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Making Decision on Selection of Optimal Stock Portfolio Employing Meta Heuristic Algorithms for Multi-Objective Functions Subject to Real-Life Constraints

Ali Sepehri [a], Hassan Ghodrati Ghazaani [b,*], Hossein Jabbari [c], Hossein Panahian [b]

[a] Department of Industrial Management- Financial, Kashan Branch, Islamic Azad University, Kashan, Iran
[b] Department of Management, Kashan Branch, Islamic Azad University, Kashan, Iran
[c] Department of Accounting, Kashan Branch, Islamic Azad University, Kashan, Iran

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ABSTRACT

The purpose of this study is to utilize data envelopment analysis and metaheuristic algorithms to make investment decisions and select an optimal stock portfolio, considering real-life constraints and multi-objective functions. The statistical population for this research comprises 183 selected companies from the Tehran Stock Exchange involved in capital decision-making and optimal capital composition. Eventually, 42 companies were identified as justifiable investment options. After assessing the risk and return of efficient companies, the study formulated a multi-objective model based on the investor's budget limitations, requirements, and expectations to determine the investment composition. To achieve optimal decisions, a modified genetic metaheuristic algorithm and MATLAB software with dual operators were employed. Sensitivity analysis revealed that eliminating the risk minimization function enhanced the decision's return level but increased risk. Conversely, eliminating the maximizing return function improved decision-making risk but reduced investment return. Eliminating investment requirements and expectations increased returns and investment risk while involving more companies in the optimal investment portfolio.

1 Introduction

Due to the significance of stock portfolio optimization for investors, this study aims to provide improved tools for solving this problem with a new approach. Many researchers today acknowledge the inefficiency of financial markets and recommend the adoption of new investment approaches instead of relying solely on traditional methods based on historical data. One crucial issue in capital markets that investors, both natural and legal entities, need to address is the selection of an optimal investment combination or portfolio with considerations of risk and return [1].

A review of existing research literature reveals that while heuristic and metaheuristic methods such as genetic algorithms have been extensively employed in optimal portfolio selection, previous studies only focused on basic criteria of risk and return, either individually or jointly, and neglected other factors influencing capital decision-making, such as stock liquidity. Additionally, these simulations

* Corresponding author. Tel.: *09131625634*
E-mail address: *dr.ghodrati42@gmail.com*

often only considered the constraint of investment composition or budget, disregarding other real constraints like investment requirements [12]. Therefore, this study introduces an innovative aspect in response to the needs and realities of capital decision-making by selecting the optimal combination based on two dimensions: the multiplicity of target criteria and practical limitations in determining the selection environment. This represents a step towards making the capital decision-making environment more objective and embracing a multidimensional approach to investment. The research literature concerning optimal combination selection in investment generally defines it as a form of capital decision-making, where the decision-maker chooses the best combination among various investment options using judgment methods, mathematical modeling, or simulations.

Previous studies, although evaluating different combinations and ultimately determining the optimal combination, often neglected the initial decision-making algorithm in their modeling and definition. By defining the initial environment and investment selection environment as "decision-taking" or "decision assistance," wherein a suitable environment is created to facilitate capital decision-making, this step can be explicitly addressed, paving the way for optimal portfolio selection or "decision-making." Many studies, including those conducted by previous researchers [11, 17], have explored various mathematical optimization, simulation, judgment, or multi-criteria methods for portfolio optimization without differentiating between decision-taking and decision-making stages. Based on the aforementioned research problem, the present study aims to address the following key question: What are the results of investment decision-making based on the data envelopment analysis (DEA) model before selecting the final stock portfolio, and how does the selection of the optimal stock portfolio using real-constraint simulation algorithms based on real constraints (MCA) impact the outcomes?

2 Literature Review

Individual investors, brokers and fund managers invest billions of dollars in various sectors every year. Therefore, proper security selection for financial investment comes into prominence since generating profit throughout all market climates and minimizing losses during market downturns are desired. The most common investment strategy is building a portfolio consisting of different securities in order to spread the risk. Traditional portfolio analysis requires the evaluation of return and risk conditions of individual securities and may not provide success due to its subjective nature. In 1952, performing an analysis of the impact of risk, Markowitz presented a revolutionary approach for portfolio theory called Mean-Variance (MV) model [11] and initiated the era of modern portfolio theory. Using covariance as a risk measure is the key of this revolution since it was a spark that triggered quantitative finance. Following this milestone, the MV model has been a standard decision-making approach to structure and measure the performance of portfolios [8] that quantitatively focus on the investment alternatives utilizing the covariance between securities based on return-risk trade-off [6]. Thanks to his pioneering works in finance theory, in 1991, Markowitz was awarded the Nobel Prize for economics. Along with the developments in computational power, a growing number of researchers from not only financial fields, but also computer scientists and mathematicians have engaged a great attention to portfolio optimization (PO), evident by the vast number of publications in scientific journals to deal with mean-variance portfolio optimization (MVPO).

New constraints, objectives, and solution approaches have been developed to address shortcomings of the early MV model [6]. Thus, a significant number of academic papers has been reached by accumulation of successive additions to MV model. Aouni, et al. [2] reviewed the lexicographic, weighted, polynomial, stochastic and fuzzy goal programming models and pointed out the lack in developing computerized decision support systems to accomplish a helpful tool to facilitate the decision-making process in portfolio optimization. In a study on Portfolio-Optimization conducted by Darabi and Baghban [5] has been the Clayton-copula along with copula theory measures, Portfolio Optimization is one of the activities in investment funds. They used copula as an alternative measure to model the dependency structure in research. In this regard, given the weekly data pertaining to the early 2002 until the late 2013, They used Clayton-copula to generate an optimized portfolio for both copper and gold. Finally, the Sharpe ratio obtained through this method has been compared with the one obtained through Markowitz mean-variance analysis to ascertain that Clayton-copula is more efficient in portfolio-optimization. According to the study conducted by Miryekemami et al. [13], decision making has always been affected by two factors: risk and returns. Considering risk, the investor expects an acceptable return on the investment decision horizon.

Accordingly, defining goals and constraints for each investor can have unique prioritization. They developed several approaches to multi criteria portfolio optimization. The maximization of stock returns, the power of liquidity of selected stocks and the acceptance of risk to market risk are set as objectives of the problem. In order to solve the problem of information in the Tehran Stock Exchange in 2017, 45 sample stocks have been identified and, with the assumption of normalization of goals, a genetic algorithm has been used. The results show that the selected model provides a good performance for selecting the optimal portfolio for investors with specific goals and constraints. Rezaei and Elmi [14] showed that the reaction of stock price in the stock market was modelled by the behavioural finance approach. The population of this study included the companies listed on the Tehran Stock Exchange.

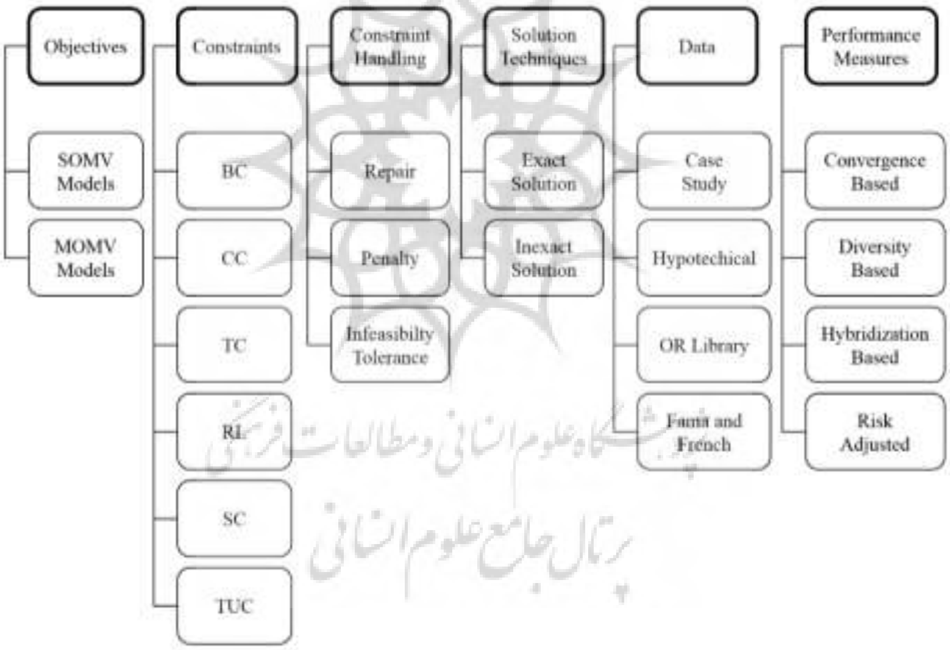

Fig. 1: Classification of models and applications on MVPO [7]:
Single-objective MV (SOMV); Multi-objective MV (MOMV); Boundary Constraints (BC); Cardinality Constraints (CC); Transaction Costs (TC); Roundlot Constraints (RL); Sector capitalization Constraints (SC); Turnover Constraints (TUC); Methodologies to deal with various constraints on MVPO (Constraint handling); Various performance indicators to test the efficiency of proposed algorithms (Performance Measures).

In order to forecast the stock price, the final price data of the end December, March, June, and September 2006-2015 and the stock prices of 2014 and 2015 were analysed as the sample. In this study, Bayes' rule was used to estimate the probability of the model change. Through this rule, the probability of an event can be calculated by conditioning the occurrence or lack of occurrence of another event. The results of model estimation showed that there is the probability of being placed in high-fluctuated regimes (overreaction) and low- fluctuated (under-reaction of stock price despite the shocks entered to the stock market. In modelling with the 4-month final prices, it was proved that the real stock price had no difference from the market price.

2.1 Category Selection

The categorization applied in this review is formed based on different properties of deterministic models and applications on MVPO. The categories are formed according to the following criteria: models, constraints, constraint handling, solution techniques, performance measures and data. The main categories used to classify the publications are shown in Fig. 1.

2.2 Consideration of Real-World constraints

Despite all the advantages of the original MV model, it falls short in real world applications. The original MV model considers only one hard constraint, setting the sum of asset weights to one meaning that sum of invested amounts must be equal to the total budget. Therefore, the MV model needs additional constraints to solve realistic PO problems. The real-world constraints that have been added to the MV model are summarized in Table 1.

Table 1: Constraints for realistic portfolio optimization [3]

Boundary constraints (BC)	Impose lower and/or upper bounds on the values of each asset weight, also known as buy-in threshold constraint.
Cardinality constraints (CC)	Related to the number of assets invested in the portfolio, may be fixed to a certain value, may also ensure that the number of assets is between the desired range.
Transaction costs (TC)	Investors pay a fee called transaction costs when they sell or buy stocks influencing the total profit.
Roundlot (minimum lots) constraint (RL)	Ensure that the amount invested in a security is multiples of the minimum transaction lot.
Sector capitalization constraints (SC)	Impose the assets which belong to the sector with more capitalization value to have more shares in the final portfolio.
Turnover constraint (TUC)	Sets the turnover rate of an asset from current period to next period which is especially useful in multi period portfolio optimization models.

Table 1 lists the classification of publications according to constraint types. Although transaction costs make more sense on multi-period portfolio optimization, researchers widely considered this constraint on single-period portfolio optimization for modelling purposes. If the selected portfolio is readjusted several times according to the determined investment horizon, TUC which is only valid for multi period portfolio optimization problems is often added into the model. While the original MV model is presented with a quadratic objective function and linear constraints, several researchers added real life constraints which introduce non-linearity and nonconvexity to MVPO.

For example, TC may be a nonlinear and nonconvex function of a difference in holdings of new and existing portfolio [10] which increases the complexity of the MVPO. However, TUC are to be formulated as a linear equation, the problem can still be solved by QP [16]. On the other hand, CC together with BC leads to a non-convex search space [9]. Therefore, QP cannot be efficiently utilized and hence, researchers often head towards to inexact techniques in realistic cases.

3 Research Method

In the present study, on the one hand, we relied on mathematical modelling with the approach of data envelopment analysis in evaluating financial efficiency and also modelling the optimal combination of investment in financially efficient companies based on quadratic or nonlinear mathematical planning, by following similar research in the field of finance or operations research, as well as relying on annual cross-sectional data or, in some cases, performance averages. Based on this assumption, the statistical population of the present study is defined as companies listed in the Tehran Stock Exchange Organization with the study period examining the financial period ending March 19. Other criteria included non-affiliation to loss-making companies, especially consecutive losses over several periods. Under the said criteria we gathered and examined financial data for 183 companies listed on the Tehran Stock Exchange.

3.1 Determining the Optimal Composition of Investment

Without loss of generality, a general multi-objective optimization problem is represented by

$$\text{Minimize } F(x) = (f_1(x), f_2(x), \ldots, f_m(x)) \tag{1}$$

S. T:

$$x = (x_1, x_2, \ldots, x_N) \in X$$

where, $x \in X$ represents a decision vector in N-dimensional space X. If f_is' are conflicting objectives, then a point at which all m objectives reach their minimum value do not exist. Hence, in a non-trivial case of multi-objective optimization, a set of non-dominated solutions with best trade-offs among objectives, called Pareto optimal set, is desirable.

A decision vector x is said to dominate another decision vector y if,

1 $f_i(x) \leq f_i(y)$ holds $\forall i \in \{1, 2, \ldots, m\}$

2. $f_i(x) < f_i(y)$ holds for at-least one index $\forall i \in \{1, 2, \ldots, m\}$

This is represented by $F(y) \leq F(x)$. Then, Pareto optimal set (P) is defined as

$P = \{x \in X \mid \forall y \in X, \text{ for which } F(x) \leq F(y)\}$

Weighted sum approach of tackling multi-objective optimization problem by combining multiple conflicting objectives into a single objective is parameterized by weight parameters. A single run of the algorithm does not provide the set of optimal trade-offs between objectives. Further, if the problem under consideration is a general non-convex problem, then a set that best approximates the Pareto optimal set is pursued. MOEAs are population based search procedures, designed for getting a set of approximate optimal trade-off solutions. There are no underlying assumptions on the structure of objectives and constraints of the problem, this enables them for handling complex problems to approximate a set of solutions in proximity of true Pareto front. MOEAs have been successfully applied in a variety of real-world problems e.g., optimal power flow problems, water distribution systems, remote medical resource assignment, wireless sensor networks etc [15]. In this study, CCPO with several hard (equality) constraints are handled using MOEAs. Four MOEAs are adapted by incorporating proposed candidate generation method and repair mechanism. In following subsections, a general structure used for modifying MOEAs is discussed in detail.

Step 1: Encoding:

A single real vector of size N represents a candidate solution or weights of a portfolio. Decision variables z_{is}' are handled by the candidate generation method implicitly. Apart from better space complexity, a further advantage of this representation is that the recombination process can be applied effectively only to the selected assets for efficient subspace exploration.

Step 2. Population initialization:

The population is initialized randomly. Following steps are required to generate a candidate solution or a portfolio in the initial population.

1. Cardinality k is chosen randomly with equal probability from k_1 to k_2 i.e. $k \in \{K_1, K_2, \dots, k_N\}$.

2. k-p positions out of N-p positions in the index set $\{1, 2, \dots, N\} \setminus I_p$, are selected randomly for allocation.

3. For each selected position in the above step and given p preassigned asset positions in set Ip, uniformly distributed random variable between their respective bounds is generated and allocated.

It is apparent from the foregoing steps that initial population would be infeasible in general. Particularly, solutions in the initial population follow pre-assignment, cardinality, floor and ceiling constraints, however, does not guarantee to satisfy budget and round-lot constraints [19]. For satisfying remaining constraints, each of the solution in the initial population is repaired using proposed repair mechanism. It is important to generate approximately equal number of candidate solutions for each feasible cardinality from k_1 to k_2. This allows a better exploration of search space by an evolutionary process. Hence, population size depends upon $k_2 - k_1$.

Step 3: Candidate generation:

Effective candidate generation forms the most important part of any evolutionary algorithm. This step directly influences exploration and exploitation capabilities of the algorithm. In the past studies, a stream of research is concentrated on developing new candidate generation methods in evolutionary computation. proposed Laplace crossover operator and power mutation operator for real-coded genetic algorithms. They established their superiority over well-known genetic operators for twenty benchmark global optimization problems. Later, Khoo [8] modified Laplace crossover operator using bounded exponential distribution that not only has better search capability but also produces offspring solutions under variable bounds.

4 Research Findings

In this section, research findings based on the application of the proposed research model in the field of decision-taking and decision-making in selecting the desired investment combination are presented.

4.1 Decision-Taking

Using knowledge analysis, inputs, and outputs affecting the financial efficiency of companies were identified. Moreover, by employing Delphi survey and Fuzzy DEMATEL**,** inputs and outputs were refined and as part of evaluating financial efficiency based on mathematical modeling data envelopment analysis, efficient companies were identified as justified investment options in the decision-taking stage. During the performance appraisal, companies with the efficiency rate of 1 or %100 were classified as efficient companies. The assumption was that with the inputs used, it was not possible to produce more outputs [4]. Other companies with a financial efficiency rating of less than 1 or lower than 100% were classified as inefficient companies.

According to Table 2, efficient companies in the present research, in comparison with other companies, were selected by relying on Meghwani and Thakur [12] model as justified investment options or justified initial environment or decision-taking, and were classified as reliable companies in the final decision-making to determine the optimal composition of investment and are therefore picked from among 183 selected companies

:

Table 2: Financially efficient companies or primary justified points in decision taking

Row	Firm Name	Code	Row	Firm Name	Code	Row	Firm Name	Code
1	Iran Yasa	7	15	Siman Dorud	64	29	Bank Parsian	147
2	Bama	9	16	Khak Chini Iran	78	30	Bank Pasargad	148
3	Behnoosh	10	17	Qand Piranshahr	91	31	Bank Dei	150
4	Palayesh Naft Isfahan	15	18	Faravari Mavad Mada-ni	94	32	Bank Saman	151
5	Palayesh Naft Tabriz	16	19	Fulad Amirkabir Kashan	97	33	Bank Gardeshgari	154
6	Petroshimi Khark	17	20	Qand Isfahan	99	34	Sarmayegozari Sanduq	161
7	Iran Tire	24	21	Qand Qazvin	100	35	Bimeh Dei	169
8	Lent Tormoz	26	22	Kashi Pars	111	36	Petroshimi Pars	177
9	Daru Exir	34	23	Sina Daru	125	37	Petroshimi Jam	178
10	Zoqalsang Negin	41	24	Madani Amlah Iran	134	38	Petroshimi Zagros	179
11	Saipa Dizel	48	25	Madani Damavand	135	39	Petroshimi Khorasan	180
12	Saipa Shisheh	49	26	Chador Melo	136	40	Petroshimi Isfahan	181
13	Siman Ilam	57	27	Naft Behran	141	41	Petroshimi Qadir	182
14	Siman Behbahan	59	28	Post Bank Iran	145	42	Petroshimi Amirkabir	183

At this stage of the analysis, i.e. the second step, namely decision-making and, in other words, the selection of the optimal combination or portfolio of investment based on real constraints among financially efficient companies as the initial justified environment, will be addressed. In this regard, first, the investment criteria were determined. Then the investment constraints were determined, and afterward, the multi-criteria optimization model was defined. The model was addressed and discussed based on hyperactive algorithms and the response sensitivity analysis. It should be noted that this modeling was inspired by the main idea in the Markowitz [11] mean-variance model.

4.2 Return Calculating

Using the background of research, including studies conducted by Roy and Shijin [15], Ban et al. [3] and Kubota and Takehara [9], the return criterion as one of the most important indicators affecting capital decision making has been selected and in calculating the average return, the model of Kalayci et al [7] was used as follows, where R represents the average stock return during the studies period. The original single objective MV model can also be rewritten as to maximize the return for a given level of risk. A portfolio obtained by solving model mentioned in paper by taking into consideration of minimum risk for a given level of return or a maximum return for a given level of risk is called efficient portfolio. However, to find an efficient portfolio, it is necessary to know either the level of risk that the investor can endure, or the desired return defined by the investor. In fact, it may not be quite possible in real world cases. So, to find the efficient portfolio among various combinations of assets in the solution space, instead of considering a single objective, researchers must consider all objectives at once. Therefore, the researchers [3, 7, 12] transformed the single-objective model into a multi-objective model. However, the used equation for calculating risk proposed by kalayci et al. [7] has been used before by some other researchers and their study report satisfying results.

$$R=\sqrt[n]{\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\dots\left(1+\frac{r_n}{100}\right)} \tag{2}$$

To calculate the average return, the geometric mean technique was used and, in this regard, $r_1, \dots, r_n$ represents the real return of stocks during the first to nth period. In this regard, the one-year performance ending 20 March 2019 for the companies under review was examined for a 12-month period.

Moreover, to calculate the return, we examined stock price changes compared to previous years.

4.3 Risk calculation

For this study, Kalayci et al. [7] model was employed and the risk criterion was calculated based on stock price changes using the following formula:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=0}^{1}(r_i - E(r))^2} \tag{3}$$

Based on our calculations, the risk and monthly returns of investment options are summarized as described in Table 3

Table 3: Summary of Risk Results and Monthly Returns

Row	Code	Return	Risk	Row	Code	Return	Risk	Row	Code	Return	Risk
1	7	1.006	0.179	15	64	1.013	0.246	29	147	1.002	0.107
2	9	1.023	0.277	16	78	1.000	0.137	30	148	1.004	0.065
3	10	1.003	0.127	17	91	1.031	0.180	31	150	0.997	0.265
4	15	1.011	0.239	18	94	1.015	0.258	32	151	1.010	0.301
5	16	1.011	0.266	19	97	1.000	0.172	33	154	1.000	0.090
6	17	0.880	0.438	20	99	1.002	0.185	34	161	1.012	0.118
7	24	1.035	0.212	21	100	1.003	0.133	35	169	1.006	0.095
8	26	1.026	0.192	22	111	1.000	0.260	36	177	1.012	0.368
9	34	1.004	0.190	23	125	1.004	0.043	37	178	1.009	0.173
10	41	1.023	0.212	24	134	1.000	0.130	38	179	1.016	0.230
11	48	0.994	0.432	25	135	1.013	0.412	39	180	1.018	0.165
12	49	1.018	0.221	26	136	1.001	0.171	40	181	1.003	0.580
13	57	1.005	0.297	27	141	1.009	0.121	41	182	1.018	0.377
14	59	1.014	0.271	28	145	1.000	0.200	42	183	1.000	0.167

In addition to the results, the description of risk findings, return and stock price for efficient companies is shown under Table 4 as follows

Table 4: Description of risk, return and latest stock price in efficient companies

Variable	Symbol	Minimum	Maximum	Average	Median	Std. deviation	Skewness	Kurtosis
Return	R	0.880	1.035	1.006	1.006	0.022	-4.650	27.276
Risk	σ	0.043	0.580	0.221	0.196	0.111	1.118	1.625
Price	P	754	61344	11671	7812	12395	1.979	5.211

4.4 Investment Combination Modelling

At this stage of decision-making to determine the optimal composition of investment based on the initial justified environment or in other words the decision taking in evaluating the financial efficiency of selected companies and introducing efficient companies as justified investment options, modeling of the composition of capital was employed as follows: 1) definition of the decision variable 2) definition of the objective function 3) identification of real constraints in decision making and 4) aggregation of the above as the final model.

Step 1: Defining the decision variable:

While following similar research by Kalayci et al. [7], in this study, the decision-making variable constituted the relative investment in an efficient company ith (for each of the 42 companies that were ultimately identified and selected as efficient companies and a viable investment option in the initial environment or decision-taking phase) that was defined as follow:

X_i: the amount of relative investment in the desired efficient company, i: 1, 2, …, 42.

Step 2: Defining the objective functions:

Based on the two criteria of risk and return, the objective functions were defined using Markowitz [11] mean-variance model as follows:

$$Max(R)=R_1X_1+R_2X_2+\ldots+R_nX_n \tag{4}$$
$$Min(\delta)=\delta_1X_1+\delta_2X_2+\ldots+\delta_nX_n$$

In this regard, R_i represents the average monthly return of the ith efficient company, X_i means the relative monthly investment in the ith efficient company, while δ_i represents the average monthly risk of the efficient company ith, R stands for the average monthly return on investment inefficient companies and finally, δ shows the average risk of investing in the efficient companies. Here the investor seeks to select a combination of investments to simultaneously have the highest return and lowest risk. By placing the performance values in Table 3, the objective functions of the first and second models for optimizing the investment mix are as follows:

$$Max\,(R) = 1.006\,X_1 + 1.023\,X_2 + 1.003\,X_3 + 1.011\,X_4 + 1.011\,X_5 + 0.880\,X_6 + 1.035\,X_7 + 1.026\,X_8 + 1.004\,X_9 + 1.023\,X_{10} + 0.994\,X_{11} + 1.018\,X_{12} + 1.005\,X_{13} + 1.014\,X_{14} + 1.013\,X_{15} + 1.000\,X_{16} + 1.031\,X_{17} + 1.015\,X_{18} + 1.000\,X_{19} + 1.002\,X_{20} + 1.003\,X_{21} + 1.000\,X_{22} + 1.004\,X_{23} + 1.000\,X_{24} + 1.013\,X_{25} + 1.001\,X_{26} + 1.009\,X_{27} + 1.000\,X_{28} + 1.002\,X_{29} + 1.004\,X_{30} + 0.997\,X_{31} + 1.010\,X_{32} + 1.000\,X_{33} + 1.012\,X_{34} + 1.006\,X_{35} + 1.012\,X_{36} + 1.009\,X_{37} + 1.016\,X_{38} + 1.018\,X_{39} + 1.003\,X_{40} + 1.018\,X_{41} + 1.000\,X_{42}$$

$$Min\,(\delta) = 0.179\,X_1 + 0.277\,X_2 + 0.127\,X_3 + 0.239\,X_4 + 0.266\,X_5 + 0.438\,X_6 + 0.212\,X_7 + 0.192\,X_8 + 0.190\,X_9 + 0.212\,X_{10} + 0.432\,X_{11} + 0.221\,X_{12} + 0.297\,X_{13} + 0.271\,X_{14} + 0.246\,X_{15} + 0.137\,X_{16} + 0.180\,X_{17} + 0.258\,X_{18} + 0.172\,X_{19} + 0.185\,X_{20} + 0.133\,X_{21} + 0.260\,X_{22} + 0.043\,X_{23} + 0.130\,X_{24} + 0.412\,X_{25} + 0.171\,X_{26} + 0.121\,X_{27} + 0.200\,X_{28} + 0.107\,X_{29} + 0.065\,X_{30} + 0.265\,X_{31} + 0.301\,X_{32} + 0.090\,X_{33} + 0.118\,X_{34} + 0.095\,X_{35} + 0.368\,X_{36} + 0.173\,X_{37} + 0.230\,X_{38} + 0.165\,X_{39} + 0.580\,X_{40} + 0.377\,X_{41} + 0.167\,X_{42} \tag{5}$$

Step 3: Applying the real constraints:

Considering the real limitations in selecting the optimal portfolio or investment combination, achieving the goals of maximizing returns and minimizing risk in choosing the investment combination, the final decision-making environment in selecting the optimal combination based on the defined investment constraints; these constraints are based on the conditions of the decision-maker and vary from person to person. Therefore, following the example of Kalayci et al [7], here we will refer to some of these limitations as an example:

a) Investment composition constraint: This constraint is affected by the type of definition of variables as a relative quantity and a relative investment in the investment portfolio or relative share of each efficient company from 1 investment unit, which when the relative investment is zero in other companies, the share of ith efficient company is equal to 1 and considering the condition that calls for real decision-making variables being non-negative as a relatively passive quantity, it is defined as follows:

$$0 \leq X_i \leq 1,\ i: 1, 2 \ldots, 42 \tag{6}$$

b) Investment budget constraint: This limitation is defined based on the present or an available budget as a ceiling or maximum amount of investment by the natural or legal person and therefore varies from person to person.

Here, based on the general assumption of limitations in investment budgets following Meghwani and Thakur [12] model, it is assumed that the investor seeks to buy a share that is relatively divided between different stocks.

In practice, it’s possible to calculate the number of final shares when the Rial budget is determined with due regard to the division of that budget on the average price of a share with the relevant optimal mix. This number will be multiplied at the relevant optimal mix and the share of each company becomes clear. When the figure is multiplied at the daily price of the said shares, it will give us the amount of share purchased from each company in Rial in the optimal composition. Accordingly, the budgetary limitations are generally defined per share as follows:

$$X_1+X_2+\ldots+X_{42}=1 \quad (7)$$

By placing the defined variables for 42 efficient companies, the above-mentioned budget limitations will be as follow:

$$X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9+X_{10}+X_{11}+X_{12}+X_{13}+X_{14}+X_{15}+X_{16}+X_{17}+X_{18}+X_{19}+X_{20}+X_{21}+X_{22}+X_{23}+X_{24}+X_{25}+X_{26}+X_{27}+X_{28}+X_{29}+X_{30}+X_{31}+X_{32}+X_{33}+X_{34}+X_{35}+X_{36}+X_{37}+X_{38}+X_{39}+X_{40}+X_{41}+X_{42}=1 \quad (8)$$

c) Minimum return corresponding bank interest constraint:

This limitation is based on the minimum risk-free return, for example, the return on investment in a one-year bank deposit, which is defined at 15% per annum and 1.25% per month according to the Central Bank. The basis of this portfolio constraint should be such that the return on investment is not less than the risk-free return:

$$R_1X_1+R_2X_2+\ldots+R_{42}X_{42}\geq 1.0125 \quad (9)$$

By substituting the average monthly return from Table (3) in the above relation, this limitation is defined as follows:

$$1.006\,X_1+1.023\,X_2+1.003\,X_3+1.011\,X_4+1.011\,X_5+0.880\,X_6+1.035\,X_7+1.026\,X_8+1.004\,X_9+1.023\,X_{10}+0.994\,X_{11}+1.018\,X_{12}+1.005\,X_{13}+1.014\,X_{14}+1.013\,X_{15}+1.000\,X_{16}+1.031\,X_{17}+1.015\,X_{18}+1.000\,X_{19}+1.002\,X_{20}+1.003\,X_{21}+1.000\,X_{22}+1.004\,X_{23}+1.000\,X_{24}+1.013\,X_{25}+1.001\,X_{26}+1.009\,X_{27}+1.000\,X_{28}+1.002\,X_{29}+1.004\,X_{30}+0.997\,X_{31}+1.010\,X_{32}+1.000\,X_{33}+1.012\,X_{34}+1.006\,X_{35}+1.012\,X_{36}+1.009\,X_{37}+1.016\,X_{38}+1.018\,X_{39}+1.003\,X_{40}+1.018\,X_{41}+1.000\,X_{42}\geq 1.0125 \quad ..... (10)$$

D) Minimum return corresponding market interest:

This limitation is determined based on the average performance in the capital market and is based on the assumption that the overall investment composition should be determined in such a way that the minimum return on investment composition is not lower than the average return in the market.

$$R_1X_1+R_2X_2+\ldots+R_{42}X_{42}\geq \overline{R} \quad (11)$$

Therefore, considering the industries corresponding to efficient companies, for each industry, there will be a limitation as follows and with due consideration of the real data:

$$1.006\,X_1+1.023\,X_2+1.003\,X_3+1.011\,X_4+1.011\,X_5+0.880\,X_6+1.035\,X_7+1.026\,X_8+1.004\,X_9+1.023\,X_{10}+0.994\,X_{11}+1.018\,X_{12}+1.005\,X_{13}+1.014\,X_{14}+1.013\,X_{15}+1.000\,X_{16}+1.031\,X_{17}+1.015\,X_{18}+1.000\,X_{19}+1.002\,X_{20}+1.003\,X_{21}+1.000\,X_{22}+1.004\,X_{23}+1.000\,X_{24}+1.013\,X_{25}+1.001\,X_{26}+1.009\,X_{27}+1.000\,X_{28}+1.002\,X_{29}+1.004\,X_{30}+0.997\,X_{31}+1.010\,X_{32}+1.000\,X_{33}+1.012\,X_{34}+1.006\,X_{35}+1.012\,X_{36}+1.009\,X_{37}+1.016\,X_{38}+1.018\,X_{39}+1.003\,X_{40}+1.018\,X_{41}+1.000\,X_{42}\geq 1.006 \quad (12)$$

e) Maximum risk limitation concerning the capital market: This limitation is determined based on the average performance in the capital market and is based on the assumption that the total investment composition should be determined in such a way that the maximum risk in the investment combination does not exceed the average risk in the capital market.

$$\delta_1X_1+\delta_2X_2+\ldots+\delta_nX_n\leq \overline{\delta} \quad (13)$$

Therefore, this limitation will be defined according to the risk of companies and its average will be calculated concerning efficient companies according to the actual data:

$$0.179\,X_1+0.277\,X_2+0.127\,X_3+0.239\,X_4+0.266\,X_5+0.438\,X_6+0.212\,X_7+0.192\,X_8+0.190\,X_9$$

$$+0.212\,X_{10} +0.432\,X_{11} +0.221\,X_{12} +0.297\,X_{13} +0.271\,X_{14} +0.246\,X_{15} +0.137\,X_{16} +0.180\,X_{17} +0.258\,X_{18} +0.172\,X_{19} +0.185\,X_{20} +0.133\,X_{21} +0.260\,X_{22} +0.043\,X_{23} +0.130\,X_{24} +0.412\,X_{25} +0.171\,X_{26} +0.121\,X_{27} +0.200\,X_{28} +0.107\,X_{29} +0.065\,X_{30} +0.265\,X_{31} +0.301\,X_{32} +0.090\,X_{33} +0.118\,X_{34} +0.095\,X_{35} +0.368\,X_{36} +0.173\,X_{37} +0.230\,X_{38} +0.165\,X_{39} +0.580\,X_{40} +0.377\,X_{41} +0.167\,X_{42} \leq 0.221 \quad \dots \quad (14)$$

Step 4: Final model:

Considering the definition of variables, objective functions and investment constraints, the final model of the optimal investment composition will be shown in Table 5:

Table 5: Mathematical Model of Optimal Investment Composition

The final model of the optimal combination of investment in efficient companies
X_i: the amount of relative investment in the desired efficient company, i: 1, 2, …, 42
$\text{Max}(R) = 1.006\,X_1 + 1.023\,X_2 + 1.003\,X_3 + 1.011\,X_4 +1.011\,X_5 +0.880\,X_6 +1.035\,X_7 + 1.026\,X_8 +1.004\,X_9 +1.023\,X_{10} +0.994\,X_{11} +1.018\,X_{12} +1.005\,X_{13} +1.014\,X_{14} +1.013\,X_{15} +1.000\,X_{16} +1.031\,X_{17} +1.015\,X_{18} +1.000\,X_{19} +1.002\,X_{20} +1.003\,X_{21} +1.000\,X_{22} +1.004\,X_{23} +1.000\,X_{24} +1.013\,X_{25} +1.001\,X_{26} +1.009\,X_{27} +1.000\,X_{28} +1.002\,X_{29} +1.004\,X_{30} +0.997\,X_{31} +1.010\,X_{32} +1.000\,X_{33} +1.012\,X_{34} +1.006\,X_{35} +1.012\,X_{36} +1.009\,X_{37} +1.016\,X_{38} +1.018\,X_{39} +1.003\,X_{40} +1.018\,X_{41} +1.000\,X_{42}$
$\text{Min}(\delta) = 0.179\,X_1 + 0.277\,X_2 + 0.127\,X_3 +0.239\,X_4 +0.266\,X_5 +0.438\,X_6 +0.212\,X_7 +0.192\,X_8 +0.190\,X_9 +0.212\,X_{10} +0.432\,X_{11} +0.221\,X_{12} +0.297\,X_{13} +0.271\,X_{14} +0.246\,X_{15} +0.137\,X_{16} +0.180\,X_{17} +0.258\,X_{18} +0.172\,X_{19} +0.185\,X_{20} +0.133\,X_{21} +0.260\,X_{22} +0.043\,X_{23} +0.130\,X_{24} +0.412\,X_{25} +0.171\,X_{26} +0.121\,X_{27} +0.200\,X_{28} +0.107\,X_{29} +0.065\,X_{30} +0.265\,X_{31} +0.301\,X_{32} +0.090\,X_{33} +0.118\,X_{34} +0.095\,X_{35} +0.368\,X_{36} +0.173\,X_{37} +0.230\,X_{38} +0.165\,X_{39} +0.580\,X_{40} +0.377\,X_{41} +0.167\,X_{42}$
S. T:
$0 \leq X_i \leq 1$, i: 1, 2…, 42
$X_1 + X_2 + X_3 +X_4 +X_5 +X_6 +X_7 +X_8 +X_9 +X_{10} +X_{11} +X_{12} +X_{13} +X_{14} +X_{15} +X_{16} +X_{17} +X_{18} +X_{19} +X_{20} +X_{21} +X_{22} +X_{23} +X_{24} +X_{25} +X_{26} +X_{27} +X_{28} +X_{29} +X_{30} +X_{31} +X_{32} +X_{33} +X_{34} +X_{35} +X_{36} +X_{37} +X_{38} +X_{39} +X_{40} +X_{41} +X_{42} = 1$
$1.006\,X_1 + 1.023\,X_2 + 1.003\,X_3 +1.011\,X_4 +1.011\,X_5 +0.880\,X_6 +1.035\,X_7 +1.026\,X_8 +1.004\,X_9 +1.023\,X_{10} +0.994\,X_{11} +1.018\,X_{12} +1.005\,X_{13} +1.014\,X_{14} +1.013\,X_{15} +1.000\,X_{16} +1.031\,X_{17} +1.015\,X_{18} +1.000\,X_{19} +1.002\,X_{20} +1.003\,X_{21} +1.000\,X_{22} +1.004\,X_{23} +1.000\,X_{24} +1.013\,X_{25} +1.001\,X_{26} +1.009\,X_{27} +1.000\,X_{28} +1.002\,X_{29} +1.004\,X_{30} +0.997\,X_{31} +1.010\,X_{32} +1.000\,X_{33} +1.012\,X_{34} +1.006\,X_{35} +1.012\,X_{36} +1.009\,X_{37} +1.016\,X_{38} +1.018\,X_{39} +1.003\,X_{40} +1.018\,X_{41} +1.000\,X_{42} \geq 1.0125$
$1.006\,X_1 + 1.023\,X_2 + 1.003\,X_3 +1.011\,X_4 +1.011\,X_5 +0.880\,X_6 +1.035\,X_7 +1.026\,X_8 +1.004\,X_9 +1.023\,X_{10} +0.994\,X_{11} +1.018\,X_{12} +1.005\,X_{13} +1.014\,X_{14} +1.013\,X_{15} +1.000\,X_{16} +1.031\,X_{17} +1.015\,X_{18} +1.000\,X_{19} +1.002\,X_{20} +1.003\,X_{21} +1.000\,X_{22} +1.004\,X_{23} +1.000\,X_{24} +1.013\,X_{25} +1.001\,X_{26} +1.009\,X_{27} +1.000\,X_{28} +1.002\,X_{29} +1.004\,X_{30} +0.997\,X_{31} +1.010\,X_{32} +1.000\,X_{33} +1.012\,X_{34} +1.006\,X_{35} +1.012\,X_{36} +1.009\,X_{37} +1.016\,X_{38} +1.018\,X_{39} +1.003\,X_{40} +1.018\,X_{41} +1.000\,X_{42} \geq 1.006$
$0.179\,X_1 + 0.277\,X_2 + 0.127\,X_3 +0.239\,X_4 +0.266\,X_5 +0.438\,X_6 +0.212\,X_7 +0.192\,X_8 +0.190\,X_9 +0.212\,X_{10} +0.432\,X_{11} +0.221\,X_{12} +0.297\,X_{13} +0.271\,X_{14} +0.246\,X_{15} +0.137\,X_{16} +0.180\,X_{17} +0.258\,X_{18} +0.172\,X_{19} +0.185\,X_{20} +0.133\,X_{21} +0.260\,X_{22} +0.043\,X_{23} +0.130\,X_{24} +0.412\,X_{25} +0.171\,X_{26} +0.121\,X_{27} +0.200\,X_{28} +0.107\,X_{29} +0.065\,X_{30} +0.265\,X_{31} +0.301\,X_{32} +0.090\,X_{33} +0.118\,X_{34} +0.095\,X_{35} +0.368\,X_{36} +0.173\,X_{37} +0.230\,X_{38} +0.165\,X_{39} +0.580\,X_{40} +0.377\,X_{41} +0.167\,X_{42} \leq 0.221$

4.5 Stock Portfolio Optimization

To adopt the desired option in capital decision making, at this stage of analysis, by relying on the genetic algorithm and the following meta-heuristic algorithm, we sought to optimize the stock portfolio in the form of determining the optimal investment composition of efficient companies to achieve maximum average returns. At the same time, the lowest risk and real limitations in decision-making were taken into consideration.

Step 1: optimization algorithm:

In this regard, the correction mechanism used in studies such as those of Ban et al. [3], Kuehn et al. [10] and Sen et al [17] have been developed to manage budget limitations and budget ceiling. Another corrective mechanism for considering the amount of budget Investment, capability, and ceiling constraints in the research of Ban et al. [3] have been developed and the algorithm used is shown in Table 6:

Table 6: The Meta-Heuristic Algorithm for Optimizing Stock Composition [3]

Modified algorithm of Ban et al. [3] in stock portfolio optimization	

Proposed repair mechanism
rocedure Repair(ω, ϑ)
$\delta \leftarrow 0$
$I_{nz} = \{i \mid \omega_i > 0\}$
$r_i = (\omega_i \bmod \vartheta_i) \quad \forall i \in I_{nz}$
$I_{LBV} = \{i \mid \omega_i - r_i < l_i\}$
if $\lvert I_{LBV} \rvert = 0$ then
$\omega_i \leftarrow \omega_i - r_i$
else
$\omega_i \leftarrow \omega_i + (\vartheta_i - r_i) \quad \forall i \in I_{LBV}$
$\omega_i \leftarrow \omega_i - (\omega_i \bmod \vartheta_i) \quad \forall i \in I_{nz}$
end if
$\beta = \sum_{i \in I_{nz}} \omega_i$
if $\beta > 1$ then
$a_i \leftarrow l_i + \vartheta_i - (l_i \bmod \vartheta_i) \quad \forall i \in I_{nz}$
$\omega_i \leftarrow a_i + \vartheta_i + \frac{\omega_i - a_i}{\sum_{i \in I_{nz}}(\omega_i - a_i)}\left(1 - \sum_{i \in I_{nz}} a_i\right) \quad \forall i \in I_{nz}$
else
$a_i \leftarrow u_i - (u_i \bmod \vartheta_i) \quad \forall i \in I_{nz}$
$\omega_i \leftarrow a_i - \frac{a_i - \omega_i}{\sum_{i \in I_{nz}}(a_i - \omega_i)}\left(\sum_{i \in I_{nz}} a_i - 1\right) \quad \forall i \in I_{nz}$
end if
	$r_i = (\omega_i \bmod \vartheta_i) \quad \forall i \in I_{nz}$
	$\delta \leftarrow \sum_{i \in I_{nz}} r_i$
	$I = \{i \mid \delta > \vartheta_i\}$
	$\vartheta_{min} \leftarrow \min\{\vartheta_i \mid i \in I\}$
	Choose an index k from $\{i \mid \vartheta_i = \vartheta_{min} : i \in I\}$
	$I_D \leftarrow 0$
	while $\delta \geq \vartheta_{min}$ do
	$I \leftarrow I \setminus I_D$
	if $\omega_k + \vartheta_{min} \leq u_k$ then
	$\delta \leftarrow \delta - \vartheta_{min}$
	$\omega_k \leftarrow \omega_k + \vartheta_{min}$
	else
	$I \leftarrow I \setminus \{k\}$
	$I_D \leftarrow I_D \cup \{k\}$
	end if
	$I \leftarrow \{i \mid \delta > \vartheta_i\}$
	$\vartheta_{min} \leftarrow \min\{\vartheta_i \mid i \in I\}$
	Choose an index k from $\{i \mid \vartheta_i = \vartheta_{min} : i \in I\}$
	end while
	end procedure

Step 2: Determining the optimization parameters:

Relying on the meta-heuristic algorithm and using the data mining process of the genetic algorithm, the optimization parameters including the number of generations, the number of iterations, base population, etc were defined as follows.

In this research, a binary genetic algorithm was used. In other words, the genetic operation was not applied directly on the variables themselves, but the coding method in base 2 was used. Also, the production of the first generation was carried out randomly. The initial population size used in this study was 100. The condition of stopping in the algorithm used was aimed at keeping the objective function constant for to allow reaching the maximum number of generations, which was considered 200 in this method. The number of elite chromosomes that entered the next generation was estimated at 3.5% of the population. In order to scale the value of the fitness function, a ranking scale was employed. The selection Tournament method was adopted to determine how to select chromosomes. The intersection rate, which represents the percentage of the population affected by the intersection operator, was considered to be 0.8 at best in the selection of the portfolio. The jump rate, which represents the percentage of the population affected by the jump operator, was assumed to be 0.1. Using the above parameters and assuming equal investment in all companies, (0.0238) as a justified starting point in the MATLAB software and the following numbers z_1, z_2, z_3, z_4, z_5, z_6 which played a major role in the fitness function, was calculated.

Step 3: simulation:

Using MATLAB software, the formulated model and the modified algorithm were performed with each of the three functions of the simulation process, and after 250 generations of simulation for the selected operators, the simulation operation was stopped. A summary of the performance for each stimulation is presented below and in the next section, the best answer is shown based on the calculations performed.

A) Selecting the optimal portfolio with the Tournament genetic operator

Following the assigned steps and by determining the assumptions and parameters, the algorithm can be simulated by MATLAB software with a repetition rate of 250 generations and an initial population of 150 by default. At this stage, using MATLAB software, the proposed meta-heuristic algorithm with the Tournament operator was used and simulation was carried out to build an optimal portfolio. Fig. 2 illustrates the rate of change of the fitting function in 250 generations.

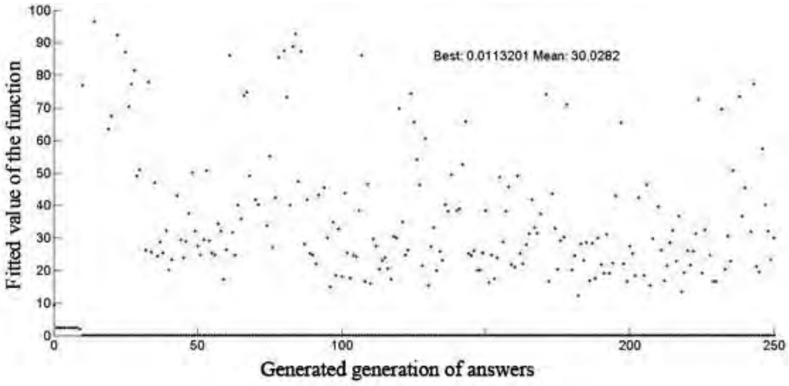

Fig. 2: Tournament Performance Changing Trend in Each Generation of Simulation.

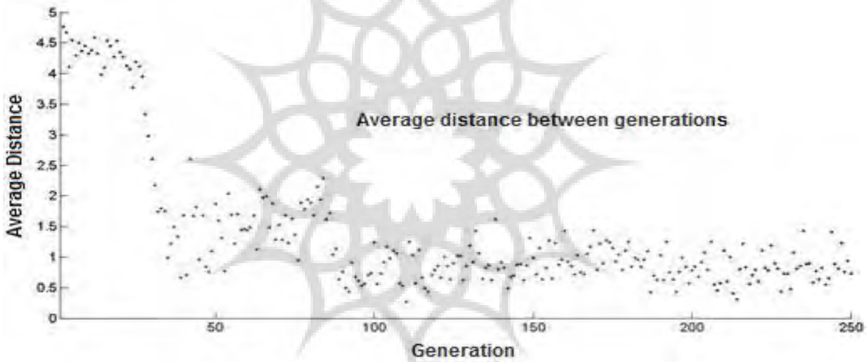

Fig. 3: Intergenerational Interval in The Tournament Operator.

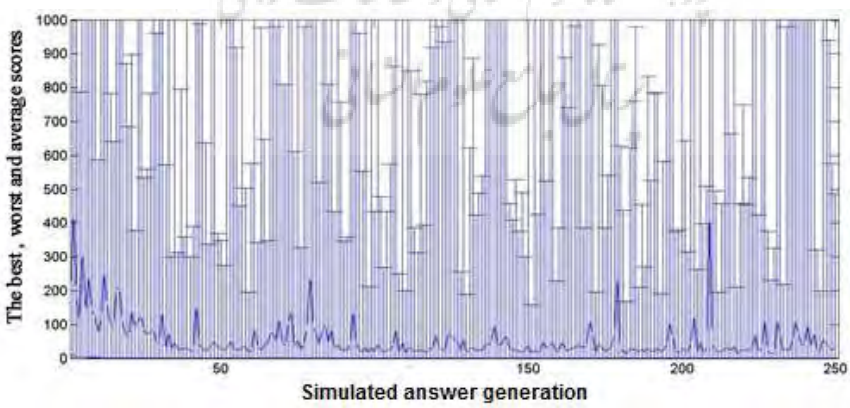

Fig. 4: The Best and Worst Answer and the Average Score in the Tournament Operator.

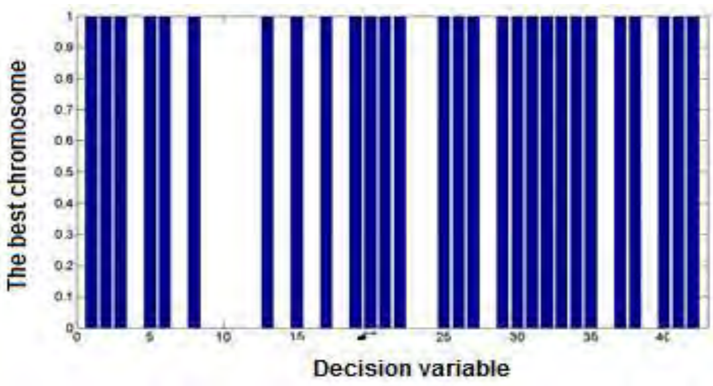

Fig. 5: The final Chromosome in the Tournament Operator.

Accordingly, Fig. 3 illustrates the interval between each generation of the proposed genetic algorithm compared to the previous generation of answers over 250 generations. Moreover, Fig. 4 shows the amount of best, worst, and average value of the fit function in each generation, using the genetic algorithm with the Tournament operator function. Fig. 5 depicts the selected chromosome (optimal portfolio) in the genetic algorithm after 250 generations:

B) Selecting the Optimal Portfolio with the Roulette Wheel Genetic Operator

Following the assigned steps and by determining the assumptions and parameters in the process of implementing the genetic algorithm simulation, the algorithm simulation by MATLAB software with a repetition rate of 250 generations and an initial population of 150 was carried out by default. Using MATLAB software, the proposed meta-heuristic algorithm with Roulette Wheel operator is employed and Fig. 6 illustrates the extent of changes of the fitting function corresponding to 250 generations.

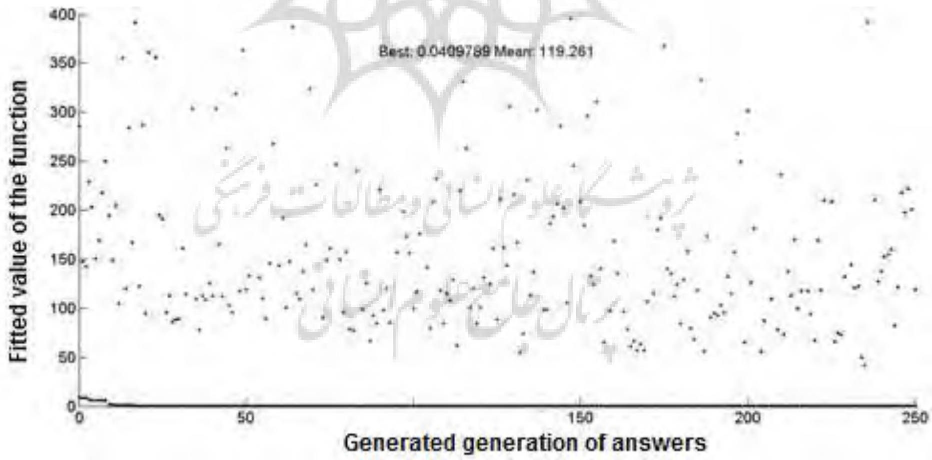

Fig. 6: The process of Changing the Performance of the Roulette Wheel Per Generation of Simulation of Answers.

Accordingly, Fig. 7 illustrates the space between each generation of the proposed genetic algorithm compared to the previous generation of responses with the Roulette Wheel operator over 250 generations:

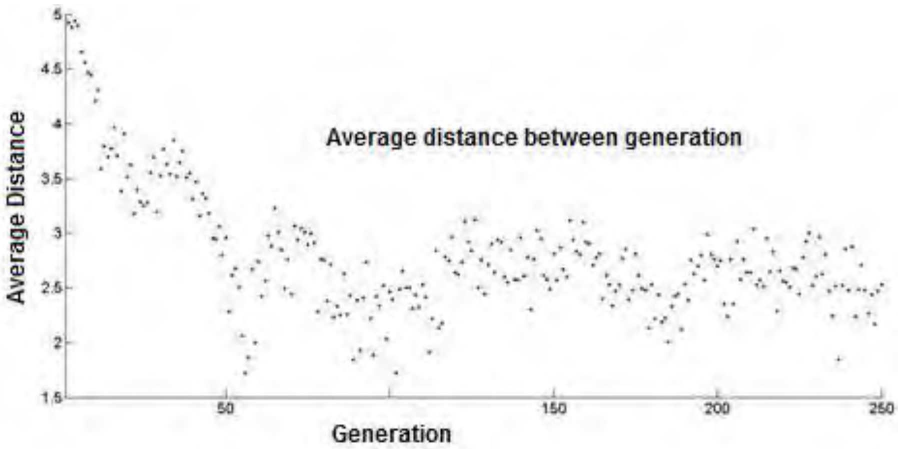

Fig. 7: Intergenerational Distance in the Roulette Wheel Operator.

Besides, Fig. 8 shows the best, worst, and an average value of the fitting function in each generation, using the genetic algorithm with the Roulette Wheel operator function:

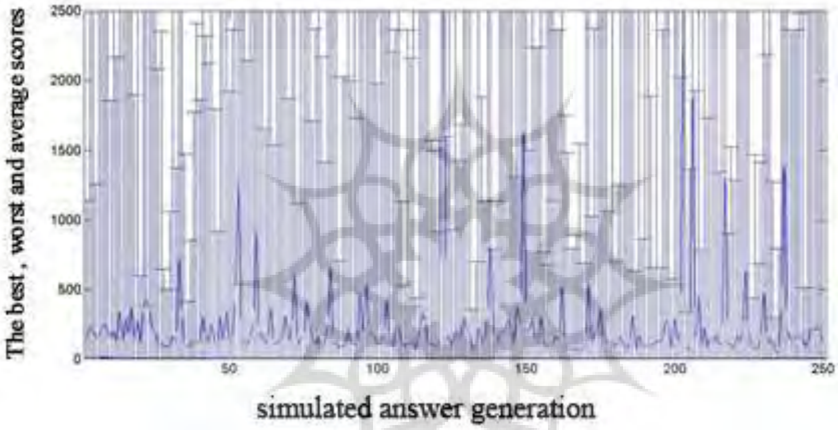

Fig. 8: The Best and Worst Answer and the Average Score in Roulette Wheel Operator.

Finally, in Fig. 9, the selected chromosome (optimal portfolio) is depicted using the genetic algorithm and the Roulette Wheel operator after 250 generations:

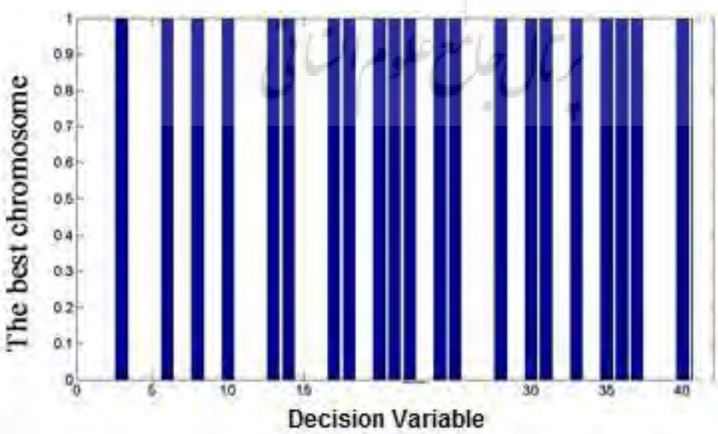

Fig. 9: The final chromosome in the Roulette Wheel operator.

C) Comparison of the performance of R&T genetic operators

According to the values of return and risk obtained in the simulation using two operators selected Tournament and Roulette Wheel, it is clear that the return of the Tournament operator is somewhat better but has a higher level of risk. In any case, based on the performance efficiency of the first operator is better and the performance of the two algorithms is compared in Fig. 10:

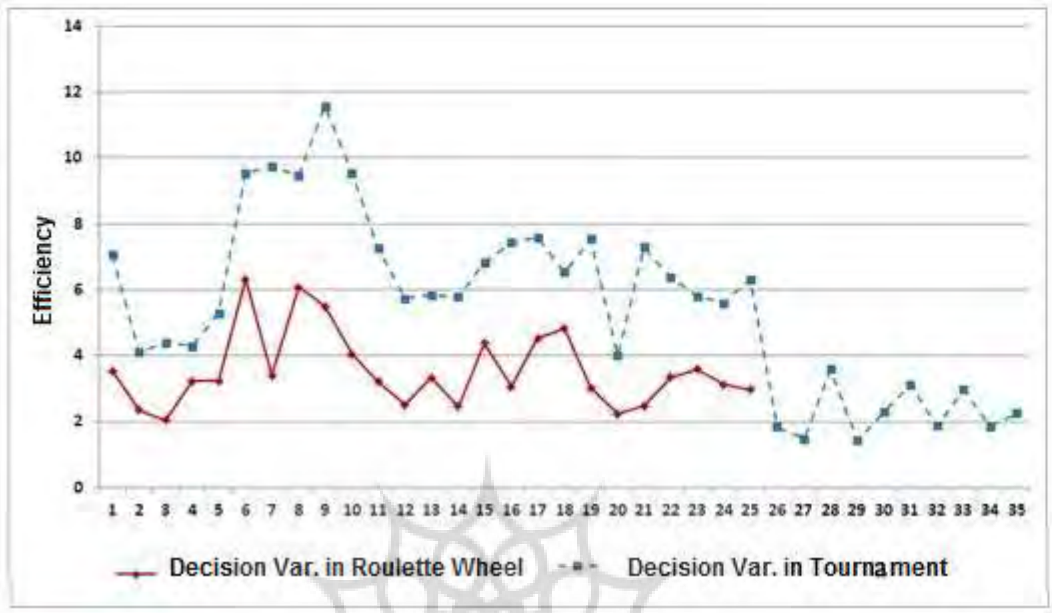

Fig. 10: Comparing the efficiency of dual operators.

Step 4: Decision making:

Finally, according to a more efficient operator, i.e. the Tournament operator, the optimal decision-making and in other words, the optimal combination of investment or the optimal portfolio of stocks were obtained in the form of Table 7:

Table 7: Deciding on the choice of investment mix

Variable	Code	Answer	Variable	Code	Answer	Variable	Code	Answer
X_{01}	7	0.085	X_{15}	64	0	X_{29}	147	0
X_{02}	9	0	X_{16}	78	0	X_{30}	148	0
X_{03}	10	0	X_{17}	91	0.092	X_{31}	150	0
X_{04}	15	0	X_{18}	94	0	X_{32}	151	0
X_{05}	16	0	X_{19}	97	0	X_{33}	154	0
X_{06}	17	0	X_{20}	99	0	X_{34}	161	0.072
X_{07}	24	0.105	X_{21}	100	0	X_{35}	169	0.103
X_{08}	26	0.070	X_{22}	111	0	X_{36}	177	0
X_{09}	34	0	X_{23}	125	0	X_{37}	178	0.099
X_{10}	41	0.101	X_{24}	134	0	X_{38}	179	0
X_{11}	48	0	X_{25}	135	0	X_{39}	180	0.098
X_{12}	49	0.078	X_{26}	136	0	X_{40}	181	0
X_{13}	57	0	X_{27}	141	0.097	X_{41}	182	0
X_{14}	59	0	X_{28}	145	0	X_{42}	183	0
Min Risk: 0.169								
Max Return: 1.018								

4.6 Sensitivity Analysis

To have relative confidence in the reliability of the simulation results, at this stage of the analysis of the findings, the sensitivity of the answers was analysed based on the evaluation of changes in the investment model. This analysis is based on estimating the confidence interval and also examining the change in constraints and examining its impact on the answers.

a) Estimation of Confidence Level

In this form, sensitivity analysis is a parametric (numerical) evaluation to re-examine the research model and its results. In the sensitivity analysis, the reliability of the results was changed from 95% to 99% confidence level and the optimal stock portfolio was obtained again. The results showed the changes made at this level of confidence. Fig. 11 shows the selected chromosomes at both levels of confidence after 250 generations. As illustrated in Table 7, the value of the fit function corresponds to a return of 1.018 and a risk level of 0.169, indicating no change in the event of changes in the confidence range.

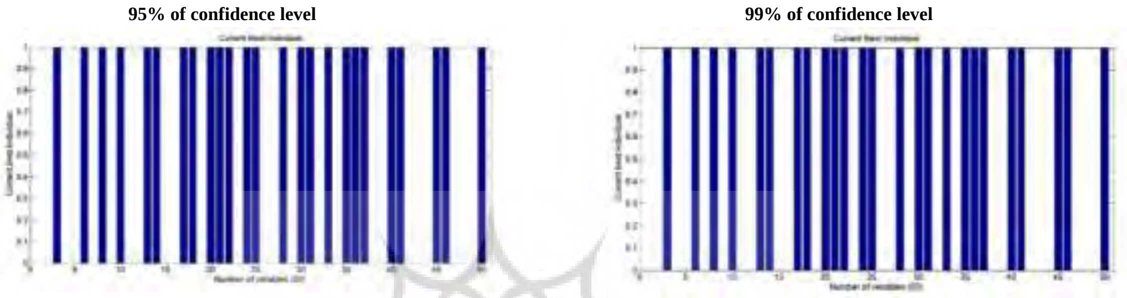

Fig. 11: Selective Chromosomes at Two Levels of Reliability.

b) Return Targeting

In this state of analysis, sensitivity analysis is performed to eliminate the second objective function, i.e. risk minimization, and the optimization of the investment combination model based on maximizing the return on investment. In other words, the decision-making model is optimized as a single-objective function. The optimization results in this step are summarized in Table 8.

Table 8: Optimal Composition of Return-Based Investment

Variable	Code	Answer	Variable	Code	Answer	Variable	Code	Answer
X_{01}	7	0.024	X_{15}	64	0.075	X_{29}	147	0
X_{02}	9	0.054	X_{16}	78	0	X_{30}	148	0
X_{03}	10	0	X_{17}	91	0.027	X_{31}	150	0
X_{04}	15	0.053	X_{18}	94	0.029	X_{32}	151	0.026
X_{05}	16	0.062	X_{19}	97	0	X_{33}	154	0
X_{06}	17	0	X_{20}	99	0	X_{34}	161	0.048
X_{07}	24	0.039	X_{21}	100	0	X_{35}	169	0.060
X_{08}	26	0.023	X_{22}	111	0	X_{36}	177	0.049
X_{09}	34	0	X_{23}	125	0	X_{37}	178	0.051
X_{10}	41	0.035	X_{24}	134	0	X_{38}	179	0.048
X_{11}	48	0	X_{25}	135	0.038	X_{39}	180	0.061
X_{12}	49	0.060	X_{26}	136	0	X_{40}	181	0
X_{13}	57	0	X_{27}	141	0.036	X_{41}	182	0.039
X_{14}	59	0.062	X_{28}	145	0	X_{42}	183	0
Min Risk: 0.231								
Max Return: 1.027								

The evaluation results in comparison with the initial model show that the level of total return has increased from 1.018 to 1.027 but based on the return criterion, the investment risk has increased from 0.169 to 0.231. In other words, the return on investment has increased by 0.88%, but by ignoring the risk, the total risk level has increased by 36.69%.

c) Risk targeting

At this stage, a sensitivity analysis was carried out to eliminate the first objective function, i.e. maximizing returns, and optimizing the investment mix model based on minimizing investment risk. In other words, the decision-making model is optimized as a single-objective function. The optimization results in this step are summarized in Table 9 as follows:

Table 9: The Optimal Combination of Risk-Based Investment

Variable	Code	Answer	Variable	Code	Answer	Variable	Code	Answer
X_{01}	7	0.085	X_{15}	64	0	X_{29}	147	0
X_{02}	9	0	X_{16}	78	0	X_{30}	148	0
X_{03}	10	0	X_{17}	91	0.092	X_{31}	150	0
X_{04}	15	0	X_{18}	94	0	X_{32}	151	0
X_{05}	16	0	X_{19}	97	0	X_{33}	154	0
X_{06}	17	0	X_{20}	99	0	X_{34}	161	0.028
X_{07}	24	0.105	X_{21}	100	0	X_{35}	169	0.037
X_{08}	26	0.070	X_{22}	111	0	X_{36}	177	0
X_{09}	34	0	X_{23}	125	0	X_{37}	178	0.025
X_{10}	41	0.101	X_{24}	134	0	X_{38}	179	0
X_{11}	48	0	X_{25}	135	0	X_{39}	180	0.026
X_{12}	49	0.078	X_{26}	136	0	X_{40}	181	0
X_{13}	57	0	X_{27}	141	0.097	X_{41}	182	0
X_{14}	59	0	X_{28}	145	0	X_{42}	183	0.048
Min Risk: 0.152								
Max Return: 1.014								

The results of the evaluation in comparison with the initial model show that the level of total risk has decreased from 0.169 to 0.152 but based on the risk criterion, the return on investment has decreased from 1.018 to 1.014. In other words, the level of investment risk has decreased by 10.05 percent, but by ignoring the return on decision-making, the total return has decreased by 0.39 percent.

d) Elimination of Investment Requirements

At this stage from sensitivity analysis, we removed the real constraints or investment requirements concerning risk and return and only considered the budget constraints and the relative definition of the share in investment and simultaneous targeting on return maximization and risk minimization. The optimization results at this stage are summarized in Table 10.

The evaluation results in comparison with the initial model show that the level of total return has increased from 1.018 to 1.036 but without considering the requirements and expectations in investment, the investment risk has increased from 0.169 to 0.225. In other words, the level of investment risk has increased by 33.14 percent, but by ignoring the investor's expectations and investment requirements according to the conditions of the capital market and risk-free investment in decision-making, the total return has increased by 1.77 percent. Although in model-based optimization, investment in some efficient companies has been zero, in this case, investment in all companies has been more or less.

Table 10: Optimal Investment Composition Based on the Elimination of Investment Requirements.

Variable	Code	Answer	Variable	Code	Answer	Variable	Code	Answer
X_{01}	7	0.019	X_{15}	64	0.035	X_{29}	147	0.025
X_{02}	9	0.024	X_{16}	78	0.025	X_{30}	148	0.011
X_{03}	10	0.025	X_{17}	91	0.010	X_{31}	150	0.034
X_{04}	15	0.020	X_{18}	94	0.037	X_{32}	151	0.023
X_{05}	16	0.027	X_{19}	97	0.030	X_{33}	154	0.031
X_{06}	17	0.017	X_{20}	99	0.018	X_{34}	161	0.030
X_{07}	24	0.027	X_{21}	100	0.012	X_{35}	169	0.033
X_{08}	26	0.019	X_{22}	111	0.025	X_{36}	177	0.025
X_{09}	34	0.018	X_{23}	125	0.011	X_{37}	178	0.029
X_{10}	41	0.032	X_{24}	134	0.034	X_{38}	179	0.034
X_{11}	48	0.019	X_{25}	135	0.023	X_{39}	180	0.008
X_{12}	49	0.017	X_{26}	136	0.031	X_{40}	181	0.021
X_{13}	57	0.029	X_{27}	141	0.030	X_{41}	182	0.029
X_{14}	59	0.023	X_{28}	145	0.033	X_{42}	183	0.018
Min Risk: 0.225								
Max Return: 1.036								

5 Conclusions

A review of the research literature shows that among the classic research issues in financial theory and operations research, the issue of portfolio optimization and the optimal combination of investment in assets can be seen in abundance. Investors, especially financial institutions such as banks, insurance companies, mutual funds, always deal with the problem of managing their budget and how to allocate them optimally to select the optimal portfolio of capital in the financial market. A pioneer in this field was Markowitz [11] who developed a portfolio model based on the mean-variance criterion in the context of a quadratic optimization problem with linear constraints. This quadratic model is defined to minimize the variance in the expected return, subject to the observance of constraints such as the composition of the investment and the investment budget.

According to the proposed research model, after identifying inputs and outputs based on knowledge analysis and content analysis, refining the effective factors based on Delphi survey and fuzzy DEMATEL model and during the evaluation of financial efficiency with data envelopment analysis approach, companies with the relative size 1 or 100% in terms of percentage were considered as efficient companies. Whereas it is assumed that with the inputs employed, it was not possible to produce more outputs. Other companies that have a financial efficiency rating of less than 1 or less than 100% are considered inefficient companies. Based on the research results, efficient companies in this research were identified based on the proposed research model and by relying on the model of Meghwani and Thakur [12] as justified investment options and as a justified initial space or decision-making, in comparison with other companies. In other words, these companies were identified and classified as reliable companies in the final decision-making. In the decision-making stage, based on the geometric average risk and return of efficient companies, the efficient companies were classified and according to the real constraints related to the budget, investors' requirements and expectations compared to market performance and risk-free investment, the issue of decision making with regard to the composition of investment was determined and presented as a multi-objective model.

Using a modified meta-heuristic algorithm and genetic algorithm and MATLAB software with dual operators, the investment combination was optimized. Finally, by eliminating the return or risk functions or eliminating the investment requirements and expectations and estimating the confidence intervals, the sensitivity of the answers was analyzed. The results showed that by eliminating the risk criterion, the level of total return on decision-making increases, but the risk also increases to a greater extent. Elimination of the return criterion in optimization also showed that lower risk can be achieved as a whole, but the total return will be reduced in this case. Elimination of investment requirements and expectations also led to higher returns and more risk, but more companies became involved in optimal investment.

The results showed that it's possible to employ and use the financial efficiency results obtained based on data envelopment analysis while relying on the proposed research model as a viable investment option and proper initial space or decision-taking and determining the optimal investment composition. Accordingly, investment companies and financial analysts could employ this method to choose justified capital options and determine the desired composition of investment. The results of this study showed that the selection of the optimal investment combination is usually made from a group of listed companies in a specific industry or companies that are known as top companies. Contrary to tradition, this choice can be made through a two-step process (decision-taking and decision-making). Based on the results of the research, it is recommended to investment companies and capital analysts, to rely on the evaluation of several criteria of financial efficiency and the application of data envelopment analysis model, to determine the initial justified scope of capital decisions and go forward with the so-called decision-taking.

Accordingly, capital market analysts and capital companies are advised, without relying on personal judgments and prejudices, to select and define a set of companies suitable for investment as efficient companies compared to all other possible companies. They are also advised to introduce companies that account for 100% of the measure of financial efficiency as justified investment options. Also, the research results based on simulation of multi-criteria investment model as well as the real constraints and further sensitivity analysis of the responses showed that the best answer with due note of both risk and return criteria in decision making for targeting and the real budgetary limitations, is expectations and requirements of investment. Accordingly, analysts in the capital market are advised to use the sensitivity analysis of investment combinations with due regard to the criteria of risk, return and change in requirements and expectations of investment. The following areas are suggested to researchers for further study and further improving the present study:

- Identifying financial and non-financial performance factors affecting capital decisions based on knowledge analysis and refining them based on quantitative algorithms such as stepwise regression, search algorithms such as decision tree and comparing their explanatory power in the form of a comparative study.
- Designing and application of expert system for decision taking and capital decision making in determining the optimal composition of investment based on data envelopment analysis approaches and mathematical simulation, algorithm and modelling employed in this research.
- Decision-taking based on the assumption of geometric convexity and using logarithmic function to evaluate financial efficiency and capital decision based on branch algorithms and decision tree limit to select the optimal combination of justified space.
- Decision-taking based on entropy theory to combine different metrics of financial performance and market evaluation in order to rank companies and select top companies and finally use the supra-innovative algorithm in capital decision to select the desired investment combination.
- Modelling the distinction between decision-taking and decision-making to determine the optimal composition of investment based on abnormal criteria and fluctuations in returns and considering macro-constraints in decision-making such as government economic policies.

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