Adv. Math. Fin. App., 2023, **8**(2), P. 589-604

Advances in Mathematical Finance & Applications

www.amfa.iau-arak.ac.ir
Print ISSN: 2538-5569
Online ISSN: 2645-4610
Doi: 10.22034/AMFA.2022.1950209.1684

Applied-Research Paper

Efficiency Analysis of Banking Sector in Presence of Undesirable Factors Using Data Envelopment Analysis

Fatemeh Bozorgi Gerdvisheh[a], Mansour Soufi[a*], Alireza Amirteimoori[b], Mahdi Homayounfar[a]

[a]Department of Industrial Management, Rasht Branch, Islamic Azad University, Rasht, Iran
[b]Department of Applied Mathematics, Rasht Branch, Islamic Azad University, Rasht, Iran

ARTICLE INFO

Article history:
Received 2021-12-19
Accepted 2022-02-23

Keywords:
Efficiency,
Profitability,
Undesirable Factors, Banking Sector

ABSTRACT

Banks play a crucial role in the growth and development of an economy. A profitable banking system contributes to economic stability and efficiency, helping to mitigate the impacts of sudden macroeconomic shocks. In order to improve efficiency and profitability, banks need to identify the factors that influence their performance. Accrued liabilities are one such factor that can hinder the profitability of banks. There are various methods available to assess the profitability of banks, each with its own advantages and disadvantages. Among these methods, data envelopment analysis (DEA) is widely recommended as a common approach to evaluate different efficiencies, including cost efficiency, revenue efficiency, technical efficiency, and overall profitability. By having access to prices and weights of inputs and outputs, financial managers can gain valuable insights for evaluating efficiencies and making informed decisions in the process of strategy development. The main objective of this study is to analyze the profitability of banks by considering accrued liabilities resulting from undesirable factors. To achieve this, relevant data was collected from 33 branches of a commercial bank in Gilan province, Iran, using managerial and weak disposability approaches. The results indicate that only three branches were ranked as efficient across three dimensions: technical efficiency, cost efficiency, revenue efficiency, and profitability. Furthermore, the study suggests that the lack of these efficiencies was not correlated with the profitability of the branches.

1 Introduction

Financial systems play an important role in the economy [35], and the performance of the banking sector, as the main financial system, determines the country's economic performance [34] and has an increasing effect on economic development [3]. Before venturing into a market, financial analysts conduct an analysis to determine its viability [27]. Profitability is a crucial objective in the banking sector [33], leading to numerous studies investigating the factors that affect bank profitability [7]. In recent decades, the stability and profitability of banks have become significant concerns for policymakers and bank managers, particularly after the global financial crises in 2007 and 2008 [2]. These crises highlighted the risks and their impact on financial operations across various institutions [8]. Furthermore,

* Corresponding author. Tel.: *+989111351135*
E-mail address: *msoufi45@gmail.com*

modern banks operate in a dynamic and challenging environment due to intense competition, changing consumer preferences, technological advancements, and ongoing innovation in systems and processes [22]. A profitable banking system enhances economic stability and efficiency, mitigating the effects of sudden macroeconomic shocks. Banks must identify the factors influencing their efficiency and profitability to effectively regulate and adopt appropriate policies [3]. In the banking industry, three major approaches have been identified [38]. The first approach focuses on analyzing branch efficiency, known as the service-oriented approach. It examines the cost efficiency of a bank's branches and aids managers in decision-making regarding the bank's sustainability [9]. The second approach is the intermediation approach, which involves the conversion of deposits into profitable investments, while the third approach is the revenue approach, which assesses the financial institution's ability to generate profits efficiently [35]. There are various methods to evaluate a company's profitability, among which data envelopment analysis (DEA) has increasingly been utilized to measure the efficiency of companies with multiple inputs and outputs based on production technologies [43].
Objective measurement standards can be used to assess an organization's current operational status and provide suggestions to improve performance, guiding future operations [37]. DEA, as a non-parametric method, constructs a set of production possibilities based on observed inputs and outputs to measure the productivity of a decision-making unit (DMU). It has been employed to measure several performance concepts, including cost efficiency (CE), revenue efficiency (RE), profit efficiency (PE), and profitability (PRO). Measuring cost and revenue efficiency is crucial in corporate production analysis [20]. However, cost and revenue efficiency models, in addition to requiring input and output quantities, also rely on prices, which may limit their practical use due to the simplified assumptions in the market [44]. In the real world, as production scales up, companies make economically justified changes to their organization, processes, and input characteristics. This leads to heterogeneity in inputs and potentially different prices among companies [43]. Additionally, organizations often face uncertainty when making production decisions, as inputs, outputs, and their prices are not easily measured with precision. Economic theory also suggests that companies with higher degrees of monopoly should receive different prices due to heterogeneity in production efficiency, which is empirically evident for inputs since most companies use an upward supply curve for purchasing decisions and encounter price assumption, a necessary and sufficient condition for Pareto performance in a competitive market [29]. Furthermore, the cost-effectiveness criterion proposed by Farrell in the 1980s has limitations even when inputs and outputs are homogeneous. Camanho and Dyson [13] argue that the measure of cost efficiency only reflects input inefficiency (technical inefficiency and/or allocation inefficiency) based on prices. They propose a comprehensive solution to measure cost-effectiveness by incorporating market and input inefficiencies. When inputs and outputs are heterogeneous, an alternative cost-efficiency/revenue-efficiency model is used to account for different input/output prices among firms [21]. In the financial literature, profit efficiency is more significant than cost efficiency since it criticizes organizations not only for using more
expensive inputs to produce the same output as other units but also for making less profit using the same input. Achieving profit efficiency requires maximizing revenues in addition to minimizing costs [21]. The factors that determine the profitability of banks can be categorized into internal and external factors. Internal factors include bank-level variables such as bank size, arrears, capital, assets, ownership, etc., while external factors encompass macroeconomic variables like inflation, GDP growth, monetary policy rates, etc. [18]. Numerous studies have examined the effect of internal and external factors on bank profitability, with a particular focus on developed economies. For example, Bekhet et al. [10], Skandar et al. [45], Öhman et al. [39], Bongini et al. [11], and Garcia and Guerrero [19] have investigated these factors. DQ and Ngo [15] studied the factors influencing the profitability of 23 countries

from 2002 to 2016 and found that the profitability of banks was positively related to the number of issued cards, ATMs, sales terminals, and capital market development, while it was inversely related to market power. Using the GMM, Ercegovac et al. [16] examined the determinants of banks' profitability in the EU from 2007 to 2019 and observed a decrease in profitability only with the ratio of overdue loans and expenses. Bucevska and Misheva [12] evaluated the performance of 127 commercial banks in selected Balkan countries from 2005 to 2009 and discovered that capital had a positive effect on banks' performance, while credit risk and the European reconstruction and development index had a negative effect. No significant effects were found for size, market density, and GDP. Ozili [40] investigated the determinants of profitability in 200 banks across 18 African countries and found that liquidity and capital risk had a positive effect on profitability and bank size, while credit risk had a negative effect. Ahmad et al. [1] examined the determinants of Latin American and East Asian banks' profitability between 2003 and 2014 and highlighted the positive impact of capital, bank size, and market focus on the profitability of East Asian banks, as well as the negative impact of liquidity.
However, no significant effects were observed for credit risk and GDP. Izadikhah [25] conducted a literature review of DEA approaches to financial evaluation and identified various DEA models that can be used to evaluate financial performance. Technical efficiency, financial evaluation, productivity analysis, portfolio selection, and financial sustainability were recognized as the five most contributing functions to profitability. Moslemi et al. [36] examined risk disclosure in reports provided to financial information users and identified positive and negative risk disclosure words, with capital adequacy ratio as an additional risk indicator. They also used DEA to analyze corporate governance in banks and found that board independence, dual CEO responsibilities, and major shareholders were influential variables. In their study, the BCC model of DEA was chosen due to its output-based nature, and they assessed the efficiency of 20 banks listed on the Tehran Stock Exchange.

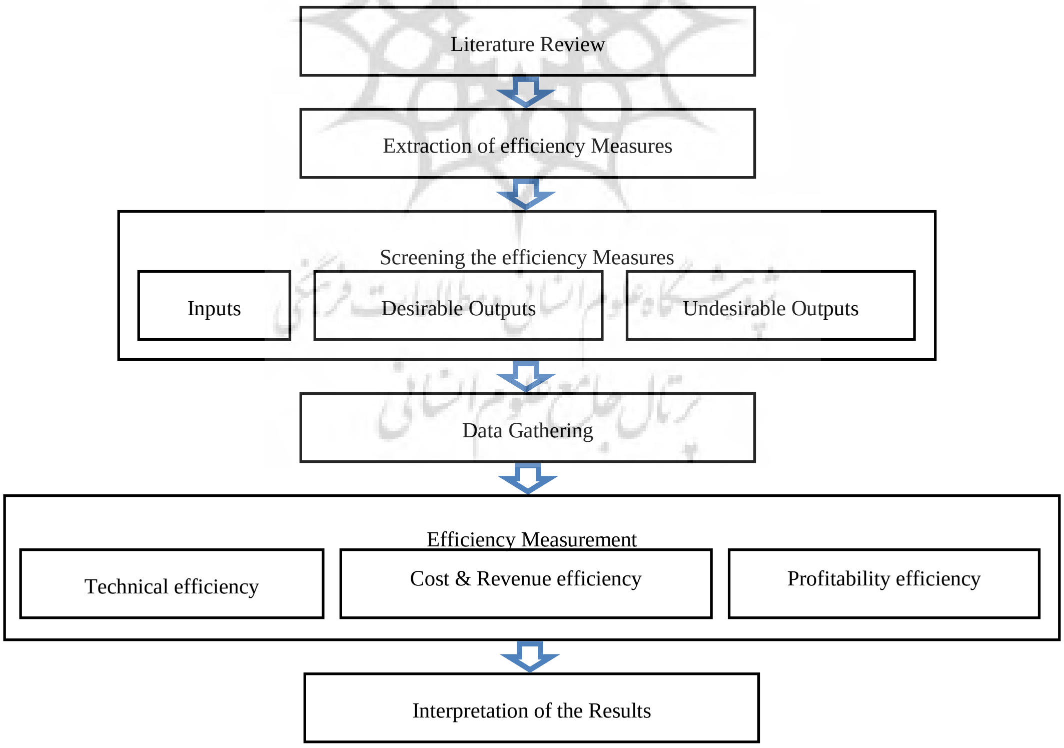

Fig 1. Research Framework

The aim of this paper is to measure the efficiency of bank branches in terms of cost efficiency, revenue efficiency, profit efficiency, and profitability while considering undesirable outputs. Additionally, this paper mathematically extends the efficiency measurement models by incorporating both weak and managerial disposability concepts. Measuring bank branch efficiencies using DEA models has gained significant attention in recent years from academics and practitioners, providing a reliable basis for decision-making. The paper is organized as follows: Section 2 examines the basic DEA models, including cost efficiency, revenue efficiency, profit efficiency, and profitability. Section 3 presents the mathematical model of profitability efficiency and its development in the presence of undesirable factors. Section 4 provides a practical study to solve the model, and the computational results and conclusions are presented in the last section. The research framework is illustrated in Figure 1.

2 Technical efficiency models with undesirable factors and managerial disposability

One main limitation of basic DEA models is the situation in which production process generates undesirable by-products such as wastes and pollutants, in addition to desirable ones [4]. These factors vary, depending on the type of industry, flight delays at the airport, overdue debts in the banking industry, and air pollutants and hazardous waste in the manufacturing industry. These undesirable outputs are produced unintentionally and inevitably along with the desired outputs and have a great impact on the efficiency and profitability of organizations [51]. Basic DEA models are incapable of measuring these outputs, so over the past three decades, modeling undesirable outputs has been a broad topic in the field of production efficiency in the DEA literature.

One of the methods in which undesirable outputs of the production process were included was presented by Kuosmanen [29] which called weak disposability. Uniform contractile factor used non-uniform contractile factor in desirable and undesirable output components of each unit [28]. Let, x_{io} be the optimal value of i-th input, v_{po} be the optimal value of p-th desirable output and w_{fo} be the optimal value of f-th undesirable output for the under-consideration unit (unit o). Suppose that x_{ij} be the i-th input of the DMU_j, v_{pj} be the p-th desirable outputs of DMU_j and w_{fj}, be the f-th undesirable output of DMU_j. In addition, suppose m_1 is the number of inputs considering managerial disposability, m_2 is the number of inputs considering weak disposability, P is the number of desirable outputs, F is the number of undesirable outputs, J is the number of DMUs, C is the price or weight of inputs, Q is the price or weight of outputs $\overline{x}_i^{(w)}$ is the optimal values of inputs considering weak disposability and $\bar{v}_r$ is the optimal values of desirable outputs considering weak disposability.

Production technology is indicated by $Y = \{(x, v, w)\}$. If θ_j representing the reduction of undesirable ttt uut is ooee by rcccc igg tee leeel of cctiii t,, tee eeefficinnt θ aan ee dffiddd ss tee bbtt emett fcctor.. If θ_j is considered the abatement factor of the Jth unit, it is decomposed into two parts to linearize $\theta_j = \mu_j + \lambda_j$. The displayed μ_j component is the part of the output that decreases with the activity level, while the display λ_j component is the part of the output of the j-th unit that remains active [42]. Using this notation, the activity analysis technology can be written as (1).

The undesirable outputs play an important role in estimating the efficiency of units. In unit evaluation, the goal is to use a method by which we can reduce undesirable outputs and increase desired outputs. Sueyoshi and Goto [48] introduced another concept of disposability called managerial disposability to reduce undesirable outputs. This principle states that the company increases an input vector to reduce undesirable outputs using technological or managerial innovations.

$$
\begin{aligned}
&T_J\left\{(x,v,w)\right\}: \\
&\textstyle\sum_{j=1}^{J}(\mu_j+\lambda_j)x_{ij}\le x_{io}\ , && i=1,\dots,m \\
&\textstyle\sum_{j=1}^{J}\lambda_j v_{pj}\ge v_{po}, && p=1,\dots,P \\
&\textstyle\sum_{j=1}^{J}\lambda_j w_{fj}= w_{fo}\ , && f=1,\dots,F \\
&\textstyle\sum_{j=1}^{J}(\mu_j+\lambda_j)=1 && \\
&\mu_j,\lambda_j\ge 0 && j=1,\dots,J
\end{aligned}
\tag{1}
$$

By increasing at least one of the input vectors, the company increases the desired outputs as much as possible and decreases the undesirable outputs. Therefore, production technology and model are shown as follows:

$$
P_T^M=\left\{(v,w),v\le\sum_{j=1}^{J}v_j\lambda_j\ ,w\ge\sum_{j=1}^{J}w_j\lambda_j\ \ ,x\le\sum_{j=1}^{J}x_j\lambda_j\ ,\sum_{j=1}^{J}\lambda_j=1,\quad j=1,\dots,J\right\}
\tag{2}
$$

Kmmmamnn's waak dioooaaii lity molll ddd oooooooa.. Gtt o's managerial disposability model differ in two constraints of inputs and undesirable outputs. In weak disposability, inputs constraint is defined by $\sum_{j=1}^{J}(\mu_j+\lambda_j)x_{ij}\le x_{io}$ and aims to reduce the inputs, but in managerial disposability, $\sum_{j=1}^{J}x_{ij}\lambda_j\ge x_{io}$ aims to increase at least one input to reduce the undesirable outputs. Also, undesirable outputs are due to the weak disposability of the boundary zone. Also, the constraint of undesirable outputs $\sum_{j=1}^{J}\lambda_j w_{fj}=w_{fo}$ in weak disposability of the border area is the efficiency of adverse outputs on the convex composition of all observed adverse outputs; in contrast, in managerial disposability, the efficiency boundary area of undesirable outputs is above or below the convex composition of the undesirable outputs observed [42].therefore Technical efficiency model with undesirable factors and both of managerial and weak disposability is shown as follows:

$$
\begin{aligned}
&MAX\ \ \varphi \\
&S.T. \\
&\textstyle\sum_{j=1}^{J}\lambda_j x_{ij}^{(M)}\ge x_{io}^{(M)}, && i=1,\dots,m_1, \\
&\textstyle\sum_{j=1}^{J}(\mu_j+\lambda_j)x_{ij}^{(w)}\le x_{io}^{(w)}, && i=1,\dots,m_2 \\
&\textstyle\sum_{j=1}^{J}\lambda_j v_{pj}\ge\varphi v_{po}\ , && p=1,\dots,p \\
&\textstyle\sum_{j=1}^{J}\lambda_j w_{fj}\le w_{fo}\ , && f=1,\dots,F \\
&\textstyle\sum_{j=1}^{J}(\mu_j+\lambda_j)=1 && j=1,\dots,J \\
&\lambda_j,\mu_j,\ge 0\quad ,
\end{aligned}
\tag{3}
$$

It is easy to show that model (3) is feasible. Because $\lambda_j=0,\ j\ne o,\lambda o=1,\mu_j=0,\forall j,j=1,\dots,J,\varphi=1$.

Constraint (1) ensures that increase in one of the inputs is based on the principle of managerial disposability to reduce undesirable outputs. Constraint (2) ensures that decrease in one of the inputs is based on the principle of weak disposability. Constraint (3) shows the increase in desirable outputs is based on the principle of weak disposability. Constraint (4) shows the increase in undesirable outputs also, is based on the principle of weak disposability and Constraint (5) represents the variable return to scale, where sum of μ and λ variables is equal to 1.

3.1 Cost and revenue efficiency model

The classical DEA model measures the efficiency of units based on inputs and outputs. However, priorities and weights can be applied to inputs and outputs, which can be analyzed to obtain more accurate information. Assume $C \in R_{+}^{m}$, is the input weight or price. Thus, the production cost of a DMU with the input-output bundle of (x_o, y_o) can be computed as $C^t x_o = \sum_{i=1}^{m} c_i x_{io}$. If we find the minimum production cost of this DMU then we can find the cost efficiency of this DMU. Note, the answer must be less than or equal to one, then we say DMU is efficient otherwise is inefficient. Also, cost efficiency index DMU_o is defined by the ratio of the lowest cost to the actual cost. $\tilde{x}_i$ Is the optimal solution of the linear programming model. Model (4) proposed the cost efficiency model considering undesirable factors and the principle of managerial and weak disposability.
It is not difficult to show that model (4) is feasible. Since, $\lambda_j = 0,\ j \neq o, \lambda o = 1, \mu_j = 0, \bar{x}_i = x_{io}$ Is a feasible solution of model (4). In addition, suppose $q \in R_{+}^{P}$ is the output weight or price. So, the real revenue DMU_o is calculated by $q^t y_o = \sum_{p=1}^{P} q_p y_{po}$. The revenue efficiency index is defined as the ratio of the optimal income to real income, so $RE_O = q^t \tilde{y}_p / q^t y_o = \sum_{p=1}^{P} q_p \tilde{y}_p / \sum_{p=1}^{P} q_p y_{po}$. Revenue efficiency is achieved if the answer to the fraction is greater than or equal to one, otherwise it is inefficient [21]. Model (5) proposed the revenue efficiency model considering undesirable factors and the principle of managerial and weak disposability.

$$
\begin{aligned}
& Min\,(\sum_{i=1}^{m_2} c_i \bar{x}_i) / (\sum_{i=1}^{m_2} c_i x_{io}^{(w)}) \\
& st: \\
& \quad \sum_{j=1}^{J} (\lambda_j + \mu_j) x_{ij}^{(w)} \leq \bar{x}_i && i = 1,\dots,m_2 \\
& \quad \sum_{j=1}^{J} \lambda_j x_{ij}^{(M)} \geq x_{io}^{(M)} && i = 1,\dots,m_1 \\
& \quad \sum_{j=1}^{J} \lambda_j v_{pj} \geq v_{po} && p = 1,\dots,P \\
& \quad \sum_{j=1}^{J} \lambda_j w_{fj} = w_{fo} && f = 1,\dots,F \\
& \quad \sum_{j=1}^{J} (\lambda_j + \mu_j) = 1 \\
& \quad \lambda_j, \mu_j, \bar{x}_i \geq 0
\end{aligned}
\qquad (4)
$$

$$
\begin{aligned}
& Max\,(\sum_{p=1}^{P} q_p \bar{v}_p)\,/\,(\sum_{p=1}^{P} q_p v_{po}) \\
& st: \\
& \quad \sum_{j=1}^{J} (\lambda_j + \mu_j) x_{ij}^{(w)} \le x_{io}^{(w)} \qquad i = 1,\dots,m_2 \\
& \quad \sum_{j=1}^{J} \lambda_j x_{ij}^{(M)} \ge x_{io}^{(M)} \qquad i = 1,\dots,m_1 \\
& \quad \sum_{j=1}^{J} \lambda_j v_{pj} \ge \bar{v}_p \qquad p = 1,\dots,P \\
& \quad \sum_{j=1}^{J} \lambda_j w_{fj} = w_{fo} \qquad f = 1,\dots,F \\
& \quad \sum_{j=1}^{J} (\lambda_j + \mu_j) = 1 \\
& \quad \lambda_j, \mu_j, \bar{v}_p \ge 0
\end{aligned}
\tag{5}
$$

The feasibility of model (5) is guaranteed, because $\lambda_j = 0,\ j \ne o,\ \lambda o = 1, \mu_j = 0,\ \bar{v}_p = v_{po}$, is a feasible solution of model (5).

3.2 Profitability model with undesirable factors

Assume we have J- DMUs and $C \in R_+^m$ is the weight or price of inputs and $q \in R_+^p$ is weight or price of output. Profitability is calculated using the method (6) considering undesirable factors and the principle of managerial and weak disposability. In this model $\tilde{x}$ and $\tilde{y}$ are the optimal values and x_{io} , y_{po} are the input and output values of DMU_O, and the functional goal of the model is to increase maximum profitability. As can be seen, model (6) is a fractional planning model that can be converted to a linear model by using cooper et al [14]. The profitability of DMU_j is maximized as follows:

$$
\begin{aligned}
& PR = Max \frac{\left(\dfrac{\sum_{r=1}^{s} p_r \tilde{v}_{ro}}{\sum_{i=1}^{m_2} c_i^w \tilde{x}_{io}^w} \right)}{\left(\dfrac{\sum_{r=1}^{s} q_r v_{ro}}{\sum_{i=1}^{m_2} c_i^w x_{io}} \right)} \\
& s.t. \quad \textstyle\sum_{j=1}^{J} (\lambda_j + \mu_j) x_{ij}^{(w)} \le \tilde{x}_i^{(w)}, \qquad i = 1,\dots,m_2, \\
& \textstyle\sum_{j=1}^{J} \lambda_j x_{ij}^{(M)} \ge x_{io}^{(M)}, \qquad i = 1,\dots,m_1 \\
& \textstyle\sum_{j=1}^{J} \lambda_j v_{rj} \ge \tilde{v}_r \ , \qquad \mathrm{r} = 1,\dots,s \\
& \textstyle\sum_{j=1}^{J} \lambda_j w_{fj} \le w_{fo} \ , \qquad f = 1,\dots,F \\
& \textstyle\sum_{j=1}^{J} (\lambda_j + \mu_j) = 1 \\
& \lambda_j, \mu_j, \tilde{x}_i^w, \tilde{v}_r, \ge 0,
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
& PR^* = Max \frac{\sum_{r=1}^{s} p_r \bar{v}_r}{\left(\dfrac{\sum_{r=1}^{s} p_r v_{ro}}{\sum_{i=1}^{m_1} c_i^{(w)} x_{io}^{(w)}} \right)} \\
& s.t. \\
& \sum_{i=1}^{m_2} c_i^{(w)} \bar{x}_i^{(w)} = 1 \\
& \textstyle\sum_{j=1}^{J} (\lambda_j + \mu_j) x_{ij}^{(w)} \le \bar{x}_i^{(w)}, \qquad i = 1, \dots, m_2, \\
& \textstyle\sum_{j=1}^{J} \lambda_j x_{ij}^{(M)} \ge T x_{io}^{(M)}, \qquad i = 1, \dots, m_1 \\
& \textstyle\sum_{j=1}^{J} \lambda_j v_{rj} \ge \bar{v}_r \ , \qquad \mathrm{r} = 1, \dots, s \\
& \textstyle\sum_{j=1}^{J} \lambda_j w_{fj} = T w_{fo} \ , \qquad f = 1, \dots, F \\
& \textstyle\sum_{j=1}^{J} (\lambda_j + \mu_j) = T \\
& \lambda_j, \mu_j, \bar{x}_i^{(w)}, \bar{v}_r, \ge 0,
\end{aligned}
\tag{7}
$$

In the objective function of model (6), the profitability of DMU_j is maximized. Clearly, this model is a linear fractional programming problem. It can easily be transformed into a linear form by using the method of cooper et al [14] as (7).

In which $\sum_{i=1}^{m} c_i \tilde{x}_i = \frac{1}{T}$, $T \tilde{x}_i = \bar{x}_i$, $T \tilde{y}_r = \bar{y}_r$, $T \lambda_k = \bar{\lambda}_k$ Suppose $PR^{\max}$ is the optimal objective value of model (7). Clearly, the optimal value of PR^* in model (7) must satisfy $PR^* \in (0, PR^{\max}]$. A sequence of linear models is solved to achieve a good approximation to PR^*. When the model is transformed into a linear form, constraint 1 shows the denominator of the objective function. The constraints 2 and 3 show inputs that are considered weak and managerially disposable, while constraints 4 and 5 highlight undesirable and undesirable outputs.

4 Empirical Example

In this section, the model proposed for calculating profitability with the presence of undesirable factors is implemented in 33 branches of Saderat bank in Gilan province, where each branch is considered as a DMU. In this research, four inputs, three desirable outputs and one undesirable output are considered ss et molll 's elemnnt.. Tsss e fatt ors wrre sll ett aa amggg mfff f ftt ors ssigg Dll hhi met.... oome similar research studies are summarized in Table 1.

Each branch in our study was considered a DMU. Data on 33 sample branches were selected and derived from operations during the first six months of 2020. We used eight variables from the data set as inputs and outputs. Proportionate to the volume of the operations, each branch uses costs (staff costs and operational cost (x_1) assuming managerial disposability with a weight of(0.4), capital (x_2) with a weight of (0.15), the number of bank accounts (x_3) with a weight of (0.2), and the number of transactions (x_4) with a weight of (0.25) assuming weak disposability.

Table 1: DEA and Banking Efficiency

Output	Input	Reference
Total loans	Fixed assets, Total deposits, Personnel expenses	Henriques et al. [23]
Loans, Investments	Personnel expenses, Deposits, Fixed assets	Kamarudin et al. [27]
Loans from customers, Other loans, Securities	Number of employees, Deposits from other banks, Client deposits	Stewart et al. [47]
Total deposits, Pre-tax income, Total credit operations	Total expenses (except personnel), Personnel expenses	Wanke et al. [54]
Total loans, Investments, Income	Total deposits, Capital	..... . and Güneş 556]
First stage: Administrative expenses, Personnel expenses. Second stage: Equity, fixed assets.	First stage: Number of agencies, Number of employees. Second stage: Administrative Expenses, Personnel expenses	Wanke and Barros [55]
Loans, Net interest income	Deposits, Personnel expenses	ee pková 44l .
Loans and net interest income. Undesirable output: Provision for credit losses	Expenses with personnel, Fixed assets, Deposits	Svitalkova [50]
Deposits, loans, Profit, Charge	branch uses costs, number of cheque accounts	Amirteimoori et al. [5]
Loans, Securities, Interbank loans.	Deposits, Number of employees, Fixed assets	Assaf et al. [6]
Total loans, Investments, Deposits.	Total expenses (except personnel), Personnel expenses, Interest expenses	Staub et al. [46]

The operational costs (excluding staff costs) include the cost of maintenance, electricity, cleaning, and security services. The staff costs are the salaries paid to all employees of the branches. Now, we introduce the desirable output variables. The first output is four main deposits (O_1) by weight (0.3). Each branch attracts funds from customers, and this is a competition between the branches. The deposit in each branch is the result of the attraction of the funds from the customers. Profit (O_2) by weight (0.25) loans (O_3) by weight (0.45), the undesirable output variable of overdue debts (O_4) by weight (0.2), these are the next four outputs used in our study. Due to the existing considerations, we were unable to provide details of small values of indicators, so a statistical summary of information about inputs and outputs is displayed in Fig. 2. The data are given in Table 2.

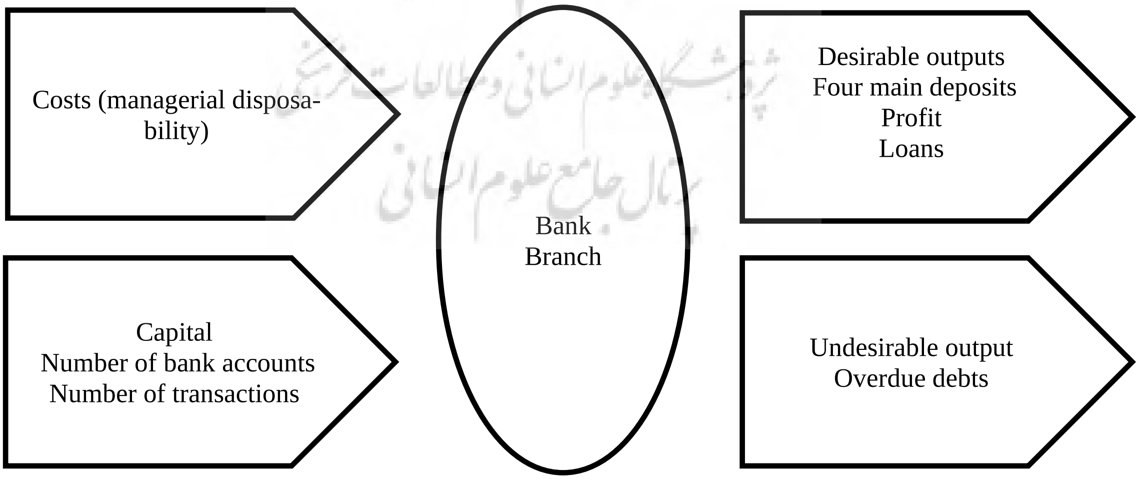

Fig 2. Input/Output Measures

Table 2: Statistical Summary of Indicators (1,000,000 IRR)

Index	Cost(x_1)	Capital (x2)	Number of accounts (x3)	Number of transac-tions (x_4)	Four main deposits (O_1)	Profit (O_2)	Loans (O_3)	Overdue debts (O_4)
Mean	4806.76	48907.01	4945.19	99562	793868.83	77925.02	732706.70	4513.45
S.D.	1057.99	11531.86	1019.90	50725	122666.10	12737.31	10982.27	1113.26
MAX	3053.99	27845.94	2999.60	72851.18	464158.78	42900.29	285028.10	4513.45
MIN	1263.62	12336.56	1230.08	16469.95	220066.97	20019.42	176462.90	2070.59

After determining the inputs and outputs of the bank, we calculate the technical efficiency, cost, income, and profitability of the 33 branches of the Saderat bank in Gilan province. The results of the model implementation in GAMS software are shown in Table 3.

Table 3: Efficiency and Profitability of Units

Units	Technical Efficiency	Cost Efficiency	Revenue Efficiency	Profitability
DMU1	0.472	0.8114	1.000	0.5115
DMU2	1.000	0.9924	1.000	0.9524
DMU3	1.000	0.6513	0.9903	0.6316
DMU4	0.734	0.5655	0.5808	0.1754
DMU5	0.649	0.5718	0.5215	0.2610
DMU6	0.552	0.5455	0.3915	0.2330
DMU7	1.000	1.000	1.000	0.7013
DMU8	1.000	0.7009	1.000	1.0000
DMU9	1.000	0.5673	0.7589	0.4954
DMU10	1.000	0.9370	1.000	0.4993
DMU11	1.000	1.000	1.000	1.0000
DMU12	1.000	1.000	1.000	1.0000
DMU13	0.757	1.000	1.000	0.2798
DMU14	1.000	1.000	1.000	0.1447
DMU15	1.000	0.8428	0.9871	0.6660
DMU16	1.000	0.9851	1.000	0.7277
DMU17	1.000	0.8210	1.000	0.6435
DMU18	1.000	1.000	1.000	1.0000
DMU19	0.841	0.7135	0.6773	0.5983
DMU20	1.000	0.8465	1.000	0.2242
DMU21	1.000	0.8022	1.000	1.0000
DMU22	1.000	0.7481	0.8426	1.0000
DMU23	0.978	0.5598	0.2088	0.1448
DMU24	0.705	0.7361	0.6313	0.5566
DMU25	0.503	0.8181	0.4355	0.3278
DMU26	1.000	0.9293	1.000	0.6470
DMU27	1.000	0.5655	0.9451	0.6570
DMU28	0.645	1.000	1.000	1.0000
DMU29	0.858	0.5652	1.000	0.8686
DMU30	1.000	0.8471	1.000	0.8212
DMU31	0.879	0.5546	0.7156	0.5174
DMU32	1.000	0.9041	0.5372	0.3441
DMU33	1.000	0.6266	1.000	0.5926

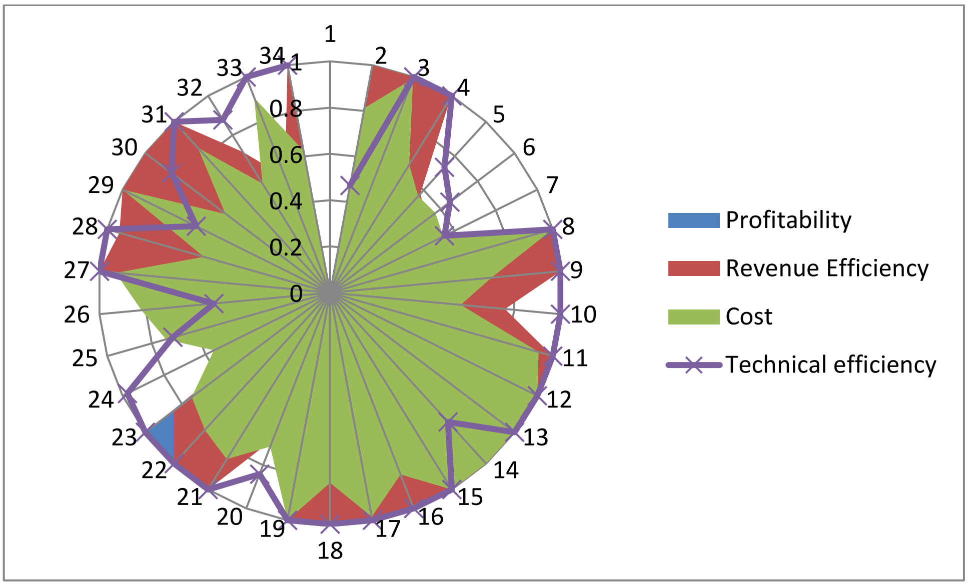

Fig. 3: The Efficiency Scores of Different Models

According to the results in Table 3, we can categorize the decision-making units. The results showed that 21 units (2, 3, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 22, 26, 27, 30, 32 and 33) were efficient, but 12 units were inefficient.

Table 4: Mean Values and Standard Deviation of Slack

	SI1	SI2	SI3	SI4	SO1	SO2	SO3	SO4
Mean	268.8415	1664.00	390.00	6691.36	22163.12	3285.06	37216.64	289.074
S. D	586.1973	6098.73	847.12	11675.28	59572.47	8406.09	92408.47	779.47

Branches that have technical efficiency do not change the amount of their inputs and outputs. However, inefficient branches are required to change the amount of their inputs and outputs to reach the efficiency limit. For example, Branch 4, which was recognized as an inefficient unit, is required to increase its costs by 333.84 units, its transaction number by 30349.72 units, and its four main deposits by 97216.22 units. Also, Branch 25, as an inefficient unit, must increase its transaction number by 4553.15 units and its loan number by 60368.44 units to reach the efficiency limit.
In terms of cost efficiency, out of the total of 33 branches, only the branches (7, 11, 12, 13, 14, 18 and 28) have cost efficiency, which means that only 21.2% of the units have cost efficiency and the rest are inefficient. According to the average and standard deviation of the available data, units to reach and remain on the cost efficiency limit must reach an average of 351017.29 units in their inputs in the variable of costs, and a standard deviation of 693507.17 units, also in the output. Including the variable of loans amount to an average of 19408.22545 and standard deviation of 37898.11589 units. The results of the mean and standard deviation of the optimal values of inputs and outputs, as well as their weights, are shown in Table 5. Based on the results of data analysis, out of the 33 branches of the bank in question, only branches (1, 2, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 20, 21, 27, 28, 29, 30, and 33) have revenue efficiency. It means that 57.57% of the units, i.e., more than half of the units, have revenue efficiency. The mean and standard deviation of optimal inputs and outputs with their respective weights in Table

5 show that the average value of branch capital to reach the efficiency boundary is 28672.57 with a standard deviation of 13774.51 units and the average value of the optimal branch fee to reach or stay on the efficiency boundary is 46567.76101 units with a standard deviation of 17687.9 units.

Table 5: Optimal Values of Indicators Using Cost Efficiency Model

Type of Eff.	Measure	Cost(x_1)	Capital (x2)	Number of accounts (x3)	Number of transactions (x_4)	Four main deposits (O_1)	Profit (O_2)	Loans (O_3)	Overdue debts (O_4)
Cost Eff.	Mean	55076.09	2467.58	24370.98	2485.06	4513.45	141637.6	22545.2	223517.38
	S. D	7529.44	396.78	9637.43	1995.79	2038.98	108921.1	11589.4	145711.50
Revenue Eff.	Mean	2768.35	28672.57	3464.62	79005.65	524774.13	46567.8	370471.3	4513.45
	S. D	1666.69	13774.51	1726.55	25721.92	203765.07	17687.9	190577.0	1113.26
Profitability	Mean	2341.75	67122.62	2374.71	32982.086	528217.84	50134.4	687759.4	4513.45
	S. D	1540.75	6050.54	464.80	11206.83	135347.23	14450.86	153801.7	1113.26

After calculating the technical efficiency, cost efficiency, and revenue efficiency, the profitability efficiency of the branches was calculated. Based on the results of data analysis, only branches (8, 11, 12, 21, 22, 28), i.e., 6 branches out of the 33 branches, have profitability performance. Also, the average of optimal values of inputs and outputs along with their weights in branch (7) states that these branches must have an average of 71322.23 units and standard deviation of 74874.8 units in the number of bank accounts, 7473.23 units and standard deviation of 75276.1 units in costs, 44474.5 units and standard deviation of 85981.14450 units in commission to reach or stay on the efficiency boundary.
According to the results, 3 branches (11, 12 and 18) out of 33 were efficient in terms of technical efficiency, cost efficiency, revenue efficiency, and profitability efficiency. This means that it has made the most of its resources to achieve efficiency and has also achieved profitability. Although branches (7 and 14) were efficient in terms of technical efficiency, cost efficiency, and revenue efficiency, they could not achieve profitability in their branches. Also among the branches, branch 28 has technically inefficient but it has cost efficiency, revenue efficiency, and profitability efficiency. Branch 21 is cost-inefficient only, meaning that it has used more input and cost to achieve technical and profitability efficiency.

5 Conclusion

Banking institutions that are both strong and viable are critical to economic growth. This holds predominantly valid for developing nations like Iran, where financial systems are generally bank based. Being the largest conduit of funds, the banking sector is responsible for efficiently mobilizing domestic savings, providing finance for investment and managing a smooth payment system facilitating working capital management. Profitability is one of the most important factors in measuring the performance of banks. The presence of undesirable factors in the structure of such processes makes them more complex and effect on performance.
The existing studies introduce different models for multi efficiency assessment. However, few studies have evaluated the performance of non-performing loans or overdue depts. with considering undesirable outputs and weak disposability. Accordingly, in this paper, we extended 4 models built on the BCC model for assessing technical, cost, revenue efficiency and profitability with the undesirable factors and

managerial disposability. The introduced model can calculate efficiency with considering both of weak and managerial disposability and undesirable factors. We examined the cost efficiency, revenue efficiency and technical efficiency, as well as profitability, of 33 branches of a commercial bank in Gilan province and the relationship between them. In addition, to evaluate the performance, the principle of weak and managerial disposability was used. In the principle of managerial disposability, the organization increases at least one of its inputs to reach the performance limit. The results of data analysis showed that 21 out of the 33 branches had technical efficiency, 20 branches had revenue efficiency, and only 7 branches had cost efficiency.

In some branches it was found that being efficient from the technical, cost, and revenue aspects does not necessarily mean the profitability of the branch. There were branches that were profitable while they were inefficient. Inefficient branches can bring their inputs and outputs closer to the optimal values expressed in the tables as averages for each performance in order to be represented on the performance boundary. For the future studies, it is suggested that the degree and size of the branches should be ssss irrr dd cccsss e rr nneees with the ii grrr drrr ee ll laaate more reuuuree.. It's rccmmmedddd db ----sider other macroeconomic indicators, such as inflation rate, sanction, etc. which affect profitability. Additionally, it is recommended to estimate the efficiency in intermediary approach with various stages, including non-controllable factors and scale elasticity. Combination of data envelopment analysis and other evaluation methods to calculate efficiency could be an interesting topic for future investigation**.**

References

[1] Ahmad, R., Koh, E., Shaharuddin, S., *Determinants of bank profitability: a comparative study of East Asia and Latin America,* Int. J. Banking, Accounting and Finance, 2016, **7**(1), P. 34–51.

[2] Ali, M., Puah, C. H., *The internal determinants of bank profitability and stability,* Management Research Review, 2018, **42**(1), P. 49–67. Doi:10.1108/MRR-04-2017-0103.

[3] Almaqtari, F. A., Al-Homaidi, E. A., Tabash, M. I., Farhan, N. H, *The determinants of profitability of Indian commercial banks: A panel data approach*, International Journal of Finance & Economics, 2019, **24**(1), P.168–185. Doi:10.1002/ijfe.1655.

[4] Amirteimoori, A., Toloie-Eshlaghi, A, Homayoonfar, M., *Efficiency measurement in Two-Stage network structures considering undesirable outputs*, International Journal of Industrial Mathematics, 2014, **6**(1), P. 65-71.

[5] Amirteimoori, A., Kordrostami, S., Khoshandam, L., *Multi-dimensional non-discretionary factors in production processes: a data envelopment analysis*, IMA Journal of Management Mathematics Advance Access, 2014, **25** (4), P. 435-448. Doi:10.1093/imaman/dpt021.

[6] Assaf, G. A., Barros, C. P., Matousek, R., *Technical efficiency in Saudi banks*, Expert Systems with Applications, 2011, **38**(5), P. 5781–5786.

[7] BAL, H., SÖNMEZER, S., *Determinants of the profitability banking sector in turkey*, oo . uş nn vvrr sssssi Dergisi, 2022, **23**(1), P. 243-254.

[8] Batten J., Vo, X.V., *Determinants of Bank Profitability Evidence from Vietnam*, Emerging Markets Finance and Trade, 2019, **55**(6), P. 1417–1428. Doi: 10.1080/1540496X.2018.1524326.

[9] Bhatia, V., Basu, S., Mitra, S.K. and Dash, P., *A review of bank efficiency and productivity*, OPSEARCH, 2018, **55**(3–4), P. 557–600.

[10] Bekhet. H. A., Mohammad, A, A., Khudari, M., *Effects of Internal and External Factors on Profitability of Jordanian Commercial Banks: Panel Data Approach*, International Journal of Financial Research, 2020, **11**(5), Doi: 10.5430/ijfr.v11n5p359.

[11] Bongini, P., Cucinelli, D., Battista, M. L. Di, Nieri, L., *Profitability shocks and recovery in time of crisis evidence from European banks,* Finance Research Letters, 2018, **30**, P. 233-239. Doi:10.1016/j.frl.2018.10.00.

[12] Bucevska, V., Misheva, B., *the Determinants of Profitability in the Banking Industry: Empirical Research on Selected Balkan Countries*. Eastern European Economics, 2017, **55**(2), P. 1–22. Doi: 10.1080/00128775.2016.1260473.

[13] Camanho, A. S., Dyson, R. G., *Cost efficiency measurement with price uncertainty: a DEA application to bank branch assessments,* European Journal of Operational Research, 2008, **161**, P. 432–446.

[14] Cooper, W., Seiford, L.M., Tone, K, *Data Envelopment Analysis, A Comprehensive Text with Models, Applications,* References and DEA-Solver Software Second Edition by Springer Science Business Media, LLC, 2007.

[15] Dq, T., Ngo, T., *The determinants of bank profitability: A cross-country analysis,* Central Bank Review Journal, 2020, **20**, P. 65–73. Doi:10.1016/j.cbrev.2020.04.001.

[16] Ercegovac, R., Klinac, I., Zdrili,, I., *Bank Specific Determinants of EU Banks Profitability after 2007 Financial Crisis*, Journal of Contemporary Management Issues, 2020, **25**(1), P. 89–102. Doi:10.30924/mjcmi.25.1.5.

[17] Fidanoski, F., Choudhry, M., Davidovic, M., Sergi, B., *What does affect profitability of banks in Croatia? Competitiveness Review*, An International Business Journal, 2018, **28**(4), P. 338–367.

[18] Gaganis, C, Galariotis, E., Pasiouras, F., Staikouras, C., *Bank profit efficiency and financial consumer protection policies,* Journal of Business Research, 2020, **118**, P. 98–116.

[19] Garcia, M. T. M., Guerreiro, J. P. S. M., *Internal and external determinants of banks' profitability: The Portuguese case,* Journal of Economic Studies, 2016, **43**(1), P. 90–107. Doi: 10.1108/JES-09-2014-01

[20] Gaur, D., Ranjan M, D., *Non-performing Assets and Profitability: Case of Indian Banking Sector, in*. sagepub. Com/ journals- permissions- India, 2020, Doi: 10.1177/0972262920914106.

[21] Ghiyasi, M., *Inverse DEA based on cost and revenue efficiency*, Computers & Industrial Engineering, 2017, 114, p. 258–263. (In Persian)

[22] Haris, M., Yao, H.X., Tariq, G., Malik, A., Mustansar, J., *Intellectual Capital Performance and Profitability of Banks: Evidence from Pakistan,* Journal Risk Financial Management, 2019, **12**(2), 56. Doi: 10.3390/jrfm12020056.

[23] Henriques, I. C., Sobreiro, V. A., Kimura, H., Mariano, E.B., *Efficiency in the Brazilian banking system using data envelopment analysis*, Future Business Journal, 2018, **4**, P. 157–178.

[24] Jahanshahloo, G. R., Soleimani-Damaneh, M., Mostafaee, A., *A simplified version of the DEA cost efficiency model,* European Journal of Operational Research, 2018, **184**, P. 814–815.

[25] Izadikhah, M., *DEA Approaches for Financial Evaluation - A Literature Review*, Advances in Mathematical Finance and Applications, 2022, **7**(1), P.1-36. Doi: 10.22034/AMFA.2021.1942092.1639

[26] Izadikhah, M., *Financial Assessment of Banks and Financial Institutes in Stock Exchange by Means of an Enhanced Two-stage DEA Mode*l, Advances in Mathematical Finance and Applications, 2021, **6**(2), P.207-232 Doi: 10.22034/amfa.2020.1896273.

[27] Kamarudin, F., Sufian, F., Nassir, A. M., *Does country governance foster revenue efficiency of Islamic and conventional banks in GCC countries?* EuroMed Journal of Business, 2016, **11**(2), P.181–211.

[28] Kao, C., Nan Hwang, S., *Measuring the effects of undesirable outputs on the efficiency of production units*, European Journal of Operational Research, 2020, **292**(3), P. 996-1003. Doi: 10.1016/j.ejor. 2020.11.026.

[29] Kuosmanen, T., Cherchye, L., Sipilainen, T., *The law of one price in data envelopment analysis: Restricting weight flexibility across firms*, European Journal of Operational Research, 2006, **170**, P. 735–757.

[30] Kuosmanen, T., Post, T., *Measuring economic efficiency with incomplete price information*, European Journal of Operational Research, 2003, **144**, P. 454–457.

[31] Kuosmanen, T., Post, T., *Measuring economic efficiency with incomplete price information: With an application to European commercial banks*, European journal of operational research, 2001, **134**, P. 43–58.

[32] Lin, C.S., Chiu, C.M., Huang, Y.C., Lang, H.C., Chen, M.S., *Evaluating the Operational Efficiency and Quality of Tertiary Hospitals in Taiwan: The Application of the EBITDA Indicator to the DEA Method and TOBIT Regression*, Healthcare, 2022, **10**, P.58. Doi: 10.3390/healthcare10010058.

[33] López-Penabad, M.C., Iglesias-Casal, A., Neto, J.F.S., *Effects of a negative interest rate policy in bank profitability and risk taking: Evidence from European banks*, Research in International Business and Finance, 2022, **60**, P. 101597.

[34] Menicucci, E., Paolucci, G., *The determinants of bank profitability: Empirical evidence from European banking sector*, Journal of Financial Reporting and Accounting, 2016, **14**(1), P. 86–115.

[35] Milenkovic, N., Radovanov, B., aaaa š, B., aa rkkḱ´c Horv,,, .. , *External Two Stage DEA Analysis of Bank Efficiency in West Balkan Countries*, Sustainability, 2022, **14**, P. 978. Doi: 10.3390/su14020978.

[36] Moslemi, A., Pourzamani, Z., Jahanshad, A., *Ranking of Banks' Risk Reporting Using Data Envelopmen Analysis*, Advances in Mathematical Finance and Applications, 2021, **6**(4), P.695-715.
Doi: 10.22034/AMFA.2021.1899631.1436.

[37] Mostafaee, A., Saljooghi F.H., *Cost efficiency measures in data envelopment analysis with data uncertainty*, European Journal of Operational Research, 2009, **197**, P. 243-252.

[38] Nourani, M., Malim, N.A.K., Mia, *A.*, *Revisiting efficiency of microfinance institutions (MFIs): An application of network data envelopment analysis*, EooL. R... E. on. Isrr.. vwn,,, 20. 1, **34**, P. 1146–1169.

[39] Öhman, P., Yazdanfar, D., *Organizational-level profitability determinants in commercial banks: Swedish evidence*, Journal of Economic Studies, 2018, **45**(6), P. 1175–1191.

[40] Ozili, P. K., *Bank Profitability and Capital Regulation: Evidence from Listed and non-Listed Banks in Africa*, Journal of African Business, 2017, **18**(2), P. 1–26. Doi:10.1080/15228916.2017.1247329.

[41] Ř—kov,, I., *Efficiency of the Czech banking sector employing the DEA window analysis approach*, Procedia Economics and Finance, 2014, **12**(1), P. 587–596.

[42] Roshdi, I., Hasannasab, M., Margaritis, D., Rouse, P., *Generalised weak disposability and efficiency measurement in environmental technologies*, European Journal of Operational Research, 2018, **266**(3), P. 1000–1012.

[43] Sahoo, B.K., Mehdiloozad, M., Tone, K., *Cost, and revenue and profit efficiency measurement in DEA: A directional distance function approach*, European Journal of Operational Research, 2014, **237**, P. 921–931.

[44] Sahoo, B. K., Tone, K., *Non-parametric measurement of economies of scale and scope in non-competitive environment with price uncertainty*, Omega: The Internatio

nal Journal of Management Science, 2013, **41**, P. 97–111.

[45] Skandar, A. S., Che-Yahya, N., Wahid, Z. A., *Determinants of Commercial Banks' Profitability in Malaysia*, Journal of Entrepreneurship and Business, 2019, **7**(1), P.27-39. Doi: 10.17687/JEB.0701.03.

[46] Staub, R. B., da Silva Souza, G., Tabak, B. M., *Evolution of bank efficiency in Brazil: A DEA approach*, European Journal of Operational Research, 2010, **202**(1), P. 204-213.

[47] Stewart, C., Matousek, R., Nguyen, T. N., *Efficiency in the Vietnamese banking system: A DEA double boot-strap approach*, Research in International Business and Finance, 2016, **36**(1), P. 96–111.

[48] Sueyoshi, T., Goto, M., *Performance Assessment of Japanese Electric Power Industry: DEA Measurement with Future Impreciseness*, Energies, 2020, **13**(2), P.490-512. Doi: 10.3390/en13020490.

[49] Sueyoshi, T., Yuan, V., Goto, M., *A literature study for DEA applied to energy and environment*, Energy Economics, 2017, **62**, P.104–124.

[50] Svitalkova, Z., *Comparison and evaluation of bank efficiency in selected countries in EU*, Procedia Economics and Finance, 2014, **12**(1), P. 644–653.

[51] Toloo, M., Hanclová, J., *Multi-valued measures in DEA in the presence of undesirable outputs*, Omega, 2019, **94**, Doi.:10.1016/j.2019.01.010.

[52] Tone, K., *A Strange case of the cost and allocative efficiencies in DEA*, Journal of the Operational Research Society, 2002, **53**, P. 1225–1231.

[53] Tone, K., Sahoo, B. K., *Evaluating cost efficiency and returns to scale in the life insurance corporation of India using data envelopment analysis*, SocioEconomic Planning Sciences, 2005, **39**, P. 261- 285.

[54] Wanke, P., Azad, M. A. K., Barros, and C., *Predicting efficiency in Malaysian Islamic banks: A two-stage TOPSIS and neural networks approach*, Research in International Business and Finance, 2014, **36**(1), P.485–498.

[55] Wanke, P., Barros, C., *and Two-stage DEA: An application to major Brazilian banks*, Expert Systems with Applications, 2016, **41**(5), P. 2337–2344.

[56] mmmzz, .. , üü n,,, .. , *Efficiency comparison of participation and conventional banking sectors in Turkey between 2007–2013. Procedia*, Social and Behavioral Sciences, 2015, **195**(1), P.383–392.

[57] Zanjirdar, M. *Overview of Portfolio Optimization Models*. Advances in Mathematical Finance and Applications, 2020, **5**(4), P. 419-435. Doi: 10.22034/amfa.2020.674941