ \begin{array}{l} & \text{if a U. Cat location } \mathcal{K} \text{ is opened at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } \mathcal{K} \in \mathbf{K} \text{ and } t \in \mathcal{T} \\ \end{array} \right. \\ \mathbf{y}_{  t }^{P^-} &  \left\{ \begin{array}{l} & \text{if a D. Cat location } \mathcal{K} \text{ is closed at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } \mathcal{K} \in \mathbf{K} \text{ and } t \in \mathcal{T} \\ \end{array} \right. \\ \mathbf{y}_{  t }^{P^-} &  \left\{ \begin{array}{l} & \text{if product } i \text{ is produced at plant } j \text{ at TP } t; \\ 0 & otherwis$	$\alpha_t$	Coefficient relating revenues outstanding at TP t, with $t \in \mathcal{T}$
$ \begin{array}{ll} q_{int}^{W} & \text{Inventory level of product i being held at WH $m$ at TP t, with i \in I.m \in M. \text{ and } t \in \mathcal{T} \\ q_{int}^{W} & \text{Inventory level of product i being held at DC $\mathcal{X}$ at TP t, with i \in I. \mathcal{K} \in K. \text{ and } t \in \mathcal{T} \\ p_{ijt} & \text{Product quantity i produced at plant j at TP t, with i \in I. j \in J. \text{ and } t \in \mathcal{T} \\ Product quantity i transferred from plant j to WH $m$ in TP t, with i \in I. j \in J. m \in M. \text{ and } t \in \mathcal{T} \\ p_{int}^{WD} & \text{Product quantity i transferred from WH $m$ to DC $\mathcal{K}$ in TP t, with i \in I. j \in J. m \in M. \text{ and } t \in \mathcal{T} \\ p_{int}^{WD} & \text{Quantity of product i transferred from DC $\mathcal{K}$ to $\mathcal{C}$ 2 Iduring TP t, with i \in I. \mathcal{K} \in K. 1 \in L \text{ and } t \in \mathcal{T} \\ p_{int}^{WD} & \text{Quantity of product i transferred from DC $\mathcal{K}$ to $\mathcal{C}$ 2 Iduring TP t, with i \in I. \mathcal{K} \in K. 1 \in L \text{ and } t \in \mathcal{T} \\ p_{int}^{P^-} & \text{Quantity of product i transferred from DC $\mathcal{K}$ to $\mathcal{C}$ 2 Iduring TP t, with i \in I. \mathcal{K} \in K. 1 \in L \text{ and } t \in \mathcal{T} \\ p_{it}^{P^-} & \begin{cases} 1 & \text{if a plant at location $j$ is opened at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with } j \in J \text{ and } t \in \mathcal{T} \\ y_{int}^{P^-} & \begin{cases} 1 & \text{if a W.H at location $m$ is opened at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $m \in M$ and } t \in \mathcal{T} \\ y_{int}^{W^+} & \begin{cases} 1 & \text{if a W. H at location $m$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $m \in M$ and } t \in \mathcal{T} \\ y_{int}^{W^-} & \begin{cases} 1 & \text{if a W. H at location $m$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $m \in M$ and } t \in \mathcal{T} \\ y_{int}^{W^-} & \begin{cases} 1 & \text{if a D. C at location $\mathcal{K}$ is opened at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $\mathcal{K} \in K$ and $t \in \mathcal{T} \\ y_{int}^{W^-} & \begin{cases} 1 & \text{if a D. C at location $\mathcal{K}$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $\mathcal{K} \in K$ and $t \in \mathcal{T} \\ y_{int}^{W^-} & \begin{cases} 1 & \text{if a D. C at location $\mathcal{K}$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $\mathcal{K} \in K$ and $t \in \mathcal{T} \\ y_{int}^{W^-} & \begin{cases} 1 & \text{if a D. C at location $\mathcal{K}$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $\mathcal{K} \in K$ and $t \in \mathcal{T} \\ y_{int}^{$		Decisions and Auxiliary Variables
$ \begin{array}{ll} q_{int}^{W} & \text{Inventory level of product i being held at WH $m$ at TP t, with i \in I.m \in M. \text{ and } t \in \mathcal{T} \\ q_{int}^{W} & \text{Inventory level of product i being held at DC $\mathcal{X}$ at TP t, with i \in I. \mathcal{K} \in K. \text{ and } t \in \mathcal{T} \\ p_{ijt} & \text{Product quantity i produced at plant j at TP t, with i \in I. j \in J. \text{ and } t \in \mathcal{T} \\ Product quantity i transferred from plant j to WH $m$ in TP t, with i \in I. j \in J. m \in M. \text{ and } t \in \mathcal{T} \\ p_{int}^{WD} & \text{Product quantity i transferred from WH $m$ to DC $\mathcal{K}$ in TP t, with i \in I. j \in J. m \in M. \text{ and } t \in \mathcal{T} \\ p_{int}^{WD} & \text{Quantity of product i transferred from DC $\mathcal{K}$ to $\mathcal{C}$ 2 Iduring TP t, with i \in I. \mathcal{K} \in K. 1 \in L \text{ and } t \in \mathcal{T} \\ p_{int}^{WD} & \text{Quantity of product i transferred from DC $\mathcal{K}$ to $\mathcal{C}$ 2 Iduring TP t, with i \in I. \mathcal{K} \in K. 1 \in L \text{ and } t \in \mathcal{T} \\ p_{int}^{P^-} & \text{Quantity of product i transferred from DC $\mathcal{K}$ to $\mathcal{C}$ 2 Iduring TP t, with i \in I. \mathcal{K} \in K. 1 \in L \text{ and } t \in \mathcal{T} \\ p_{it}^{P^-} & \begin{cases} 1 & \text{if a plant at location $j$ is opened at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with } j \in J \text{ and } t \in \mathcal{T} \\ y_{int}^{P^-} & \begin{cases} 1 & \text{if a W.H at location $m$ is opened at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $m \in M$ and } t \in \mathcal{T} \\ y_{int}^{W^+} & \begin{cases} 1 & \text{if a W. H at location $m$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $m \in M$ and } t \in \mathcal{T} \\ y_{int}^{W^-} & \begin{cases} 1 & \text{if a W. H at location $m$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $m \in M$ and } t \in \mathcal{T} \\ y_{int}^{W^-} & \begin{cases} 1 & \text{if a D. C at location $\mathcal{K}$ is opened at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $\mathcal{K} \in K$ and $t \in \mathcal{T} \\ y_{int}^{W^-} & \begin{cases} 1 & \text{if a D. C at location $\mathcal{K}$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $\mathcal{K} \in K$ and $t \in \mathcal{T} \\ y_{int}^{W^-} & \begin{cases} 1 & \text{if a D. C at location $\mathcal{K}$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $\mathcal{K} \in K$ and $t \in \mathcal{T} \\ y_{int}^{W^-} & \begin{cases} 1 & \text{if a D. C at location $\mathcal{K}$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ \text{with $\mathcal{K} \in K$ and $t \in \mathcal{T} \\ y_{int}^{$	$q_{iit}^{P}$	Inventory level of product i being held at plant j at TP t, with i $\in$ I, j $\in$ J. and t $\in$ T.
$ \begin{array}{ll} \begin{array}{ll} \begin{array}{ll} \begin{array}{ll} \begin{array}{ll} \\ \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \begin{array}{ll} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \begin{array}{ll} \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{ll} \end{array} \\ \end{array} $		Inventory level of product i being held at WH m at TP t, with $i \in I. m \in M$ . and $t \in T$
$\begin{array}{lll} & \operatorname{Product} \operatorname{quantity} i \operatorname{produced} \operatorname{at} \operatorname{plant} j \operatorname{at} \operatorname{TP} t, \operatorname{with} i \in I, j \in J, \operatorname{and} t \in \mathcal{T} \\ & \operatorname{with}^{T} \\ & \operatorname{Product} \operatorname{quantity} i \operatorname{transferred} from \operatorname{plant} j \operatorname{to} \operatorname{WH} m \operatorname{in} \operatorname{TP} t, \operatorname{with} i \in I, m \in M, m \in M, and t \in \mathcal{T} \\ & \operatorname{WD}_{intet} \\ & \operatorname{Quantity} of \operatorname{product} i \operatorname{transferred} from \operatorname{DC} \mathcal{K} \operatorname{to} \mathcal{C} \mathcal{Z} \operatorname{Iduring} \operatorname{TP} t, \operatorname{with} i \in I, m \in M, \mathcal{K} \in K \operatorname{and} t \in \mathcal{T} \\ & \operatorname{WD}_{intet} \\ & \operatorname{Quantity} of \operatorname{product} i \operatorname{transferred} from \operatorname{DC} \mathcal{K} \operatorname{to} \mathcal{C} \mathcal{Z} \operatorname{Iduring} \operatorname{TP} t, \operatorname{with} i \in I, m \in M, \mathcal{K} \in K, I \in L \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With}^{T} i  \text{qluantity} \operatorname{of product} i \operatorname{transferred} from \operatorname{DC} \mathcal{K} \operatorname{to} \mathcal{C} \mathcal{Z} \operatorname{Iduring} \operatorname{TP} t, \operatorname{with} i \in I, m \in M, \mathcal{K} \in K, I \in L \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With}^{T} i  \text{qluant} \operatorname{allocation} j \operatorname{is opened} \operatorname{at} \operatorname{TP} t; \\ & \operatorname{Quantity} o \operatorname{otherwise.} \\ & \operatorname{with} j \in J \operatorname{and} t \in \mathcal{T} \\ & \operatorname{Y}_{H}^{P^{-}} \\ & \left\{ \begin{array}{l} 1 & \text{if a plant at location} j \operatorname{is opened} \operatorname{at} \operatorname{TP} t; \\ & \operatorname{Quantity} o \operatorname{otherwise.} \\ & \operatorname{with} m \in M \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} m \in M \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} m \in M \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} m \in M \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} m \in M \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} m \in M \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} m \in M \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} m \in M \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} m \in K \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} \mathcal{K} \in K \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} \mathcal{K} \in K \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} \mathcal{K} \in K \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} \mathcal{K} \in K \operatorname{and} t \in \mathcal{T} \\ & \operatorname{Quantity} \mathcal{K} \in K \operatorname{and} t \in \mathcal{T} \\ & \operatorname{With} \mathcal{K} \in K \operatorname{and} t \in \mathcal{T} \\ & \operatorname{Ul}_{I} \\ & \operatorname{If a D. C at location} \mathcal{K} \operatorname{is closed} \operatorname{at} \operatorname{TP} t; \\ & \operatorname{Quantity} m \in \mathcal{K} \\ & \operatorname{Vith} \mathcal{K} \in K \operatorname{and} t \in \mathcal{T} \\ & \operatorname{Ul}_{I} \\ & \operatorname{If product} i \operatorname{Is produced} \operatorname{at plant} j \operatorname{at} \operatorname{TP} t; \\ & \operatorname{Quantity} \mathcal{K} \\ & \operatorname{Quantity} \\ & \operatorname{Quantity} \mathcal{K} \\ & \operatorname{Quantity} \\ & \operatorname{Quantity} \\ & \operatorname{Quant} \\ & Quantit$		
Number Number NumberProduct quantity i transferred from plant j to WH m in TP t, with $i \in I, j \in J, m \in M, and t \in T$ Number Number NumberProduct quantity i transferred from WH m to DC $\mathcal{R}$ in TP t, with $i \in I, m \in M, \mathcal{R} \in K$ and $t \in T$ Vint Number NumberQuantity of product i transferred from DC $\mathcal{R}$ to $\mathcal{C}$ . Z1 during TP t, with $i \in I, m \in M, \mathcal{R} \in K$ . $I \in L$ and $t \in T$ $y_{1t}^{p_{+}}$ Quantity of product i transferred from DC $\mathcal{R}$ to $\mathcal{C}$ . Z1 during TP t, with $i \in I, \mathcal{R} \in K$ . $I \in L$ and $t \in T$ $y_{1t}^{p_{-}}$ $\begin{pmatrix} 1 & \text{if a plant at location j is opened at TP t; \\ 0 & \text{otherwise.} \\ with j \in J and t \in Ty_{1t}^{W_{-}}\begin{pmatrix} 1 & \text{if a W. H at location m is opened at TP t; \\ 0 & \text{otherwise.} \\ with m \in M and t \in Ty_{mt}^{W_{-}}\begin{pmatrix} 1 & \text{if a W. H at location m is closed at TP t; \\ 0 & \text{otherwise.} \\ with m \in M and t \in Ty_{mt}^{W_{-}}\begin{pmatrix} 1 & \text{if a D. C at location m is closed at TP t; \\ 0 & \text{otherwise.} \\ with m \in M and t \in Ty_{mt}^{W_{-}}\begin{pmatrix} 1 & \text{if a D. C at location \mathcal{K} is closed at TP t; \\ 0 & \text{otherwise.} \\ with \mathcal{K} \in K and t \in Ty_{mt}^{U_{-}}\begin{pmatrix} 1 & \text{if a D. C at location \mathcal{K} is closed at TP t; \\ 0 & \text{otherwise.} \\ with \mathcal{K} \in K and t \in Ty_{mt}^{U_{-}}\begin{pmatrix} 1 & \text{if a D. C at location \mathcal{K} is closed at TP t; \\ 0 & \text{otherwise.} \\ with \mathcal{K} \in K and t \in Ty_{mt}^{U_{-}}\begin{pmatrix} 1 & \text{if a D. C at location \mathcal{K} is closed at TP t; \\ 0 & \text{otherwise.} \\ with \mathcal{K} \in K and t \in Ty_{mt}^{U_{-}}\begin{pmatrix} 1 & \text{if product i is produced at plant j at TP t; \\ 0 & \text{otherwise.} \\ \end{pmatrix}$		
$ \begin{array}{ll} \mathbf{x}_{\mathrm{int}}^{\mathrm{W}} & \operatorname{Product} \operatorname{quantity} i \operatorname{transferred} from \mathrm{WH} \textit{m} \text{ to DC}  \mathcal{K} \text{ in TP t. with } i \in \mathrm{I},  m \in \mathrm{M},  \mathcal{K} \in \mathrm{K} \text{ and } t \in \mathcal{T} \\ \mathbf{x}_{\mathrm{ik}t}^{\mathrm{DC}} & \operatorname{Quantity} of \operatorname{product} i \operatorname{transferred} from \mathrm{DC}  \mathcal{K} \text{ to } \mathrm{C},  \mathrm{Z}, \mathrm{I}  \mathrm{during}  \mathrm{TP}  \mathrm{t},  \mathrm{with}  i \in \mathrm{I},  \mathcal{K} \in \mathrm{K},  \mathrm{I} \in \mathrm{L}  \mathrm{and}  \mathrm{t} \in \mathcal{T} \\ \mathbf{y}_{\mathrm{I}}^{\mathrm{P}^{+}} & \left\{ \begin{array}{c} 1 & \text{if a plant at location } j \text{ is opened at TP}  \mathrm{t}; \\ 0 & \text{otherwise.} \end{array} \right. \\ \mathrm{with}  j \in \mathrm{J}  \mathrm{and}  \mathrm{t} \in \mathcal{T} \\ \mathbf{y}_{\mathrm{I}}^{\mathrm{P}^{-}} & \left\{ \begin{array}{c} 1 & \text{if a plant at location } j \text{ is closed at TP}  \mathrm{t}; \\ 0 & \text{otherwise.} \end{array} \right. \\ \mathrm{with}  j \in \mathrm{J}  \mathrm{and}  \mathrm{t} \in \mathcal{T} \\ \mathbf{y}_{\mathrm{II}}^{\mathrm{W}^{+}} & \left\{ \begin{array}{c} 1 & \text{if a W}, \mathrm{H}  \mathrm{at location}  m \text{ is opened at TP}  \mathrm{t}; \\ 0 & \text{otherwise.} \end{array} \right. \\ \mathrm{with}  m \in \mathrm{M}  \mathrm{and}  \mathrm{t} \in \mathcal{T} \\ \mathrm{y}_{\mathrm{III}}^{\mathrm{W}^{+}} & \left\{ \begin{array}{c} 1 & \text{if a W}, \mathrm{H}  \mathrm{at location}  m \text{ is opened at TP}  \mathrm{t}; \\ 0 & \text{otherwise.} \end{array} \right. \\ \mathrm{with}  m \in \mathrm{M}  \mathrm{and}  \mathrm{t} \in \mathcal{T} \\ \mathrm{y}_{\mathrm{III}}^{\mathrm{W}^{+}} & \left\{ \begin{array}{c} 1 & \text{if a W}, \mathrm{H}  \mathrm{at location}  m \text{ is closed at TP}  \mathrm{t}; \\ 0 & \text{otherwise.} \end{array} \right. \\ \mathrm{with}  m \in \mathrm{M}  \mathrm{and}  \mathrm{t} \in \mathcal{T} \\ \mathrm{y}_{\mathrm{III}}^{\mathrm{W}^{+}} & \left\{ \begin{array}{c} 1 & \text{if a D}, \mathrm{C}  \mathrm{at location}  \mathcal{K} \text{ is opened at TP}  \mathrm{t}; \\ 0 & \text{otherwise.} \end{array} \right. \\ \mathrm{with}  \mathcal{K} \in \mathrm{K}  \mathrm{and}  \mathrm{t} \in \mathcal{T} \\ \mathrm{y}_{\mathrm{III}}^{\mathrm{W}^{+}} & \left\{ \begin{array}{c} 1 & \text{if a D}, \mathrm{C}  \mathrm{at location}  \mathcal{K} \text{ is opened at TP}  \mathrm{t}; \\ 0 & \text{otherwise.} \end{array} \right. \\ \mathrm{with}  \mathcal{K} \in \mathrm{K}  \mathrm{and}  \mathrm{t} \in \mathcal{T} \\ \mathrm{y}_{\mathrm{III}}^{\mathrm{W}^{+}} & \left\{ \begin{array}{c} 1 & \text{if a D}, \mathrm{C}  \mathrm{at location}  \mathcal{K} \text{ is opened at TP}  \mathrm{t}; \\ 0 & \text{otherwise.} \end{array} \right. \\ \mathrm{with}  \mathcal{K} \in \mathrm{K}  \mathrm{and}  \mathrm{t} \in \mathcal{T} \\ \mathrm{u}_{\mathrm{IIII}}  \mathrm{if}  \mathrm{a}  \mathrm{D}  \mathrm{c}  \mathrm{at location}  \mathcal{K} \text{ is closed at TP}  \mathrm{t}; \\ \mathrm{o} & \text{otherwise.} \end{array} \right. \\ \mathrm{with}  \mathcal{K} \in \mathrm{K}  \mathrm{and}  \mathrm{t} \in \mathcal{T} \\ \mathrm{u}_{IIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIIII$		
$ \begin{array}{ll} x_{ikt}^{bfc} & \text{Quantity of product i transferred from DC $\mathcal{K}$ to $C.Z$ lduring TP t, with $i \in I$. $\mathcal{K} \in K$. $l \in L$ and $l \in $\mathcal{T}$ \\ y_{lt}^{p_{+}} & \left\{ \begin{array}{ll} & \text{if a plant at location $j$ is opened at TP $t$;} \\ 0 & \text{otherwise.} \\ & \text{with $j \in J$ and $l \in $\mathcal{T}$ \\ \end{array} \right\} \\ y_{lt}^{p_{-}} & \left\{ \begin{array}{ll} & \text{if a plant at location $f$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ & \text{with $j \in J$ and $l \in $\mathcal{T}$ \\ \end{array} \right\} \\ y_{mt}^{W^{+}} & \left\{ \begin{array}{ll} & \text{if a W. H at location $m$ is opened at TP $t$;} \\ 0 & \text{otherwise.} \\ & \text{with $m \in M$ and $l \in $\mathcal{T}$ \\ \end{array} \right\} \\ y_{mt}^{W^{+}} & \left\{ \begin{array}{ll} & \text{if a W. H at location $m$ is opened at TP $t$;} \\ 0 & \text{otherwise.} \\ & \text{with $m \in M$ and $l \in $\mathcal{T}$ \\ \end{array} \right\} \\ y_{mt}^{W^{+}} & \left\{ \begin{array}{ll} & \text{if a W. H at location $m$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ & \text{with $m \in M$ and $l \in $\mathcal{T}$ \\ \end{array} \right\} \\ y_{mt}^{W^{+}} & \left\{ \begin{array}{ll} & \text{if a D. C at location $m$ is opened at TP $t$;} \\ 0 & \text{otherwise.} \\ & \text{with $\mathcal{H} \in K$ and $l \in $\mathcal{T}$ \\ \end{array} \right\} \\ y_{kt}^{D^{+}} & \left\{ \begin{array}{ll} & \text{if a D. C at location $\mathcal{K}$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ & \text{with $\mathcal{K} \in K$ and $l \in $\mathcal{T}$ \\ \end{array} \right\} \\ y_{kt}^{D^{+}} & \left\{ \begin{array}{ll} & \text{if a D. C at location $\mathcal{K}$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ & \text{with $\mathcal{K} \in K$ and $l \in $\mathcal{T}$ \\ \end{array} \right\} \\ y_{kt}^{D^{+}} & \left\{ \begin{array}{ll} & \text{if a D. C at location $\mathcal{K}$ is closed at TP $t$;} \\ 0 & \text{otherwise.} \\ & \text{with $\mathcal{K} \in K$ and $l \in $\mathcal{T}$ \\ \end{array} \right\} \\ u_{ijt} & \left\{ \begin{array}{ll} & \text{if product $i$ is produced at plant $j$ at TP $t$;} \\ 0 & \text{otherwise.} \\ \end{array} \right\} \end{array} \right\}$		
$ y_{jt}^{p+} \begin{cases} 1 & \text{if a plant at location } j \text{ is opened at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } j \in J \text{ and } t \in \mathcal{T} \end{cases} $ $ y_{jt}^{p-} \begin{cases} 1 & \text{if a plant at location } j \text{ is closed at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } j \in J \text{ and } t \in \mathcal{T} \end{cases} $ $ y_{mt}^{W+} \begin{cases} 1 & \text{if a W. H at location } m \text{ is opened at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } m \in M \text{ and } t \in \mathcal{T} \end{cases} $ $ y_{mt}^{W-} \begin{cases} 1 & \text{if a W. H at location } m \text{ is opened at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } m \in M \text{ and } t \in \mathcal{T} \end{cases} $ $ y_{mt}^{W-} \begin{cases} 1 & \text{if a W. H at location } m \text{ is closed at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } m \in M \text{ and } t \in \mathcal{T} \end{cases} $ $ y_{mt}^{W+} \begin{cases} 1 & \text{if a D. C at location } \mathcal{K} \text{ is opened at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \end{cases} $ $ y_{kt}^{D-} \begin{cases} 1 & \text{if a D. C at location } \mathcal{K} \text{ is closed at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \end{cases} $ $ y_{kt}^{D-} \begin{cases} 1 & \text{if a D. C at location } \mathcal{K} \text{ is closed at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \end{cases} $ $ u_{ijt} \begin{cases} 1 & \text{if product } i \text{ is produced at plant } j \text{ at TP } t ; \\ 0 & \text{otherwise.} \end{cases} $	<sup>x</sup> imkt DC	
$ \begin{array}{ll} \begin{array}{l} & 0 & \text{otherwise.} \\ & \text{with } j \in J \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{y}_{jt}^{p-} \begin{cases} 1 & \text{if a plant at location } j \text{ is closed at TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } j \in J \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } m \in M \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } m \in M \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } m \in M \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } m \in M \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } m \in M \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } m \in M \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } m \in M \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } m \in M \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } m \in M \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \begin{array}{0} \end{array}{0} & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \\ \end{array}{0} & \text{with } \mathcal{K} \text{ otherwise.} \\ \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l} \begin{array}{l}$		
$ \begin{aligned} & \text{with } j \in J \text{ and } t \in \mathcal{T} \\ y_{jt}^{P-} & \begin{cases} 1 & \text{if } a \text{ plant } at \text{ location } j \text{ is closed } at \text{ TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } j \in J \text{ and } t \in \mathcal{T} \end{cases} \\ y_{mt}^{W+} & \begin{cases} 1 & \text{if } a \text{ W. H } at \text{ location } m \text{ is opened } at \text{ TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } m \in M \text{ and } t \in \mathcal{T} \end{cases} \\ y_{mt}^{W-} & \begin{cases} 1 & \text{if } a \text{ W. H } at \text{ location } m \text{ is closed } at \text{ TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } m \in M \text{ and } t \in \mathcal{T} \end{cases} \\ y_{mt}^{W-} & \begin{cases} 1 & \text{if } a \text{ W. H } at \text{ location } m \text{ is closed } at \text{ TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } m \in M \text{ and } t \in \mathcal{T} \end{cases} \\ y_{triv}^{D-} & \begin{cases} 1 & \text{if } a \text{ D. C } at \text{ location } \mathcal{K} \text{ is opened } at \text{ TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \end{cases} \\ y_{kt}^{D-} & \begin{cases} 1 & \text{if } a \text{ D. C } at \text{ location } \mathcal{K} \text{ is closed } at \text{ TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \end{cases} \\ y_{kt}^{U-} & \begin{cases} 1 & \text{if } a \text{ D. C } at \text{ location } \mathcal{K} \text{ is closed } at \text{ TP } t; \\ 0 & \text{otherwise.} \\ & \text{with } \mathcal{K} \in K \text{ and } t \in \mathcal{T} \end{cases} \\ u_{ijt} & \begin{cases} 1 & \text{if } p \text{ roduct } t \text{ is produced } at \text{ plant } j \text{ at TP } t; \\ 0 & \text{otherwise.} \end{cases} \end{cases}$	y <sub>jt</sub> .	
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0 otherwise.	11	(1 if product <i>i</i> is produced at plant <i>i</i> at TP <i>t</i> :
	uijt	

$\mathbf{z}_{jmt}^{PW}$	$\begin{cases} 1 & \text{if plant } j \text{ supplies W. H } m \text{ at TP } t; \\ 0 & \text{otherwise.} \end{cases}$
	with $j \in J$ . $m \in M$ and $t \in \mathcal{T}$
$\mathbf{z}_{\mathrm{mkt}}^{\mathrm{WD}}$	$\begin{cases} 1 & \text{if W. H } m \text{ supplies D. C } \mathcal{K} \text{ at TP } t; \\ 0 & \text{otherwise.} \end{cases}$
	with $m \in M$ . $\mathcal{K} \in K$ and $t \in \mathcal{T}$
$\mathbf{z}_{\mathbf{klt}}^{\mathrm{DC}}$	(1 if D. C $\mathcal{K}$ supplies C. Z l at TP t;
<sup>2</sup> klt	o therwise.
	with $\mathcal{K} \in K$ . $l \in L$ and $t \in \mathcal{T}$
$w_{jst}^{P-}$	1 if plant j was opened at TP s and closed at TP t 0 otherwise.
	with $j \in J$ and $s$ and $t \in \mathcal{T}$
Рт	
w <sup>P+</sup> <sub>jst</sub>	1 if plant j was opened at TP s and is still open at TP t 0 otherwise.
	with $\mathcal{K} \neq J$ and s and t $\in \mathcal{T}$
147	
w <sup>W–</sup> <sub>mst</sub>	$\begin{cases} 1 & \text{if W. H } m \text{ was opened at TP s and closed at 'TP t,} \\ 0 & \text{otherwise.} \end{cases}$
	with $m \in M$ and s and $t \in T$
147.1	
$w_{mst}^{W+}$	$\begin{cases} 1 & \text{if W. H } m \text{ was opened at time period s and is still open at TP f;} \\ 0 & \text{otherwise.} \end{cases}$
	with $m \in M$ and s and $t \in \mathcal{T}$
D	
$w_{kst}^{D+}$	$\begin{cases} 1 & \text{if D. C } \mathcal{K} \text{ was opened at TP s and is still open at TP t;} \\ 0 & \text{otherwise.} \end{cases}$
	with $\mathcal{K} \in K$ and s and $t \in \mathcal{T}$
D-	
w <sub>kst</sub> <sup>D-</sup>	1 If D. C. R. was opened at TP s and closed at TP t; 0 otherwise.
	with $\mathcal{K} \in K$ and s and $t \in \mathcal{T}$
ncp <sub>t</sub> rp <sub>t</sub>	New capital entries from shareholders during the TP t, with $t \in \mathcal{T}$ Repaid amount to the bank during the TP t, with $t \in \mathcal{T}$
CA <sub>t</sub>	Current assets during the TP t, with $t \in \mathcal{T}$
bt	Bank debts during the TP t, with $t \in \mathcal{T}$
DPV <sub>t</sub>	Depreciation value at TP t, with $t \in \mathcal{T}$
CS <sub>t</sub>	Cost of sales at time period t, with $t \in \mathcal{T}$
C <sub>t</sub> FAI <sub>t</sub>	Cash during the time period t, with $t \in T$ Investment of fixed assets during the TP t, with $t \in T$
FAD <sub>t</sub>	Divestment of fixed assets during the TP t, with $t \in \mathcal{T}$
IP <sub>t</sub>	Interest paid(interest expense) during the TP t, with $t \in \mathcal{T}$
IC <sub>t</sub>	Cost of holding inventory during the TP t, with $t \in \mathcal{T}$
LTD <sub>t</sub>	Long-term debt during the TP t, with $t \in \mathcal{T}$
IV <sub>t</sub>	Value of inventory at TP t, with $t \in \mathcal{T}$
NOI <sub>t</sub> PC <sub>t</sub>	Non-operating income during the TP t, with $t \in \mathcal{T}$ Cost of production during the TP t, with $t \in \mathcal{T}$
NFA <sub>t</sub>	Net fixed assets during the TP t, with $t \in \mathcal{T}$
REV <sub>t</sub>	Revenues from sales during the TP t, with $t \in \mathcal{T}$
TCt	Cost of transportation during the TP t, with $t \in \mathcal{T}$

(DC: Distribution Center, WH: Warehouse, CZ: Customer Zone, TP: Time Period)

#### **Objective Function**

As presented in the previous sections, strategic and operational decisions in supply chain management impact company's financial performance and, consequently, the financial value created for shareholders. Shareholder value is the value delivered to the equity owners of a corporation; it is created when earnings exceed the total costs of invested capita. In accordance with it, in this work, the shareholder value analysis (SVA) as an objective function has been applied in order to maximize shareholder value created with the supply chain network configuration.

SVA calculates the shareholder value (or equity value) by deducting the long-term liabilities value at the end of the project lifetime (LTDT) from the firm value for the time period under analysis. Equation (1) shows the objective function. (1)

$$maxSVA = DFCF - LTD_T$$

Now, we explain DFCF, LTDT and other components involved to calculate them.

As given by equation (2), the discounted free cash flow (DFCF) is obtained by adding the summation of the discounted free cash flows (FCFFt) to the terminal value of a firm (VT) over the planning period. (2)

$$DFCF = \sum_{t \in \mathcal{T}} \frac{FCFF_t}{(1+r_t)^t} + \frac{V_T}{(1+r_T)^T}$$

Note that T shows the number of time periods of the planning horizon.  $(r_T)$  is a parameter to show the discount rate and cost of capital and represents the time value of money and investment risk.  $V_T$ shows the final value of the firm, that is, the value of total future cash flows, beyond the planning horizon. In this study,  $V_T$  is calculated by the growing perpetuity model, which presumes that free cash flows grow at a fixed rate (g) constantly. Equation (3) shows how the terminal value of the firm is calculated.

$$V_T = \frac{FCFF_{T+1}}{r_T - g} \qquad \forall t \in \mathcal{T}$$
(3)

Because we estimate  $FCFF_{T+1}$  based on an adjustment to FCFF from the last period of the planning horizon, making it grow at the fixed rate q (see Equation (4)), therefore modification in the FCFF is needed since we have assumed stability beyond the planning horizon. This means that non-operating income is considered zero and new investments will be offset by depreciation.

$$FCFF_{T+1} = [(REV_T - CS_T - DPV_T)(1 - TR_T) - \Delta WC_T](1 - g)$$
(4)

# • Free Cash Flow To The Firm (FCFF)

The free cash flow to the firm represents the quantity of cash flow from operations after accounting for depreciation expenses, taxes, working capital, and investments. It is calculated by equation (5) which deducts the net fixed asset investment (FAIt – DPVt) and the changes in working capital ( $\Delta WCt$ ) from the operating income after taxes. In this equation, (REVt) is the revenue, the non-operating income (NOI), the cost of sales (CSt), and depreciation (DPVt).

Note that operating earnings are a taxable revenue; it means that in order to get net income, taxes must be subtracted from incomes. The tax rate (TRt) is according to current tax laws.

As shown in equation (5), depreciation is considered a cost because it decreases taxable income, and it is not related to a real payment (cash outflow). This means that in order to calculate the  $(FCFF_t)$ , depreciation has to be added again.

$$FCFF_t = (REV_t + NOI_t - CS_t - DPV_t)(1 - TR_t) - (FAI_t - DPV_t) - \Delta WC_t. \quad \forall t \in \mathcal{T}$$
(5)

Next, the free cash flow components will be explained in more detail.

Revenues

The revenues  $(REV_t)$  coming from selling products/providing services are calculated as shown in equation (6):

$$REV_{t} = \sum_{i \in I.I \in L} PR_{ilt}O_{ilt} \qquad \forall t \in \mathcal{T}$$
(6)

## • Non-Operating Income

The non-operating income  $(NOI_t)$  is the portion of a firm's income that is derived from activities not related to its core business operations including gains/losses from property or property sales [41]. Therefore, in a period that physical assets are not sold, the non-operating income will be zero. In this model, we have assumed that if there is a decision to close a facility, it will be sold. As shown in equation (7), the  $NOI_t$  consists of three income components derived from the sale of plants, warehouses, or distribution centers. The profit or loss from selling a plant is the difference between the cash inflow resulting from alienation and calculated by the market price of the plant for the period  $(A_{jt}^{P})$  minus the

$$NOI_{t} = \sum_{j \in J} (A_{jt}^{P} - C_{jt}^{P-}) y_{jt}^{P-} - \sum_{S=1}^{t} C_{js}^{P+} (1 - ACDPR_{st}) w_{jst}^{P-} + \sum_{m \in M} (A_{mt}^{W} - C_{mt}^{W-}) y_{mt}^{W-} - \sum_{S=1}^{t} C_{ms}^{W+} (1 - ACDPR_{st}) w_{mst}^{W-} + \sum_{k \in K} (A_{kt}^{D} - C_{kt}^{D-}) y_{kt}^{D-} - \sum_{S=1}^{t} C_{ks}^{D+} (1 - ACDPR_{st}) w_{kst}^{D-}. \quad \forall t \in \mathcal{T}$$

$$(7)$$

#### • Cost of Sales

As expressed in equation (8), the cost of sales (CSt) represents all the expenditures that are needed for producing and delivering products to customers. It consists of four parts: costs of production (PCt), costs of transportation (TCt), costs of inventory holding (ICt), and changes in inventory value  $(IV_t - IV_{t-1})$ .

$$CS_t = PC_t + TC_t + IC_t - (IV_t - IV_{t-1}) \quad \forall t \in \mathcal{T}$$
(8)

Production costs have a fixed and variable part, as follows:

$$PC_{t} = \sum_{i \in I} \sum_{j \in J} \left( C_{ijt}^{VPP} p_{ijt} + C_{ijt}^{FPP} u_{ijt} \right) \quad \forall t \in \mathcal{T}$$

$$\tag{9}$$

In equation (9),  $C_{ijt}^{\nu PP}$  and  $C_{ijt}^{\nu PP}$  represent the variable and fixed cost of production, respectively, at plant j, in time period t. Also,  $p_{ijt}$  is the quantity of product i produced in plant j at time period t and  $u_{ijt}$  is a binary value which has the value 1 if the product i is produced in plant j at the time period t and zero otherwise.

Equation (10) shows the transportation costs which include three parts with fixed and variable costs; these costs are incurred during transporting products from plants to warehouses, distribution centers, and customer zones.

$$TC_{t} = \sum_{i \in I} \sum_{j \in J} \sum_{m \in M} \left( C_{ijmt}^{VTPW} x_{ijmt}^{PW} + C_{ijmt}^{FTPW} z_{jmt}^{PW} \right)$$

$$+ \sum_{i \in I} \sum_{m \in M} \sum_{k \in K} \left( C_{imkt}^{VTWD} x_{imkt}^{WD} + C_{imkt}^{FTWD} z_{mkt}^{WD} \right)$$

$$+ \sum_{i \in I} \sum_{k \in K} \sum_{l \ k \ L} \left( C_{iklt}^{VTDC} x_{iklt}^{DC} + C_{iklt}^{VTDC} z_{ikt}^{DC} \right) \quad \forall t \in \mathcal{T}$$

$$(10)$$

Equation (11) shows the total inventory holding costs and it has three parts related to the average quantity held at each facility (plants, warehouses, and distribution centers) during the time period.

$$IC_{t} = \sum_{i \in I} \sum_{j \in J} \left( C_{ijt}^{IP} \frac{q_{ijt}^{P} + q_{ijt-1}^{P}}{2} \right) + \sum_{i \in I} \sum_{m \in M} \left( C_{imt}^{IW} \frac{q_{imt}^{W} + q_{imt-1}^{W}}{2} \right) + \sum_{i \in I} \sum_{k \in K} \left( C_{ikt}^{ID} \frac{q_{ikt}^{D} + q_{ikt-1}^{D}}{2} \right) \qquad \forall t \in \mathcal{T}$$

$$(11)$$

Based on accounting principles, the value of inventory is calculated by historical cost; in this case, equation (13) shows the production price for each product at each time period.

$$IV_{t} = \sum_{i \in J} \sum_{j \in J} \sum_{m \in M} \sum_{k \in K} C_{ijt}^{VPP} (q_{ijt}^{P} + q_{imt}^{W} + q_{ikt}^{D}) \qquad \forall t \in \mathcal{T}$$
(12)

#### • Depreciation

The value of fixed assets such as plants, warehouses, and distribution centers should be modified for devaluation. Based on this accounting rule, the total depreciation value at the time period t  $(DPV_t)$  is calculated by the summation of the depreciated value of plants, warehouses, and distribution centers that are operating during the time period t [42]. In this model, we assume that fixed assets existing before the planning horizon have been completely depreciated.

$$DPV_{t} = \sum_{j \in J} \sum_{s=1}^{t} DPR_{st}C_{js}^{P+}W_{jst}^{P+} + \sum_{m \in M} \sum_{s=1}^{t} DPR_{st}C_{ms}^{W+}W_{mst}^{W+}$$

$$+ \sum_{k \in K} \sum_{s=1}^{t} DPR_{st}C_{ks}^{D+}W_{kst}^{W+} \quad \forall t \in \mathcal{T}$$
In equation (12)  $W_{st}^{P+}W_{st}^{W+} = 0$  binom unichlap set to 1 if a facility append at the time

In equation (13),  $W_{jst}^{P+}$ ,  $W_{mst}^{W+}$ , and  $W_{kst}^{W+}$  are binary variables set to 1 if a facility opened at the time period s is still open at the time period t.

#### • Fixed Assets Investment

Fixed assets are long-term tangible properties that a firm owns and utilizes in its operations to generate income [52]. In our model, (*FAIt*) represents fixed assets investment at the time period t which is the needed finance to establish facilities (plants, warehouses, and distribution centers) in the time period t:

$$FAI_{t} = \sum_{j \in J} C_{jt}^{P+} y_{jt}^{P+} + \sum_{m \in M} C_{mt}^{W+} y_{mt}^{W+} + \sum_{k \in K} C_{kt}^{D+} y_{kt}^{D+} \quad \forall t \in \mathcal{T}$$
(14)

### Changes In Working Capital

The changes in working capital ( $\Delta WCt$ ) are obtained by the difference between the working capital in two successive periods. The working capital is calculated by adding receivable accounts to the value of inventory and deducting payable accounts. It is assumed that the accounts receivable and the accounts payable are a portion of the revenues and of the operational costs, respectively, at the end of time period t. Therefore,  $\Delta WC_t$  can be obtained as follows:

$$\Delta WC_{t} = (\alpha_{t} REV_{t} - \alpha_{t-1} REV_{t-1}) + (IV_{t} - IV_{t-1}) - [\mu_{t}(PC_{t} + TC_{t} + IC_{t}) - \mu_{t-1}(PC_{t-1} + TC_{t-1} + IC_{t-1})] \quad \forall t \in \mathcal{T}$$
(15)

Note that  $\alpha_t$  and  $\mu_t$  represent the amount of revenues and payments (in percentage), respectively, which are outstanding in the current time period and defined by the company policy on payables and receivables.

### • Long-Term Liabilities Calculation

Long-term liabilities are represented by long-term debt  $(LTD_t)$  which is incurred to finance fixed assets investments, and calculated by equation (16). This is a function of the previous period debt value and current period loans  $(B_t)$  and bank repayments  $(RP_t)$ .

$$LTD_t = LTD_{t-1} + B_t - RP_t \qquad \forall t \in \mathcal{T}$$
(16)

# **3.2 The Model Constraints**

The model constraints can be categorized into two groups that should be satisfied as financial constraints and operational constraints.

# **3.2.1 Financial Constraints**

Financial ratios are one of the beneficial parts of financial statements which prepare standard tools to evaluate the overall financial condition of a company's performance, efficiency, liquidity, and leverage. The financial constraints enforce financial ratios in order to support the financial health of the corporation. This study used the ratio categories defined by Jin et al. [40] ,Breally et al. [45] and Borges at al. [46] and sets upper/lower limits value for them.

### **3.2.1.1 Performance Ratios**

Performance ratios measure the financial performance of the company [47]. In this study we considered two common measures, that is, return on equity (ROE) and return on assets (ROA). Equations (20) and (21) present the least values of  $ROE_t$  and  $ROA_t$  that have to be satisfied in each time duration.

# • Return on Equity (ROE)

Return on equity illustrates the marginal investment income of shareholders and is calculated by dividing the net income by shareholders' equity. The net income  $(NI_t)$  is what the business has left over after all expenses. Also,  $(EBIT_t)$  is named earnings before interests and taxes. They are calculated by equations (17) and (18):

$$EBIT_t = REV_t + NOI_t - CS_t - DPV_t \qquad \forall t \in \mathcal{T}$$
(17)

$$NI_t = (EBIT_t - IR_t * LTD_t)(1 - TR_t) \qquad \forall t \in \mathcal{T}$$
(18)

$$E_t = E_{t-1} + (EBIT_t - IR_t * LTD_t)(1 - TR_t) + NCP_t \quad \forall t \in \mathcal{T}$$
(19)

According to the previous descriptions, the *ROE* equation can be written as:  $(EBIT_t - IR_t * LTD_t)(1 - TR_t)$ 

$$\frac{BIT_t - IR_t * LTD_t)(1 - TR_t)}{E_t} \ge ROE_t \qquad \forall t \in \mathcal{T}$$
(20)

## • Return on Assets (ROA)

The return on assets (ROA) is a measure of financial performance and represents the percentage of how profitable a company's assets are for generating revenue. It is calculated by equation (21). Note that in this equation, (NOPAT),  $(NFA_t)$  and  $(CA_t)$  are the net operating profit after taxes, net fixed assets, and the current assets, respectively.

$$\frac{EBIT_t(1 - TR_t)}{+CA_t} \ge ROA_t \qquad \forall t \in \mathcal{T}$$
(21)

Equation (22) shows how the current net fixed assets  $(NFA_t)$  are calculated from those of the previous period, which are increased/decreased in an amount equal to the value of the investment  $(FAI_t)$ /divestment  $(FAD_t)$  in fixed assets of depreciation in time period t, as follows:

$$NFA_t = NFA_{t-1} + FAI_t - FAD_t - DPV_t \qquad \forall t \in \mathcal{T}$$
 (22)

Investment expresses the ownership of fixed assets, while divestment represents sales fixed assets. In this study, we have assumed that before the planning horizon, existing assets were completely depreciated, also ( $FAD_t$ ) shows the net value (accounting value of the asset after depreciation) of the assets bought during the planning horizon and until-time period t:

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$$FAD_{t} = \sum_{s=1}^{t} \left[ \sum_{j \in J} C_{js}^{P+} (1 - ACDPR_{st}) W_{jst}^{P-} + \sum_{m \in M} C_{ms}^{W+} (1 - ACDPR_{st}) W_{mst}^{W-} + \sum_{k \in K} C_{ks}^{D+} (1 - ACDPR_{st}) W_{kst}^{D-} \right] \quad \forall t \in \mathcal{T}$$

$$(23)$$

 $DPV_t$  and  $FAI_t$  reffer to equations (13) and (14). Current assets are any assets that can be expected to be sold, consumed, or exhausted by the operations of a business. In this study, current assets  $(CA_t)$  consist of : cash and banks  $(C_t)$ ; accounts receivable, here represented as a percent of the revenues  $(\alpha_t REV_t)$ , and inventory value  $(IV_t)$ :

$$CA_{t} = C_{t} + \alpha_{t} REV_{t} + IV_{t}$$
  $\forall t \in \mathcal{T}$  (24)

Equation (25) represents the cash function at each time period ( $C_t$ ). The cash at time period t is the available cash in the previous period, cash inflows, and cash outflows. Cash inflows come from different sources:

- Customer and receivables( $\alpha_{t-1}REV_{t-1}$ ) and product sales( $(1 \alpha_t)REV_t$ ),
- Fixed assets sales,
- New capital entries  $(NCP_t)$ ,
- Loans of the period to finance investments  $(B_t)$ .
- Also, cash outflows come from different sources:
- Repayments of debt to the bank  $(RP_t)$ ,
- Costs of interest; are calculated by multiplying an interest rate by the debt of the period  $(IR_tLTD_t)$ ,
- Accounts payable  $(\mu_{t-1}(PC_{t-1} + TC_{t-1} + IC_{t-1}))$  and payments to suppliers  $((1 \mu_t)(PC_t + TC_t + IC_t))$ ,
- Payment of income taxes of the previous period,
- The amount invested in new assets.

$$\begin{split} C_{t} &= C_{t-1} + \alpha_{t-1} \text{REV}_{t-1} + (1 - \alpha_{t}) \text{REV}_{t} \\ &+ \left[ \sum_{j \in J} (A_{jt}^{P} - C_{jt}^{P-}) y_{jt}^{P-} + \sum_{m \in M} (A_{mt}^{W} - C_{mt}^{W-}) y_{mt}^{W-} + \sum_{k \in K} (A_{kt}^{D} - C_{kt}^{D-}) y_{jt}^{D-} \right] \\ &+ NCP_{t} + B_{t} - RP_{t} - IR_{t}LTD_{t} - \mu_{t-1} (\text{PC}_{t-1} + \text{TC}_{t-1} + \text{IC}_{t-1}) - (1 - \mu_{t})(\text{PC}_{t} + M_{t-1}^{D-}) + \frac{1}{2} \left[ (P_{t-1}^{D-} + P_{t-1}^{D-}) + (P_{t-1}^{D-}) +$$

 $TC_t + IC_t) - TR_{t-1}(EBIT_{t-1} - IR_{t-1}LTD_{t-1}) - FAI_t$ 

Note that  $(\text{REV}_t)$  is defined in equation (6) and income taxes are due only if there is a taxable income.

 $\forall t \in \mathcal{T}$ 

# 3.2.1.2 Efficiency Ratios

Efficiency ratios measure how well the company utilizes its different assets. These ratios allow the company to evaluate its efficiency. In this study, we considered profit margin (PMR) and asset turnover (ATR) as efficiency ratios [48].

# Profit Margin (PMR)

Profit margin measures the proportion of sales that finds its way into profits. It is defined as the ratio of net income to sales and must attain a minimum value at each time duration  $(PMR_t)$ ; its ratios are given by equation (26):

$$\frac{(EBIT_t - IR_t LTD_t)(1 - TR_t)}{REV_t} \ge PMR_t \qquad \forall t \in \mathcal{T}$$
(26)

# • Asset Turnover (ATR)

Asset turnover displays the incomes generated per monetary unit of total assets, measuring how hard the firm's assets are working. It is given by the ratio of sales revenue to total assets in time period t. Equation (27) shows asset turnover ratios.

(25)

(28)

(29)

$$\frac{REV_t}{NFA_t + CA_t} \ge ATR_t \qquad \forall t \in \mathcal{T}$$
(27)

#### **3.2.1.3 Liquidity Ratios**

Liquidity ratios determine how quickly assets can be converted into cash. The liquidity ratios analysis helps the company to evaluate its ability to keep more liquid assets [49].

### • Current Ratio (CUR)

Current ratio is the ratio of current assets to its current liabilities and must attain a minimum value  $(CUR_t)$ . Equation (28) shows current ratio constraint:

$$\frac{CA_t}{STD_t} \ge CUR_t \qquad \forall t \in \mathcal{I}$$

As in our model, short-term loans are negligible, thus short-term debt  $(STD_t)$  is due to accounts payable and taxes, as follows:

$$STD_t = \mu_t(PC_t + TC_t + IC_t) + (EBIT_t - IR_tLTD_t)TR_t \quad \forall t \in T$$

# • Quick Ratio (QR)

Quick ratio is the ratio of current assets (except inventory) to its current liabilities which must satisfy a threshold value  $(QR_t)$  as follows:

$$\frac{C_t + \alpha_t REV_t}{STD_t} \ge QR_t \quad \forall t \in \mathcal{T}$$
(30)

• Cash Ratio (CR)

The cash ratio is the ratio of its current liabilities which must satisfy a threshold value  $(CR_t)$  as follows:

$$\frac{C_t}{STD_t} \ge CR_t \qquad \forall t \in \mathcal{T}$$
(31)

### **3.2.1.4 Leverage Ratios**

Leverage ratios assess the firm's ability to meet the financial obligations [50].

# • Long Term Debt to Equity Ratio (LTDR)

It provides an indication on how much debt a company is using to finance its assets. This ratio must be below a given limit:

$$\frac{LTD_t}{E_t} \ge LTDR_t \quad \forall t \in \mathcal{T}$$
(32)

#### Total Debt Ratio (TDR)

The total debt ratio provides an indication on the total amount of debt relative to assets; it is obtained by dividing total debt by total assets and must be lower a given limit:

$$\frac{STD_t + LTD_t}{NFA_t + CA_t} \ge LTD_t \quad \forall t \in \mathcal{T}$$
(33)

# • Cash Coverage Ratio (CCR)

The cash coverage ratio measures the firm's capacity to meet interest payments in cash, thus it must satisfy a given lower limit:

$$\frac{\text{EBIT}_{t} + DPR_{t}}{\text{IR}_{t}LTD_{t}} \ge CCR_{t} \quad \forall t \in \mathcal{T}$$
(34)

### 3.2.1.5 Other Financial Constraints

Equation (35) shows that new capital entries are limited to the quantity that company partners desire to invest in the company:

$$NCP_t \le CP_t \qquad \forall t \in \mathcal{T} \tag{35}$$

Commonly, banks constrain the repayment (RPt) to be at least the interest costs to barricade a growing debt:

$$RP_t \ge IR_t LTD_t \qquad \forall t \in \mathcal{T}$$

Furthermore, because repayments (RPt) are part of the debt, in each period they must satisfy the constraint (37):

 $RP_t \ge LTD_t \qquad \forall t \in \mathcal{T}$ 

For each time period, the company may limit the amount borrowed to the percentage of the value of investments, as follows:

 $B_{t} \leq \gamma_{t} FAI_{t} \qquad \forall t \in \mathcal{T}$ 

# 3.2.2 Operational Constraints

# 3.2.2.1 At The Plant Level

Equations (39) and (40) show that production constraints enforce the production quantities in each time period, each plant, and for each product to be in a specified range.

$$p_{ijt} \leq P_{ij}^{max} \sum_{\substack{s=0 \\ t}} w_{jst}^{P+} \qquad \forall i \in I. j \in J. \text{ and } t \in \mathcal{T}$$

$$p_{ijt} \leq P_{ij}^{min} \sum_{\substack{s=0 \\ t}} w_{jst}^{P+} \qquad \forall i \in I. j \in J. \text{ and } t \in \mathcal{T}$$

$$(39)$$

$$(40)$$

Production quantities are also collectively limited by the available quantity of each time period, each resource, and each plant (constraint (41). Note that the availability of the resources is fixed over time.

$$\sum_{i \in I}^{t} \rho_{ije} p_{ijt} \le R_{je} \qquad \forall j \in J. e \in E. \text{ and } t \in \mathcal{T}$$

$$(41)$$

Because production has a fixed cost, in equation (42), a binary variable (uijt) is used to show the existence of production that assumes the value 1 whenever some non-zero quantity is produced.  $p_{iit} \le Mu_{iit}$   $\forall i \in I, j \in J$ . and  $t \in \mathcal{T}$  (42)

Plants might send all or part of the products to the warehouses that have been established. This is stated by equations (43) and (44):

$$\forall i \in I \text{ and } t \in T \tag{43}$$

(12)

(36)

(37)

(38)

(20)

$$\sum_{i \in I} \sum_{j \in J} x_{ijmt}^{PW} \le M \sum_{s=0}^{t} w_{mst}^{W+}$$

$$m \in M$$
. and  $t \in \mathcal{T}$ 

The total production quantity sent by each plant to each warehouse must satisfy the transport capacity, which is shown by equation (45) (Note that M is a sufficiently large number).

A

$$\sum_{i \in I} x_{ijmt}^{PW} \le Q_{jm}^{PW}. Z_{jmt}^{PW} \qquad \forall j \in J. m \in M. \text{ and } t \in \mathcal{T}$$

$$\tag{45}$$

Equation (46) is for inventory balance at each plant and each product in each time period. The available inventory is calculated by the available inventory in the previous period, plus the produced quantity

(48)

in the current period minus the quantity sent to warehouses.

$$q_{ijt}^{P} = q_{ijt-1}^{P} + p_{ijt} - \sum_{m \in M} x_{ijmt}^{PW} \quad \forall i \in I. j \in J. \text{ and } t \in \mathcal{T}$$

$$(46)$$

Equation (47) shows that at each plant and in each time period, inventory for each product is limited.

$$q_{ijt}^{P} \le I_{ijt}^{max}$$
  $\forall i \in I, j \in J. and t \in \mathcal{T}$  (47)

Finally, the proper auxiliary variables associated with the closing/remaining open status of the facilities should be set to confirm the accuracy of the opening and closing decisions in the model. During the whole planning period, if a plant was not initially open, it can only be opened at most once (equation (48)).

$$\sum_{\mathbf{t}\in\mathcal{T}}y_{j\mathbf{t}}^{\mathbf{P}+} \leq 1 \qquad \forall \mathbf{j}\in \mathbf{J}$$

Throughout the planning period, a plant can be closed at most once if it was opened before (equations (49) and (50)).

$$\sum_{t \in \mathcal{T}} y_{jt}^{P-} \le 1 \qquad \forall j \in J \qquad (49)$$

$$y_{jt}^{P-} \le \sum_{s=0}^{t-1} y_{js}^{P+} \qquad \forall j \in J \text{ and } t \in \mathcal{T} \qquad (50)$$
It is impossible for a plant to be opened and closed in the same time period (equation (51))

$$y_{jt}^{P+} + y_{jt}^{P-} \le 1 \qquad \forall j \in J \text{ and } t \in \mathcal{T}$$
(51)

Equation (52) illustrates that if a plant was opened in the time period s and then closed in the time period t, therefore all decision variables: opening  $(y_{js}^{P+})$ , closing  $(y_{jt}^{P-})$ , and closing status  $(w_{jst}^{P-})$  should be set to 1.

$$y_{js}^{P+} + y_{jt}^{P-} \le w_{jst}^{P-} + 1$$
  $\forall j \in J. \ s = 0. \dots T - 1. \text{ and } t = s + 1. \dots T$  (52)

If only a closing decision was made, the closing status variable would be set to 1:

$$w_{jst}^{P-} \le y_{jt}^{P-}$$
  $\forall j \in J. \ S = 0. ... T - 1. \ And \ t = s + 1. ... T$  (53)

Also, the opening status variable  $(w_{ist}^{P+})$  would be set to 1 if an opening decision was made:

$$w_{jst}^{P+} \le y_{js}^{P+}$$
  $\forall j \in J. \ s \in \mathcal{T}. \ and \ t = s. ... \ \mathcal{T}$  (54)

If a plant was opened in the time period s and is yet open in the time period t, in any of the periods in the internal s+1 and t, a closing decision would be impossible:

$$w_{jst}^{P+} - y_{js}^{P+} + \sum_{v=s+1}^{t} y_{jv}^{P-} \le 0 \qquad \forall j \in J. \ s = 0....\mathcal{T} - 1. \ \text{and} \ t = s + 1....\mathcal{T}$$
(55)

# 3.6.2.2 At The Warehouse Level

Equations (56) and (57) show that the stored quantities in each warehouse for each product and time period to be within a pre-specified range.

$$\sum_{i \in I} q_{imt}^{W} \le W_m^{max} \sum_{s=0}^t W_{mst}^{W+} \qquad \forall m \in M \text{ and } t \in \mathcal{T}$$
<sup>(56)</sup>

$$\sum_{i} q_{imt}^{W} \ge W_{m}^{\min} \sum_{s=0}^{t} W_{mst}^{W+} \qquad \forall m \in M \text{ and } t \in \mathcal{T}$$
(57)

Active warehouses might send all or part of their products to distribution centers in operation as stated by equations (58) and (59).

$$\sum_{i \in I} \sum_{k \in K} x_{imkt}^{WD} \le M \sum_{s=0}^{t} W_{mst}^{D+}. \qquad \forall m \in M \text{ and } t \in \mathcal{T}$$

$$\sum_{i \in I} \sum_{m \in M} x_{imkt}^{WD} \le M \sum_{s=0}^{t} W_{kst}^{D+} \qquad \forall k \in K \text{ and } t \in \mathcal{T}$$
(58)
(59)

Equation (60) shows that the total quantity sent by warehouses to distribution centers in each time period, if any, must satisfy the transport capacity.

$$\sum_{i \in I} x_{imkt}^{WD} \le Q_{mk}^{WD} Z_{mkt}^{WD} \qquad \forall m \in M. \ k \in K \ and \ t \in \mathcal{T}$$
Foundation (61) is for inventory belonge at werehouses and shows that for each werehouse and see

Equation (61) is for inventory balance at warehouses and shows that for each warehouse and each product in each time period, the available inventory is calculated by the available inventory in the previous period plus the quantity received from the plants in the current period minus the quantity sent to distribution centers.

$$q_{\text{imt}}^{W} = q_{\text{imt}-1}^{W} + \sum_{j \in J} x_{ijmt}^{PW} - \sum_{k \in K} x_{imkt}^{WD} \quad \forall i \in I. \ m \in M. \ k \in K \ and \ t \in \mathcal{T}$$
(61)

Moreover, for each product, safety stock is defined in each time period at each warehouse (see equation (62)).

$$q_{\text{imt}}^{W} \ge SS_{\text{imt}}^{w} \sum_{s=0}^{t} W_{\text{mst}}^{W+} \qquad \forall i \in I. \ m \in M. \ k \in K \ and \ t \in \mathcal{T}$$

$$(62)$$

Now the proper auxiliary variables associated with the closing / remaining open status of the facilities should be set to confirm the accuracy of the opening and closing decisions in the model. Equations (63) to (66) show that during the whole planning period, firstly, if a warehouse was not initially open, it could only be opened at most once. Secondly, it also could be closed at most once if it was opened before. Finally, a warehouse cannot be opened and closed in the same time period.

$$\sum_{t \in \mathcal{T}} y_{mt}^{W+} \le 1 \qquad \forall m \in M$$
(63)
$$\sum_{t \in \mathcal{T}} y_{mt}^{W-} \le 1 \qquad (64)$$

$$\sum_{t \in \mathcal{T}} y_{mt} \leq 1 \qquad \forall m \in M \text{ and } t \in \mathcal{T}$$

$$(65)$$

Equation (67) illustrates that if a plant was opened in the time period s then closed in the time period t, therefore all decision variables: opening  $(y_{ms}^{W+})$ , closing  $(y_{mt}^{W-})$ , and closing status  $(w_{mst}^{W-})$  should be set to 1.

$$y_{ms}^{W+} + y_{mt}^{W-} \le w_{mst}^{W-} + 1 \quad \forall m \in M. \ s = 0.... \ T - 1. \ and \ t = s + 1.... \ T$$
 (67)

If only a closing decision was made, a closing status variable would be set to 1:

$$W_{\text{mst}}^{\text{W}-} \le y_{\text{mt}}^{\text{W}-} \qquad \forall m \in M. \, s = 0. \dots \mathcal{T} - 1. \, and \, t = s + 1. \dots \mathcal{T}$$

$$\tag{68}$$

The opening status variable  $(W_{mst}^{W+})$  would be set to 1 if an opening decision was made:

$$W_{mst}^{W+} \le y_{ms}^{W+} \qquad \forall m \in M. \, s \in \mathcal{T}. \, and \, t = s + 1. \dots \mathcal{T}$$
(69)

If a warehouse was opened in the time period s and is yet open in the time period t, in any of the periods in the internal s+1 and t, a closing decision is impossible:

$$W_{mst}^{W+} - y_{ms}^{W+} + \sum_{v=s+1}^{t} y_{mv}^{W-} \le 0 \qquad \forall m \in M. \, s = 0....\mathcal{T} - 1. \, and \, t = s + 1....\mathcal{T}$$
(70)

# 3.6.2.3 At the Distribution Center Level

t

Equations (71) and (72) show that the stored quantities in each distribution center for each product and time period must be within a pre-specified range.

$$\sum_{i \in I} q_{ikt}^{D} \le D_{k}^{max} \sum_{\substack{s=0 \\ t}} W_{kst}^{D+} \qquad \forall k \in Kand \ t \in \mathcal{T}$$

$$\sum_{i \in I} q_{ikt}^{D} \ge D_{k}^{min} \sum_{\substack{s=0 \\ s=0}}^{t} W_{kst}^{D+} \qquad \forall k \in Kand \ t \in \mathcal{T}$$
(72)

Active distribution centers might send all or part of their products to customer zones as stated by equation (73).

$$\sum_{i \in I} \sum_{l \in L} x_{iklt}^{DC} \le M \sum_{s=0}^{t} W_{kst}^{D+} \qquad \forall k \in K \text{ and } t \in \mathcal{T}$$
(73)

Equation (74) shows that the total quantity sent by distribution centers to customer zones in each time period, if any, must satisfy the transport capacity.

$$\sum_{i \in I} x_{iklt}^{DC} \le Q_{kl}^{DC} Z_{klt}^{DC} \qquad \forall k \in K. \ l \in L. \ and \ t \in \mathcal{T}$$
(74)

Note that customer zones do not hold inventory, so the total product received by each customer zone from the distribution centers has to be the same as the market demand (see equations (75)).

$$\sum_{k \in K} x_{iklt}^{DC} = O_{ilt}. \qquad \forall i \in I. \ l \in L. \ and \ t \in \mathcal{T}$$
(75)

Equation (76) is for inventory balance at distribution centers. It shows that for each distribution center and each product in each time period, the available inventory is calculated by the inventory available in the previous period, plus the quantity received from the warehouses minus the quantity sent to the customer zones.

$$q_{ikt}^{D} = q_{ikt-1}^{D} + \sum_{m \in M} x_{imkt}^{WD} - \sum_{k \in K} x_{iklt}^{DC} \quad \forall i \in I. \ m \in M. \ and \ t \in \mathcal{T}$$
(76)

Also, at each warehouse, safety stock is defined for each product and time period (see equation (77)).  

$$q_{ikt}^{D} \ge SS_{ikt}^{D}$$
  $\forall i \in I. \ m \in M. \ k \in K. \ and \ t \in \mathcal{T}$  (77)

Now the proper auxiliary variables associated with the closing / remaining open status of the facilities should be set to confirm the accuracy of the opening and closing decisions in the model. Equations (78) to (81) show that during the whole planning period, firstly, if a distribution center was not initially open, it could only be opened at most once. Secondly, it could also be closed at most once if it was opened before. Finally, a distribution center cannot be opened and closed in the same time period.

$$\sum_{t \in \mathcal{T}} \mathbf{y}_{kt}^{D+} \le 1 \qquad \forall k \in K$$
<sup>(78)</sup>

$$\sum y_{kt}^{D-} \le 1 \qquad \forall k \in K \tag{79}$$

$$\begin{aligned}
\sum_{t \in \mathcal{T}} & \\
y_{kt}^{D-} \leq \sum_{s=0}^{t-1} y_{ks}^{D+} & \forall k \in K. \text{ and } t \in \mathcal{T} \\
y_{kt}^{D+} + y_{kt}^{D-} \leq 1 & \forall k \in K. \text{ and } t \in \mathcal{T}
\end{aligned}$$
(80)
$$(80)$$

$$(81)$$

Equation (82) illustrates that if a plant was opened in the time period s then closed in the time period **t**, therefore, all decision variables: opening  $(y_{ks}^{D+})$ , closing  $(y_{kt}^{D-})$ , and closing status  $(w_{kst}^{D-})$ 

should be set to 1.

$$y_{ks}^{D+} + y_{kt}^{D-} \le w_{kst}^{D-} + 1 \qquad \forall k \in K. \ s = 0....\mathcal{T} - 1. \ and \ t = s + 1....\mathcal{T}$$
(82)  
If only a closing decision was made, a closing status variable would be set to 1:

$$w_{kst}^{D-} \le y_{kt}^{D-} \qquad \forall k \in K. \ s = 0. \dots \mathcal{T} - 1. \ and \ t = s + 1. \dots \mathcal{T}$$
(83)

Also, an opening status variable  $(w_{kst}^{D+})$  would be set to 1 if an opening decision was made:  $w_{kst}^{D+} \le y_{ks}^{D+}$   $\forall k \in K. \ s = 1....\mathcal{T}. and \ t = s....\mathcal{T}$  (84)

If a distribution center was opened in the time period s and is yet open in the time period t, in any of the periods in the internal s+1 and t, a closing decision would be impossible:

$$w_{kst}^{D+} \le y_{ks}^{D+} + \sum_{v=s+1}^{t} y_{kv}^{D-} \le 0 \quad \forall k \in K. \ s = 0....\mathcal{T} - 1. \ and \ t = s + 1....\mathcal{T}$$
(85)

# 4 Model Implementation and Numerical Results

# 4.1 Case Study Description

In order to evaluate the applicability and efficiency of the developed model presented in the previous section, we applied the data of a real company which is located in the UK and studied by Longinidis and Georgiadis [5]. Note that, because of some data incongruity and missing data, their case study could not be directly applied and we have considered the following assumptions regarding the missing information:

- This company has three plants in three different locations and four possible locations for warehouses and six potential locations for distribution centres.
- Each plant is able to produce six of seven products within its limitations of production capacity. Each plant also holds about two times the average annual demand as initial inventories.
- In each time duration, each warehouse and also distribution centres have an upper and lower bound handling capacity and need safety stock.

> Initial inventories are considered about two times the average annual demand.

- Safety stock for each product at each facility is equal to the total quantity transferred from the facility during a period of 15 days.
- Product flows among plants, warehouses, distribution centres and customer zones have upper bounds.
- > Prices and demands of products in each customer zone are known.
- > The company has a 4-year planning horizon.
- > Before the planning horizon, balance sheet data are integrated into the optimization process.
- All tangible assets have been deprecated. Short-term liabilities (accounts payables and taxes of previous profits) should be paid in one year.

> The real value of cash has been calculated, instead of considering it as a percent of net income.

# 4.2 Comparison Between Basic Model And Developed Models

Now, in order to display the improvements in the proposed model, we compared the results of the basic model presented by Longinidis and Georgiadis [5] with our developed models which have a new objective function, accurate calculations, and additional financial considerations. All the problems were solved by Branch and Reduce Optimization Navigator (BARON) solver in GAMS software on a personal computer with core i5 CPU 2.50 GHz and 8 GB of RAM on Windows 8.

### 4.2.1 Basic Model with Traditional approach

The basic model was considered with the same decision-making assumptions and objective function presented by Longinidis and Georgiadis [5]. Its objective is to maximize the company's net created value which is measured by Economic Value Added (EVA) index. The model for the problem was solved and the total value created amounts to 85,855,590 monetary units. The optimal results of the basic model will be used to compare them with results obtained from other developed models. In this way, it is possible to show the advantages of the proposed approach clearly.

# 4.2.2 The First Developed Model With New Objective Function

According to what is explained in section 2, SVA is one of the most accepted methods to measure the value of a company. SVA determines the financial value of a company by looking at the returns it provides for its stockholders and is based on the view that the objective of company managers is to maximize the wealth of company stockholders. SVA calculates the shareholder value by deducting the value of long-term liabilities at the end of planning horizon from the value of the firm for the time period. In this study, the final value of the company is obtained by discounted free cash flow (DFCF) method with a fixed growth rate (0.5%).

Now, in the first stage of developing the model, Shareholder Value Analysis (SVA) is applied as an objective function in the basic model. The model was solved and the total value created amounts is 86,855,590 monetary units. The optimal network structure is shown in Figure 3. The total production quantities for the whole planning horizon is only 1407 units: plant 1 and plant 3 produce 809 and 598, respectively; plant 2 does not produce at all. Therefore, reducing inventory was clearly shown and had these results: i) decreasing production quantities to reduce the product quantities in stock. ii) the large flows lead to establishing a new distribution center to meet customers' demands. In order to reduce the need for working capital, SVA tends to reduce the inventory. Therefore, the produced quantity by SVA model is smaller than the EVA model.

This feature of SVA model also makes a large number of flows between warehouses and distribution centers and between distribution centers and customer zones. The total quantities transported from plants to warehouses for both models are compared in Table 2.



Figure 3: Network Structure and Produced Products for The Developed Model

Table 2: Total Products Tra	nsported from Plants	to Warehouses
-----------------------------	----------------------	---------------

	W1	W2	W3 W4		W1	W2	W3	W4
Plant 1	7901		San Julia	Plant 1	7471			
Plant 2		6210	ومفاقات وراي	Plant 2	1.97	1498		
Plant 3			3502	Plant 3	,		3201	
Develope	ed Model wi	ith New Obje	ctive Function	Basic Moo	lel with Tra	ditional Appr	oach	

According to Table 2, by SVA model, warehouse 1 receives more products supplying distribution centers 1 and 6. Similarly, warehouse 2 receives more quantity, therefore it supplies distribution centers 1, 2, 5, and 6. But by EVA model, warehouse 2 just supplied distribution center 2.

Table 3: Total Products Transported From Warehouses to Distribution Centers

	DC1	DC2	DC3	DC4	DC5	DC6		DC1	DC2	DC3	DC4	DC5	DC6
W1	5298					2543	W1	7471					
W2	105	2303			508	3321	W2		1498				
W3	161		3298				W3			3201			
W4							W4						
Developed Model with New Objective Function				Dasia	Model u	ith Tradit	ional An	nroaah					

Developed Model with New Objective Function

Basic Model with Traditional Approach

As shown in Tables 3 and 4, by applying the model with SVA as the objective function, inventory was stored in five distribution centers (all distribution centers except 4), therefore, total flows between distribution centers and customer zones are much larger than total flows transported when EVA was the objective function.

Note that since distribution center 6 has the lowest inventory cost among others, it received most of the inventory transferred from warehouses to distribution centers. It receives 5864 units but it only supplies customer zone 6 with 531 units and 5333 units are kept as inventory. Also, the model with SVA as the objective function tends to reduce the inventory quantities to decrease the need for working capital. Only 878 units stay at the plants as inventory.

	CZ1	CZ2	CZ3	CZ4	CZ5	CZ6	CZ7	CZ8		CZ1	CZ2	CZ3	CZ4	CZ5	CZ6	CZ7	CZ8
DC1	1349		114	1672	123	904	1443		DC1	1349			2018	1241	1413	1458	
DC2		1516						728	DC2		1498						
DC3			1498	346	620			816	DC3			1498					1559
DC4									DC4								
DC5					508				DC5	1							
DC6						531			DC6				<b>X</b>				
Devel	Developed Model with New Objective Function					Basic	Model	with Tra	ditional	Approac	ch						

Table 4: Total Products Transported from Distribution Centers to Customer Zones (SVA Base Model)

4.2.3 The Second Developed Model With with a Value-Based Approach

Now, in the second phase of model development, we add new financial aspects to the previous version of the model to make it similar to real conditions. These new features include the possibility of closing and opening facilities at any time period of the planning horizon, repayments obligation to the bank, adding the possibility of new capital entries from shareholders, and adoption of an accounts payable policy. To better understand the effect of these aspects, we explained them separately.

First, to test the possibility of closing and opening facilities at any time period, we considered two times of the establishment price of each facility as selling prices (Table 9). The value created for shareholders is 87,397,697 monetary units which is 0.88% larger than the value created by the basic model which is the gains resulting from selling the plants. Then the new model with the obligation of bank repayments created 89,407,636 monetary units, which is 3.02% larger than the value created by the model with SVA as objective function. The network structure remains the same. By repaying to the bank every year, long term debt is reduced and a lower amount is deducted from the free cash flow that was generated over the planning horizon, creating more value for shareholders.

Next, in order to consider an account payable policy, it is assumed that 60% of payments to suppliers are made in cash and 40% are made in credit. In this situation, the value created for shareholders is 88,549,322 monetary units, that is, 0.96% smaller. Because more amount of money (working capital) is needed to support operating expenses and pay suppliers, the free cash flow decreases and the value created is 858,314 monetary units lower.

Finally, we add the possibility of raising new capital from shareholders and also set a per-year limit of 60,000 monetary units for the new capital entries. This limit shows the maximum that shareholders are willing to invest in the company to receive dividends in the future. The new developed model was solved optimally and the value for shareholders increased to 92,460,308 monetary units, which is 3.18% larger than the value without these financial considerations created and 6.3% larger than the value created by the basic model. The optimal networks for time periods are shown in Figs. 4 to 7 (see appendix).

These Figures display the network structure during the planning horizon. As it can be seen, the flows between facilities and the quantities transported change during the time.

According to Figures 4 to 7, plants only produce during the first two years and their total quantity is 1394 units. The total quantity produced by the SVA model is much lower than the quantity production when EVA was the objective function. Therefore, the need for working capital and payments to suppliers is smaller. These changes lead to an increase in the value created for shareholders. Also, by using EVA as the objective function, the value of the company improves by creating higher inventories (which are a part of current assets).

Plant 2 closes at the start of the second year with a final inventory of 3341 units, reducing its initial inventory by 76%. Plant 1 and plant 3 are closed at the beginning of third year, with the final inventory of 1971 and 881 units. This means an inventory reduction of 245% and 285%, respectively. Note that products 2, 4, and 7 at plant 1 which were not sold within the planning horizon are considered as the final inventory. Also, products like 3 and 6 at plant 1 that were produced in the years 1 and 2, have no final inventory. As explained before, in accordance with the evolution of the number of flows among facilities, the product quantities transported from plants to warehouses increase from year 1 to year 2. Table 6 presented the operating costs (production, transportation, and inventory holding costs) that resulted from the decisions described above. As we can see, the largest portion of the operating costs is transportation costs (50.58%), then inventory holding costs (40.27%), and production costs (9.15%). There are production costs in the first and second years. Also, due to high inventory at the beginning of the planning horizon, there is no production in the years three and four. In these two years, from plants to warehouses and from warehouses to distribution centres, there are no transportation costs because plants are closed and the warehouses are not operating. As shown in Table 5, inventory costs decrease over time. The inventory costs at plants in in years three and four refer to products that were already in inventory at the beginning of the planning horizon and the ones customers didn't request. It is important to note that although the final inventory at the distribution centres is equal to zero, there is an inventory cost since inventory is calculated based on its average during a year.

Table 5: Production, Transpo	rtation, and Inventory	Costs for each Year Obtained	by the Developed Model with the
Value-Based Approach			

	Year 1	Year 2	Year 3	Year 4	Total
Production Costs	1013	90,102	0	0	91,115
Transportation Costs	162,717	209,856	60,417	71,303	504,293
Inventory Costs	141,402	109,542	89,502	60,991	401,437
		11 - 10 %	1.00		

According to financial decisions made by the final model, managers are provided with an accounts payable policy in Table 6. It shows that the company has enough cash (based on the initial balance sheet) and does not need bank loans. Therefore, all capital entries are captured from shareholders. As we can see, production costs by the developed model are low, since high levels of inventory and money are available for investment. Therefore, the company is in a good condition for repayments to the bank, decreasing debt and maximizing the value of the corporate which is measured by SVA.

Table 6: Financial Decisions For Eac	Year Obtained by the Developed Model	With the Value-Based Approach

Financial decisions	Year 1	Year 2	Year 3	Year 4	Total
Loans	0	0	0	0	0
New Capital Entries	60,000	60,000	60,000	60,000	240,000
Investment	300,000	0	0	0	300,000

Financial decisions	Year 1	Year 2	Year 3	Year 4	Total
Repayments	540,000	270,000	135,000	67,500	1,012,500

### 4.3 Influence of Results According to Variation in Financial Parameters

In this section, the performance of proposed model was tested by changing some important financial parameters. These parameters are important because they are suggestive of the economic environment and in many cases are accepted conditions that the company has no impact on them. The cost of capital rate at time period t ( $r_t$ ) is an important parameter. Also, one of the important financial parameters affecting the company's wealth is the tax rate ( $Tr_t$ ). Moreover, we selected the depreciation ( $DPR_{st}$ ) rate as a financial parameter for the sensitivity test.

Table 7 presents the effects on the proposed model by changing these parameters from -15% to +15%. The results illustrate that the model with new financial considerations is resistant to the changes of these financial parameters.

Doromotor	Change (%)							
Parameter	-15	-10	-5	-2	+2	+5	+10	+15
Cost of Capital								
Rate at Time	105,947,496	101,350,940	96,869,752	94,204,964	90,717,780	88,114,172	83,796,384	79,838,788
Period t (r <sub>t</sub> )					$\mathbf{A}$	×		
Tax Rate $(Tr_t)$	99,756,840	97,326,664	94,896,184	93,435,236	91,484,468	90,020,784	87,580,196	85,139,760
Depreciation								
Rate $(DPR_{st})$	93,832,792	93,377,628	92,919,880	92,644,304	92,275,780	91,998,608	91,534,324	91,070,724
rune (DI Mst)			A 10					

Table 7: Sensitivity Analysis of The Objective Function by Changing in Financial Parameter	Table 7: Sensitivity	Analysis of T	he Objective	Function by	Changing in	1 Financial	Parameters
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# 4.4 Results and Discussions

In the previous section, the optimal results of a basic model were used to compare them with the results obtained from other developed models to show the advantages of the developed models. We carried out two phases of development in order to improve the basic model: i) applying a new objective function, which maximizes the value of the company measured by the SVA method, ii) adding new financial aspects to the previous version of the model to make it more realistic.

In the first step, SVA was applied as a new objective function instead of EVA. The model with the new objective was solved and the total value created for shareholders was increased by 86,635,307 monetary units.

In the second step, the new financial aspects were integrated into the previous version of the model. The total value created by the complete version of the model was 92,460,308 monetary units which is 0.7% larger than the SVA obtained without financial aspects and 0.93% larger than the value created in the basic model. The main reasons for an increase in value creation for shareholders are due to new operational and financial aspects, which mainly show the possibility of closing facilities and bank-debt repayments. Bank repayments which reduce debt and new capital enables the company to choose better operational options. The value created by each model is reported in Table 8.

Table 8: Value Created for Shareholders by Each Model

Model

Value Created (Monetary Units)

An Optimization Model for Designing a Supply Chain Network with a Value-Based Management Approach

The Basic Model with Traditional Approach	85,855,590
The First Developed Model with New Objective Function	86,635,307
The Second Developed Model with a Value-Based Approach	92,460,308

The main reasons for an increase in the value created are due to both operational and financial aspects such as the possibility of closing facilities and bank repayments.

In terms of the type of objective function in this study instead of EVA, which is based on conventional accounting principles, SVA is applied as an objective function that is one of the most accepted methods of measuring how corporate performance relates to shareholder value. As mentioned before, the SVA for a company is calculated by adding the present value of cash flows to their terminal value, which represents the value of the company discounted at the proper cost of capital. The EVA for measuring a company's financial performance deducts its cost of capital from its net operating profit after taxes. As explained in the previous sections, since EVA is based on accounting principles, making unreasonable decisions is possible. For example, increasing current assets by higher inventories in order to make more EVA.

### **5** Conclusions and Future Research

One of the main purposes of a supply chain is to fulfill demand, improve responsiveness and profitability and build a good network to facilitate the financial success of a company. Many of the previous studies emphasize that strategic decisions such as supply chain decisions have a significant impact on shareholder value creation. Investment decisions also should be considered as critical inputs to financial planning. Since these kinds of decisions for supply chain networks play a key role in financial health of companies, therefore, financial considerations should also be regarded when modeling supply chains.

However, studies on supply chain models integrating financial aspects are limited. In these studies, financial aspects have been considered as endogenous variables or known parameters in objective functions and constraints.

In view of the above concerns, this study suggests a mathematical model that considers the physical and financial aspects of a supply chain planning problem, simultaneously. A deterministic Mixed-Integer Nonlinear Programming (MINLP) model was developed to specify the number and location of facilities and the links between them. The model also determines the quantities to be produced, stored, and transported in order to meet customers' demands as well as maximize the shareholder value measured by SVA method. In financial decisions, the amount of investment, the source of the money needed (cash, bank loan, or new capital from shareholders) and repayments to the bank were considered. To show the applicability and efficiency of the developed model, data of Longinidis and Georgiadis [5] were used. The results show that with appropriate financial decisions, creating more value for the company and its shareholders is achievable. The model could be used by supply chain managers as an effective decision tool, supporting their decisions with figures and indexes convenient for financial managers. The major contributions of this study can be summarized as follow:

- This study presents a mathematical model to solve a SCND problem that considers tactical, strategic and financial decisions simultaneously.
- Maximizing the creation of economic value for shareholders measured by shareholder value analysis (SVA) as a new objective function instead of traditional approaches such as maximizing profits or minimizing costs.
- The proposed model considers the amount of loan, bank repayment and new capital from shareholders as decision variables, therefore, it provides managers an accounts payable policy, instead of

considering that all payments should be paid in cash. Previous studies of the literature consider them as parameters.

- At the strategic level, the model specifies the location of each facility. At the tactical level, it determines the products quantities to be produced and stored to satisfy customers demand. Regarding financial decisions, the model specifies the amount of investment and their sources such as cash, bank debt or shareholders' capital as decision variables and it provides a repayment policy for managers.
- Regarding the constraints, in addition to common operational constraints, lower limit and/or upper limit values for financial ratios in order to support the financial health of the corporation. To retain a better financial performance, the proposed model provides a balance among new capital entries, loans and repayment. With consideration of large cost of new capital entries, the model imposes upper bound on it and avoid an ever-increasing debt; it considers lower bound for bank repayments. Besides, these benefits of our model provides managers with an accounts payable guideline.
- Providing the possibility of opening or closing facilities in order to deal with market fluctuations during the planning horizon.
- In contrast with basic models in previous studies which have too many assumptions, the presented model uses accounting principles with less assumptions that made it more realistic. For example, we use the net liabilities in the analysis of financial statements that balances bank loans and payments, determines the exact value of deprecation by knowing the lifetime of each asset in each time period, and applies real cash value instead of pre-determined proportion of profit.

This study can be expanded in the following directions: in order to make the model similar to real conditions, future studies can consider uncertainty in some parameters such as product prices and demand. Applying financial ratios as objective functions in the proposed model in order to find a way to increase and improve the firm soundness. The green supply chain with a closed-loop structure can be the other research trend for the model considering environmental, social, technological and economic facets; such facets can be included in the supply chain network design. The problem would get more complicated with such developments. Therefore, other solutions, such as metaheuristics, can be considered as other suggestions for futures research.

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Figure 4: Network Structure for the Complete Model in Year 1 and for the Developed Model with New Financial Aspect

Figure 5: Network Structure in Year 2 and for the Developed Model with New Financial Aspects



Figure 6: Network Structure in Year 3 and for the Developed Model with New Financial Aspects



Figure 7: Network Structure In Year 4 and for the Developed Model with New Financial Aspects

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