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Research Paper

Provide an Improved Factor Pricing Model Using Neural Networks and the Gray Wolf Optimization Algorithm

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ABSTRACT

The issue of asset pricing in the market is one of the most important and old issues in the financial world. Factor pricing models seek to be able to determine a significant relationship between return on assets based on the risk parameters of that asset. A wide range of factors can be found in the literature that can be an element for measuring the risk of an asset, but the big question is which of these models will work better. The factors studied in this research include factors that cover market risk, valuation risk, psychological (technical) market risk, profit quality risk, profitability, investment, etc. In this study, we have tried to Use machine learning techniques and optimization tools as a way to derive adaptive-robust nonlinear models that can reduce the risk of model error as much as possible. In this research, two models have been developed. In the first model, using the feature extraction technique (using a gray wolf algorithm for optimizing the input parameters) in order to optimize the “models based on a neural network”, since a non-linear and adaptable model has been developed for each asset. In the second approach, a portfolio of improved neural network-based models that are developed in the first stage is used, which can be used to minimize the risk of model error and achieve a model that is resistant to different market conditions. Finally, it can be seen that the development of these models can significantly improve the risk of error and average error of the model compared to traditional CAPM approaches and the Fama and French three-factor model

1 Introduction

When investors invest in financial assets, they expect to receive a return, but they feel that their capital is at risk, in other words, they pay attention to an element other than return, and that element is the risk. This financial principle allows investors to pay attention to both parameters of return and risk. These complexities have led to a variety of theories to determine the relationship between risk and return. It can also be said that one of the great advances in the financial literature has been the discovery of models for measuring risk in financial markets. If an accurate way can be found to assess the fair price

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relative to asset risk, it can be expected that the model could accurately valuation of a risky asset in order to improve allocating assets in the optimal portfolio, and this can increase market allocation efficiency.

Usually, pricing models with a forward-looking approach try to estimate the relationship between risk and expected return by making assumptions about the investor approach as well as investment opportunities. In the meantime, experimental models have a retrospective approach in which they can use historical conditions to extract a suitable estimate from past patterns, which can be used to estimate a suitable model for the future. Therefore, identifying an appropriate model derived from the results of the present study can be very effective in economic decisions made by various investors, including actual and potential shareholders of companies to assess the accuracy of returns forecasts and expected risk. A review of the literature and research background in Iran reveals that despite the great importance of factor models and their applications in financial issues and to explain various financial anomalies, there is very limited research on factor pricing models and new models in this area. This study has tried to propose an adaptive-robust model for calculating factor pricing and risk assessment models in Tehran Stock Exchange. Universal models have not always performed well in emerging markets. In addition, the characteristics of financial markets in Iran are unique, such as a wide range of stock liquidity, active participation of individuals, and on the other hand a large part of the market value of the total index is summarized in a limited number of stocks. These characteristics are very different from those of the United States and other developed stock markets, as well as large emerging markets. In addition to these unique features, the study of the Iranian stock market from the perspective of asset allocation is also important, given that the market has grown significantly in recent years. However, few studies examine the performance of these competing models and provide comprehensive and robust empirical evidence to determine the most appropriate asset pricing model for the Iranian stock market.

Another important point in the financial market is the nonlinear behaviors and relationships between financial phenomena. Pricing models generally attempt to model linear causal relationships, while nonlinear financial market behaviors are time-varying and unique to each financial asset. This research tries to offer an experimental model that can create the best pricing models for each asset based on different risk factors by using machine learning tools and techniques as well as applying financial concepts. In other words, this article can be considered an attempt to use data science to extract the best stock pricing elements of different companies. The main idea of this modeling is that the nature of market pricing is very complex and each financial asset has a unique pricing behavior based on its shareholder trading strategies as well as the fundamental and even psychological conditions of the market, which means there could not be considered a universal pricing method for the pricing of a stock. In this research, using library research, all risk factors, including market risk, valuation risk, liquidity risk, risk of fundamental parameters of company equity, as well as the risk of psychological and technical market factors that have been used in previous research have been tried to Be analyzed at once. The introduced model use data mining and a combination of machine learning techniques to identify the dynamics of asset pricing.

2 Scientific Background and Literature Review

The experimental performance of the capital asset pricing model (CAPM) proposed by Sharpe [35] and Lintner [24] has been very poor. Fama and French [7] reinforce CAPM with size and value factors to improve model explanatory power. Carhart, meanwhile, introduced a four-factor model that added the momentum factor to the three Fama and French factors. Over the past three decades, however, it has become increasingly clear that even the highly influential Fama-French three-factor model and the Carhart four-factor model cannot explain many of the capital market anomalies. Recently, inspired by the dividend discount valuation model by Miller and Modigliani [28] who explained that total dividend equals total net income minus the change in total equity, Fama and French [9] developed a five-factor model. This model adds profitability and investment factors to the factors of market return, size, and book value-price factor (value) of the three-factor model. Recently, Fama and French [12] reviewed a six-factor model that adds the momentum factor to the five-factor model.

Fama and French examine the five-factor model for developed stock markets, including North America (the US and Canada), Europe, Asia-Pacific, and Japan. They showed that the five-factor model recognizes the average efficiency patterns globally. This model has also been tested for other major developed markets, such as the Australian market by Elliott et al. [6] and Huynh [18] and the Japanese market by Kubota and Takehara [22]. Different regions have been reported to have different types of anomalies, and the importance of a particular factor varies from region to region. [19] [16] For example, the profitability and investment factor are strong in Europe, Asia, and the Pacific. But for Japan, profitability and investment have a weaker relationship with average returns. The relationship with the momentum factor is less evident in many emerging Asian markets, including China and Korea [4,23], but performs well in developed markets other than Japan [1,2,8]. Fama and French [10] point out that the global version of the model performs poorly, and the local version can reasonably provide a better explanation of the anomalies. A number of asset pricing studies examine US market anomalies. Karolyi [20] refers to the US "home bias" in terms of empirical finance because most of these studies cover only US markets and some other non-US countries that are more "foreign biased" than others. These countries are either developed markets or large emerging economies. Given that well-developed markets are well-integrated, there must be similar phenomena and risk factors in these markets that lead to similar findings. Therefore, off-sample testing is critical to understanding the applicability of the model, especially in emerging markets. Because these markets show different characteristics and dynamics than developed markets. Experimental research in financial markets is relatively rare. But researchers such as Hanauer and Lauterbach, Zaremba, and Maydybura [17,38] conducted a series of studies on three-, four-, five- and six-factor models.

Various studies have shown that the five-factor model cannot explain the low average returns of small stocks with low profitability and aggressive investment. [9,11,12,4] argue that such stocks are only a small part of the US market, but it is different in global markets. The Iranian stock market also has a large number of small, low-profit companies with medium (or high) investments. Iran has the characteristics of an emerging market, such as high returns with excessive volatility, low market capitalization, and high trading volume. In addition, the characteristics of financial companies in Iran are unique, such as liquidity, and active participation of individuals, a large part of the market value of the total index is summarized in a limited number of shares. These features are different compared to the US and other developed stock markets as well as emerging markets. In addition to these unique features, the study of the Iranian stock market from the perspective of asset allocation is also important given that the market has grown significantly in recent years. However, few studies examine the performance of these competing models and provide comprehensive and robust empirical evidence to determine the most appropriate asset pricing model for the Iranian stock market. Vast research has attempted to extract important factors for constructing a model for estimating asset returns. The result is various parameters that have been extracted to estimate the rate of return [13,34,26,27]. A study was conducted on the Tehran Stock

Exchange and showed that there is ambiguity in the Tehran Stock Exchange, which could make it impossible to develop a suitable functional model for pricing, at least linearly [31]. Some research like [32] shows that non-linear models can act much better than linear models in the estimating of return. When we speak about prediction with non-linear models, usually hybrid models using artificial neural networks and meta-heuristic optimization models are the so famous in this field [5,21,25,30,36]

2.1 The Capital Asset Pricing Model

One of the most prominent models in asset pricing is CAPM sharp and Lintner [35,24], which was developed separately based on using portfolio theory to reach market equilibrium. In other words, the model does not consider only the decision of a single investor but aggregates them to determine market equilibrium. we know that the current price affects the expected returns and vice versa. Given future expected dividends and assuming that markets are efficient, i.e. that the prices of assets equal their fundamental value, a high current price results in a low expected return in the next period and a low current price in a high expected return. Convention in the academic literature requires us to focus on expected returns. We know from portfolio theory that every investor j (j = 1,…, M) maximizes his expected utility by choosing an optimal portfolio, i.e. choosing optimal weights for each asset

$$max\ E\left[U^j\left(R_p\right)\right] = max\ (\textstyle\sum_{i=1}^{N} x_i \mu_i - \frac{1}{2} z_j \sum_{k=1}^{N}\sum_{i=1}^{N} x_i x_k \sigma_{ik}) \tag{1}$$

For all j=1,…, M with the restriction $\sum_{i=1}^{N} x_i = 1$, z_j is absolute local risk aversion. The Lagrange function for solving this problem can easily be obtained as

$$L_j = \sum_{i=1}^{N} x_i \mu_i - \frac{1}{2} z_j \sum_{i=1}^{N}\sum_{i=1}^{N} x_i x_k \sigma_{ik} + \lambda \left(1 - \sum_{i=1}^{N} x_i\right) \tag{2}$$

The first-order conditions for a maximum are given by

$$\frac{\partial L_j}{\partial x_i} = \mu_i - z_j \sum_{k=1}^{N} x_k \sigma_{ik} - \lambda = 0 \tag{3}$$

$$\frac{\partial L_j}{\partial \lambda} = 1 - \sum_{i=1}^{N} x_i = 1 \tag{4}$$

If assume the optimal portfolio is the market portfolio, Solving the above equations for μ_i gives,

$$\mu_i = \lambda + z_j Cov\left[R_i, R_p\right] \tag{5}$$

If we put a riskless asset that means an asset with $\sigma_{ip} = 0$, hence we can interpret λ as the expected return of an asset that is uncorrelated with the market portfolio. As the riskless asset is uncorrelated with any portfolio, we can interpret λ as the risk-free rate of return r. to put everything in a nutshell, the CAPM model just introduces and proves a risk factor for pricing models. The covariance risk of any assets to the optimal portfolio return (which usually assumes a market portfolio) is called systematic risk. The CAPM explains the expected returns only by a single variable, the risk of an asset relative to the market. Besides these theoretical critiques, empirical investigations show some deficits in the CAPM. To overcome these shortages to explain the returns model need to add some new factors like

Fama & French model. It is reasonable to assume that other factors may as well influence the expected returns. We will therefore discuss the Arbitrage Pricing Theory as an alternative to the CAPM framework forward.

2.2 The Arbitrage Pricing Model

The aggregation of all risk into a single risk factor (market risk) is one of the critical points in the concept of the CAPM. The CAPM model is a simplified model that could be proper for a well-diversified portfolio but not for a single asset. It is well observable that assets are not only driven by general factors like market movement but that industry or country-specific influences also have a large impact on returns. This section presents an alternative to the CAPM, the Arbitrage Pricing Theory (APT) as first introduced by Ross [33]. The basic concept in APT is "the law of one price"; That is, two assets that are similar in risk and return cannot be sold at different prices. The definition of incorrectly priced securities in a way that generates risk-free profits is called "arbitrage". An arbitrage opportunity arises when an investor can form a portfolio with zero investment volume so that he can make a safe (risk-free) return. Zero investment portfolio; This means that there is no need for the investor to use his own money in order to invest. Arbitrage opportunity arises when "the law of one price" is not observed; That is, an asset is exchanged at different prices. The main assumptions of APT are as follows:

1. Capital markets are highly competitive.
2. Investors always prefer more wealth to less wealth.
3. The process of generating a return on assets can be modeled as a multi-factor linear model

this model can be empirically valid. As noted before many empirical factors models find evidence that other variables are able to explain the observed returns better than the market risk. The APT could be a framework to find a justification for their results on a sound theoretical basis. The main issue in APT is the detection of factors and the measurement of sensitivities. Although APT has fewer assumptions than CAPM; It also has two special assumptions:

1. Investors agree on the factors that are systematically important in the pricing of assets (Homogeneity of beliefs).
2. There is no arbitrage opportunity (risk-free profit).

At a first glance, we could interpret the APT as a generalization of the CAPM to a multi-beta model. But it has clearly to be pointed out that the models differ substantially in their assumptions. The CAPM is concerned to find an equilibrium of the market by holding optimal portfolios as implied by portfolio theory, whereas the APT finds this equilibrium by ruling out arbitrage possibilities. Assume that asset markets are completely competitive, and assume that investors believe that the return on assets is derived from a k-factor model so that the return on the asset i is as follows:

$$r_i = E_i + b_{ij}\delta_j + \cdots + b_{ik}\delta_k + \varepsilon_i \tag{6}$$

E_i is the expected return, δ_j are common risk factors and b_{ij} is the sensitivity of the ith asset return to fluctuations of the jth risk factor and ε_i is the residual risk of ith asset that $E\{\varepsilon_i|\delta_j\} = 0$ for all j. with optimization of the model in a competitive market that there is not any arbitrage you can prove that the equation of expected return of the ith asset is like below:

$$E_i = \lambda_0 + b_{i1}\lambda_1 + \cdots + b_{ik}\lambda_k \tag{7}$$

$\lambda_0$is risk free asset return and parameters $\lambda_1, \lambda_2, \ldots., \lambda_k$ are the risk premium related to risk factors $\delta_1, \ldots., \delta_k$

2.3 Gray wolf optimization algorithm

The important point about the optimization problem is that it should pay special attention to the constraint of the model and determine the optimal answer as far as possible based on the constraints. Certain optimization models such as the available direction approach, descending gradient model, etc. have been among these models. However, due to the limited performance and complexity of the constraints of these algorithms, they have not been very practical in real-world problems that have high complexity of constraints and search space, such as structural optimization problems, economic optimization problems, and engineering design problems. [15]. In this space, many metaheuristic algorithms were developed to solve the problem of optimizing problems with many constraints. These models promise acceptable convergence speed, higher accuracy, stability, and better performance. The Gray Wolf Optimization Algorithm is one of the newest meta-heuristic optimization approaches that use the leadership and unique hunting mechanism of gray wolves. This optimization approach has a high ability to bypass local optimal [37]. Also, this approach has a high ability to converge to the general optimal solution. In general, the GWO method is a specialized optimization model for extrapolation problems.
First, the gray wolf optimization model was introduced by Mirjalili [29]. This algorithm is inspired by the unique search and hunting behavior of gray wolves. This model considers the four social classifications of gray wolves, alpha, beta, gamma, and omega. Alpha wolves are the leader and manage all the wolves and direct them toward the goal. He is also responsible for controlling the entire hunting process and making all kinds of important decisions such as hunting, maintaining order and movement, and the whole group sleeping. And beta wolves are the best alternatives to the alpha group, and it takes feedback from other wolves and gives it to the alpha wolf. And the wolf of the third group, which is the gamma wolf, is responsible for controlling and maintaining the cohesion of the omega wolf herd.
If the distance from the three alpha, beta, and gamma is calculated as D_α و D_β و D_δ and for each wolf X an equation of 1 is calculated. For each wolf, the effect of each of the alpha, beta, and gamma wolves is displayed as X_1 و X_2 و X_3, which is present in Equation 2.

$$D_\alpha = |C_1.X_\alpha - X| \quad , D_\beta = |C_2.X_\beta - X| \quad , D_\delta = |C_3.X_\delta - X| \tag{8}$$

$$X_1 = |X_\alpha - A_1.D_\alpha| \quad , X_2 = |X_\beta - A_2.D_\beta| \quad , X_3 = |X_\delta - A_3.D_\delta| \tag{9}$$

$$\mathrm{A} = (2a.r_1) - a \ , \ C = 2.r_2 \tag{10}$$

$$X(t+1) = (X_1 + X_2 + X_3)/3 \tag{11}$$

The number of control parameters of the model, ie a, A and C, are calculated by Equation 10, in which r_1 , r_2 are random vectors in the interval [0, 1] that this vector can cause wolves to any area between a wolf and Prey access. Also, the control parameter a, which is the effect of superior wolves on the flock, moves linearly during the number of repetitions from 2 to zero. In other words, this parameter can enhance the extrapolation and interpolation of the model. At the beginning of the movement, the wolves are searching for a larger space, and as they get closer to the end of the path, they gather around the target and focus on the target. The following figure can be drawn schematically.

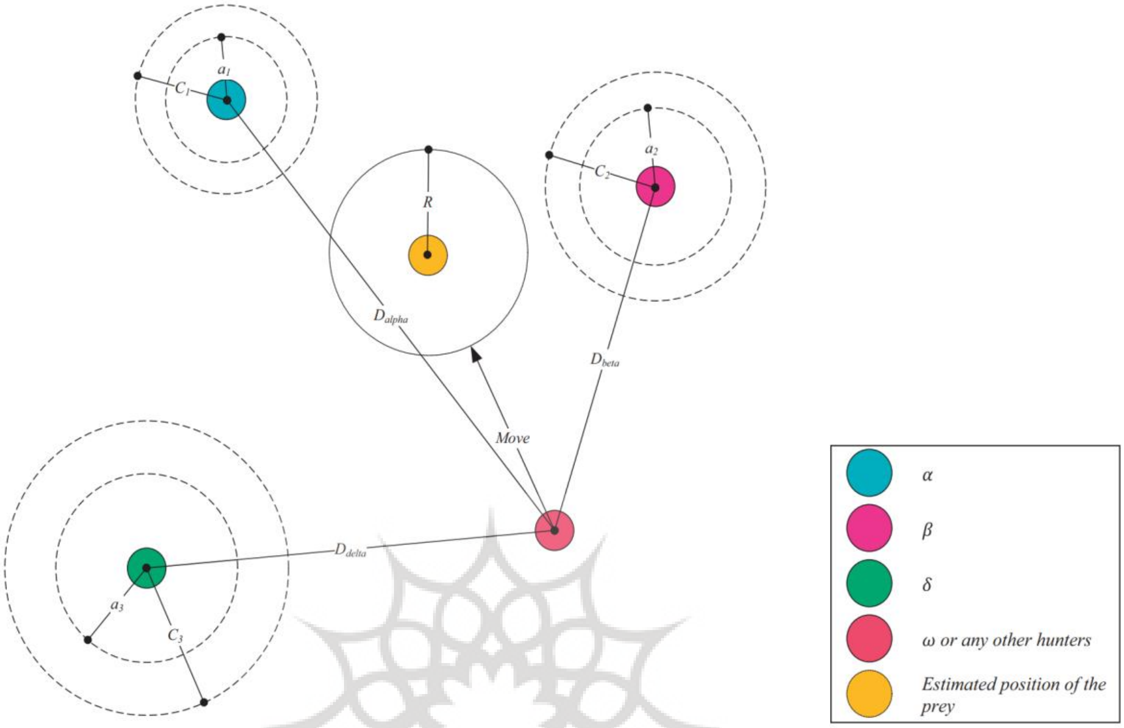

Fig. 1: Schematic view of gray wolf optimization [29]

As can be seen, omega wolves walk in the path of prey at every stage, but in their range of motion, they may also search around. In simpler terms, each of the wolves is constantly searching for all the places around them and moving toward the superior wolves. 1 shows that in practice, the movement performed will be an outward movement and in search of other points. This is shown in figure 1. Despite the novelty of the gray wolf model in the optimization models, it can be seen that this model has been used in different examples.

2.4 Artificial Neural Networks

Neural networks are one of the entrenched concepts in the world of machine learning, which tries to extract hidden patterns between input and output by creating a structure like a brain. Each neural network is made up of a set of neurons, which is the smallest processing element. Each neuron takes some inputs and, by initial processing, produces outputs, each of which can be the output of another layer or each of which can be the input of another layer of neurons. Each neuron is schematic as follows:

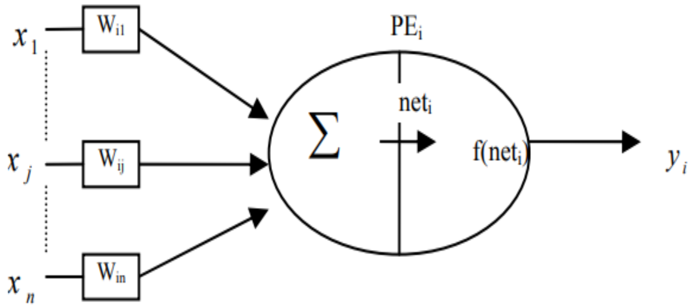

Fig. 2: Schematic structure of a neuron

Where Ws are the estimated weights of the model for each input and then the multiple of the weights with the inputs is added together with a bias element and converted into a function that is generally a saturated function such as hyperbolic tangent, sigmoid, and so on. Multi-layer feedforward networks are networks in which only the output of each neuron can be transmitted to the neurons of the next layer, while in a set of neural networks known as feedback networks, the output of each neuron can be Re-turned to the same layer or even previous layers. Multilayer perceptron, or MLP, is one of the most widely used neural network architectures in the form of feed and includes an input layer, middle layer (hidden layer), and output layer. The output layer is generally an aggregation layer. The multilayer perceptron can be represented schematically as follows:

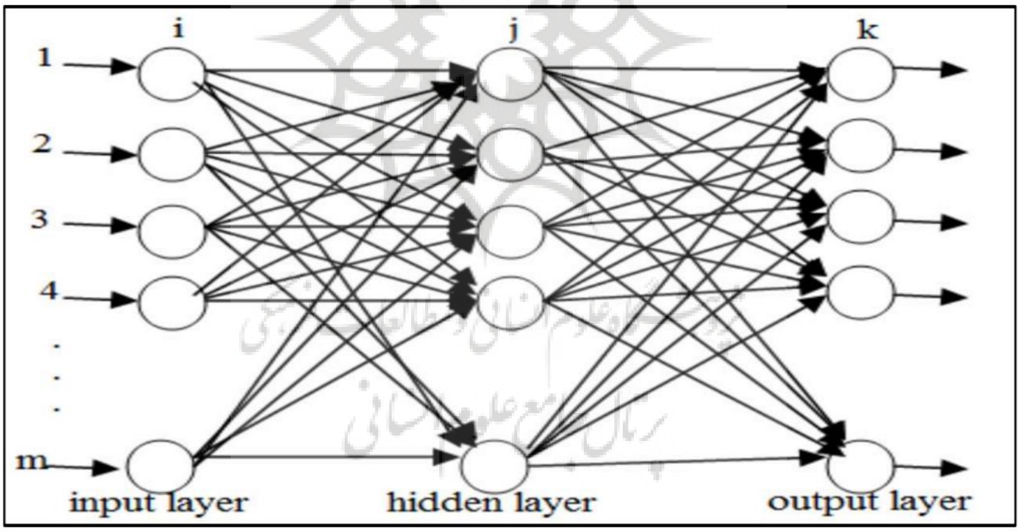

Fig. 3: Multilayer perceptron network

3 Research Method and Model Description

There are different views and theories about the research method. In short, four approaches can be named:

1. Expand and improve existing theories

2. Comparison of different theoretical perspectives
3. Investigation of a specific phenomenon using various theoretical perspectives
4. Investigation of a documented and repetitive phenomenon in new environments and conditions [14]

The present research method is descriptive and belongs to the first group. In other words, this paper has tried to extract models that can significantly improve the performance of traditional pricing models by developing an adaptive-robust model. In the present study, first, by examining library sources, the factors that have been considered more than other factors in the financial and pricing literature have been extracted. For data extraction, a period of 5 years leading up to 2020 has been used from the perspective of the research location, the Tehran Stock Exchange selected. The data used were used by Noavaran amin software and Bourseview output which are two famous data providers of the Tehran stock exchange's data, and Python programming language and Jupyter software were also used for model development and processing.

3.1 Review the proposed model

The study of factor pricing models is always based on the concept of factor. The models used generally follow the following equation:

$$R_i = \alpha + \beta_1\lambda_1 + \beta_2\lambda_2 + \cdots + \beta_n\lambda_n \tag{12}$$

In this equation, λ_n is called the risk factor element, and $\beta_n$is called the sensitivity to factor. In other words, the factor itself represents the risk factor and beta practically shows the degree of dependence on that risk element. In the above formula, alpha is the intercept and represents the risk-free return on the asset under consideration (based on the apt model), and R_i represents the expected return on the asset. In simpler terms, pricing models seek to describe the expected return on assets based on the risk elements in the market.

Table 1: Lists the factors used in the model

Receivable turnover	60 day moving average Gap	TEDPIX(overall index)
Payable turnover	Absolut 60 day moving average Gap	Total equal weighted Index
Debt ratio	Momentum 5 day	PE
Long-term debt ratio	Momentum 20 day	PB
Current ratio	Momentum 60 day	PS
Quick ratio	liquidity	Market value
FCFF to sales ratio	ROA	CCI(20)
FCFF to net earning ratio	ROE	RSI(14)
CAPX to revenue ratio	Basic earning power	ATR
CFO to debt ratio	Gross margin	20 day moving average Gap
CFO to revenue ratio	Operating margin	Absolut 20 day moving average Gap
	Net margin	5 day moving average Gap
	Asset turnover	Absolut 5 day moving average Gap

In various studies, the risk factor has been considered as the difference between two ends of a risk element in such a way that, for example, concerning market risk, the risk factor is displayed as $R_m - R_f$, in other words, the return on an asset with Market risk (eg index) and return on risk-free assets, which in other words can indicate the risk exposure to the market risk element. The same issue was used by Fama and French in introducing the three-factor model and then by other researchers. In their research to calculate each factor, Fama and French first performed clustering on market stocks based on the risk factors, for example, placing the market in five portfolios of companies of different sizes, so that the largest companies in portfolio number 5 and the smallest companies are in portfolio number 1 and other stocks are in other categories. It then subtracted the returns of each end of the spectrum from each other so that it could calculate the risk element. Therefore, it could calculate the effect of the element at both ends of the spectrum and describe the return based on the risk element.

If assume λ_i is the risk factor based on the δ_i parameter then we can divide all companies based on sorting δ_i parameter in n parameter portfolio with the expected return of $r_{\delta_i,1}$, $r_{\delta_i,2}$,...., $r_{\delta_i,\mathrm{n}}$ then we can propose λ_i as:

$$\lambda_i = r_{\delta_i,\mathrm{n}} - r_{\delta_i,1} \tag{13}$$

In the present study, a similar approach has been used to calculate the risk elements. In this way, for each risk element, all the review space is classified into 5 different categories based on the risk element, and then the daily return of the portfolio with the highest risk element is reduced from the daily return of the portfolio with the lowest risk element. The risk is calculated for that day. For this research, 37 risk elements have been used, which are identified in Table 1.

As can be seen, in this research, an attempt has been made to examine all the data and factors that have been studied in the literature. Also, two elements of the total index and total equal-weighted index have been used to measure market risk. Since the weighting of the total index of the Tehran Stock Exchange is weighted based on the size of the company, it can show a different interpretation of market risk, while the equal weight index can show a different interpretation of the market, so both indicators as elements. Market risk is used. Indicators and technical indicators have also been used, which can be classified into three categories. Indicators indicate fluctuations, indicating momentum as well as indicators indicating saturation or the beginning of the movement in the market. In other words, an attempt has been made to observe the effect of market psychological parameters in the developed model. Liquidity is a very important risk element that has been considered in the model under discussion and the concept of liquidity has been considered as changes in share price per unit of the trading volume. The parameters of investment profitability, which are used in the 5-factor model of Fama and French, are also seen in the model. Another category of parameters used is parameters related to share valuation, such as the price-to-earnings ratio. On the other hand, elements related to company performance, credit risk, liquidity management as well as dividend quality have also been examined. As mentioned earlier, the model used in this research is a machine learning approach to extract and develop an adaptive-robust model for the asset pricing problem. The algorithm used can be divided into three main steps:

1. Pre-processing and preparation of data and extraction of risk factors:

In the initial stage of the research, it is first necessary to integrate all the data and to be able to extract risk factors. As mentioned earlier, risk parameters are calculated as the return difference between the two ends of the spectrum arranged according to the risk element. It should be noted that in the calculations, all stock price data have been adjusted based on dividends and capital increases, which means that dividends and capital increases alone will not identify excess profits or losses, and the effect of these parameters has been eliminated. Unusual changes and singularity points of each stock have also been removed. From another perspective, all data are normalized in the range of -1 to 1 so as not to cause saturation error after entering the neural network. Also, to test the robustness and meaningfulness of the extracted model, it was necessary to divide the data into three categories: primary education, secondary education, and testing in 10 different scenarios. In other words, trials in 10 different random scenarios are divided into three categories: 60% primary training dataset, 30% secondary training dataset(selecting the best set of NN models and making an optimized portfolio of models), and 10% testing. Primary training datasets are used to train neural networks to build various neural network models with different input parameters. The secondary training dataset is used to validate the developed models and select the best set of neural networks models that are developed with the primary training dataset and used to optimize a portfolio of neural networks models with the minimum error risk of pricing estimation, and the third dataset is used to test all of models and portfolio of models. Since the "sample bias" can be a threat to the research result when using data and numeric solutions; to overcome the sample bias more robustly, in this research, all datasets were used in 10 different scenarios of primary, secondary, and test training datasets that shuffle randomly. In another word, we repeated our research process 10 times with different datasets and then merged them to test and validate the research results.

2. Using the Gray Wolf algorithm to extract neural networks and select input factors

In the second stage, the data extracted from the first stage are taught in the form of dynamic neural network models by the basic training dataset. The neural networks used in this research are multilayer perceptron networks that are trained in the back-propagation method. The neural network model inputs are used as optimization parameters in the gray wolf model, which tries to achieve the optimal inputs with the best performance. In general, it can be said that the neural network used has a structure such as the following:

$$R_i = NN(\lambda_1, \lambda_2, \dots, \lambda_n) \tag{14}$$

The neural network has a hyperbolic tangent transfer function and two hidden layers designated 7 and 3 neurons. As mentioned, the inputs of your model are searched by the gray wolf algorithm to search for a very large state space. The model fitness function has three elements: the absolute mean value of the error, the standard deviation of the error, and also a punitive element for the number of input variables. In other words, this model not only seeks the least errors but also focuses on error distribution and the absence of pricing anomalies as an important element in the model. Also, the mentioned model tries to minimize the number of input variables of the model by using the punitive element and makes the addition of each variable only subject to the appropriate improvement of the model. The optimization problem is defined as follows:

Find risk factor vector which $S \epsilon \{\lambda_1, \lambda_2, \dots, \lambda_n\}$, provided that

$$\text{minimize } F(S) = \sum_{i=1}^{s\_Num} a_1\, mean(e_i) + a_2\, std(e_i) + a_3\, len(S) \tag{15}$$

Where S_Num is the number of scenarios for segmenting training and test data. Also, e_i is the error between the return which is estimated by the neural network model, and the real return on the secondary training dataset in the i-th scenario. This part of the model tries to create an optimal model with high adaptability to time and any asset that can create the best output in different modes with the least processing power. The factors extracted for each asset will be different from the other assets

3. Using the Gray Wolf algorithm to create a portfolio with minimal error risk based on extracted networks and factors

Each of the models developed in the second stage is a nonlinear model based on the neural network that has a certain risk of error. In other words, the accuracy of each model developed in the second part can be seen as a problem of portfolio building and diversification. With this approach, a set of pricing strategies can be achieved that is more resistant to model error risk. In other words, in this approach, an attempt has been made to deal with a stable and resistant structure to the occurrence of efficiency anomalies with a linear combination of nonlinear models developed in the previous stage. The final extended

$$R_i = \beta_1 NN(S_1) + \beta_2 NN(S_2) + \cdots + \beta_n NN(S_n) + \epsilon \tag{16}$$

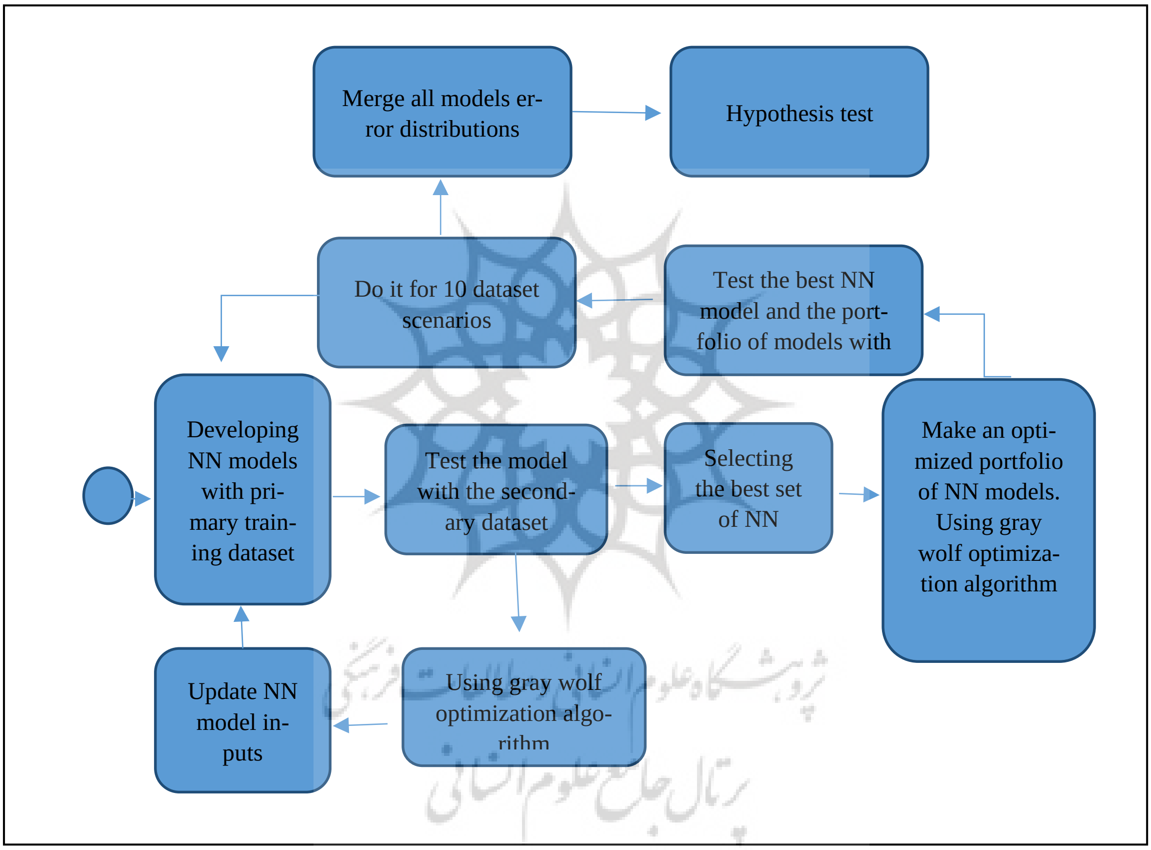

Fig. 4: Schematic figure of process of making the models and validate them

In the mentioned model, another gray wolf optimization algorithm tries to calculate the beta weights. In the mentioned optimization model, in addition to maintaining the competitiveness of the model in the error-index, an attempt is made to achieve a structure with a minimum standard deviation of error by minimizing the combination of models. In other words, it is expected that the correlation between errors in different neural network models can create a space that improves the risk of errors. In other words, the mentioned optimization problem can be shown as follows:

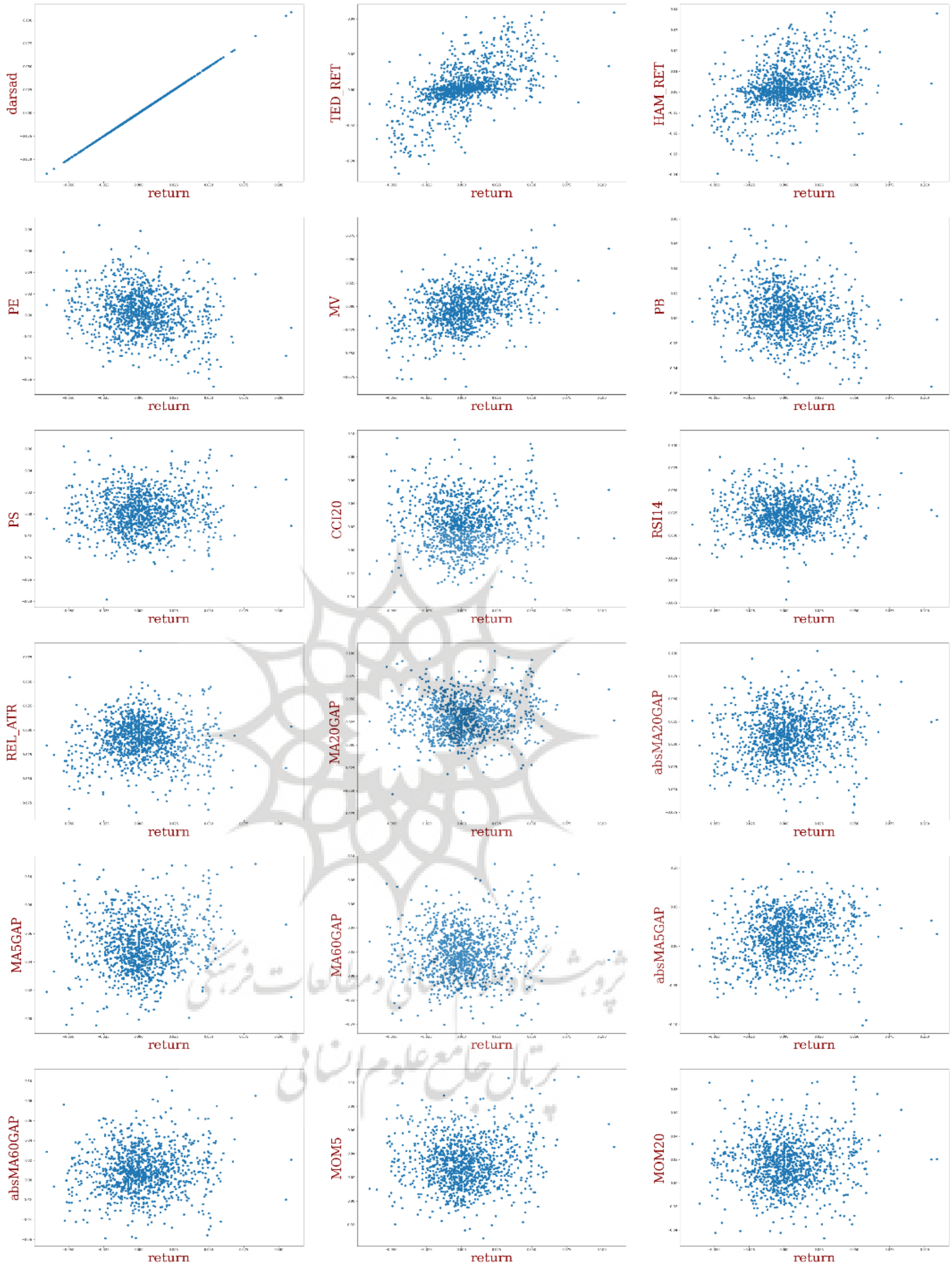

Fig. 5: The scatter plot of Folad share return and the risk factors

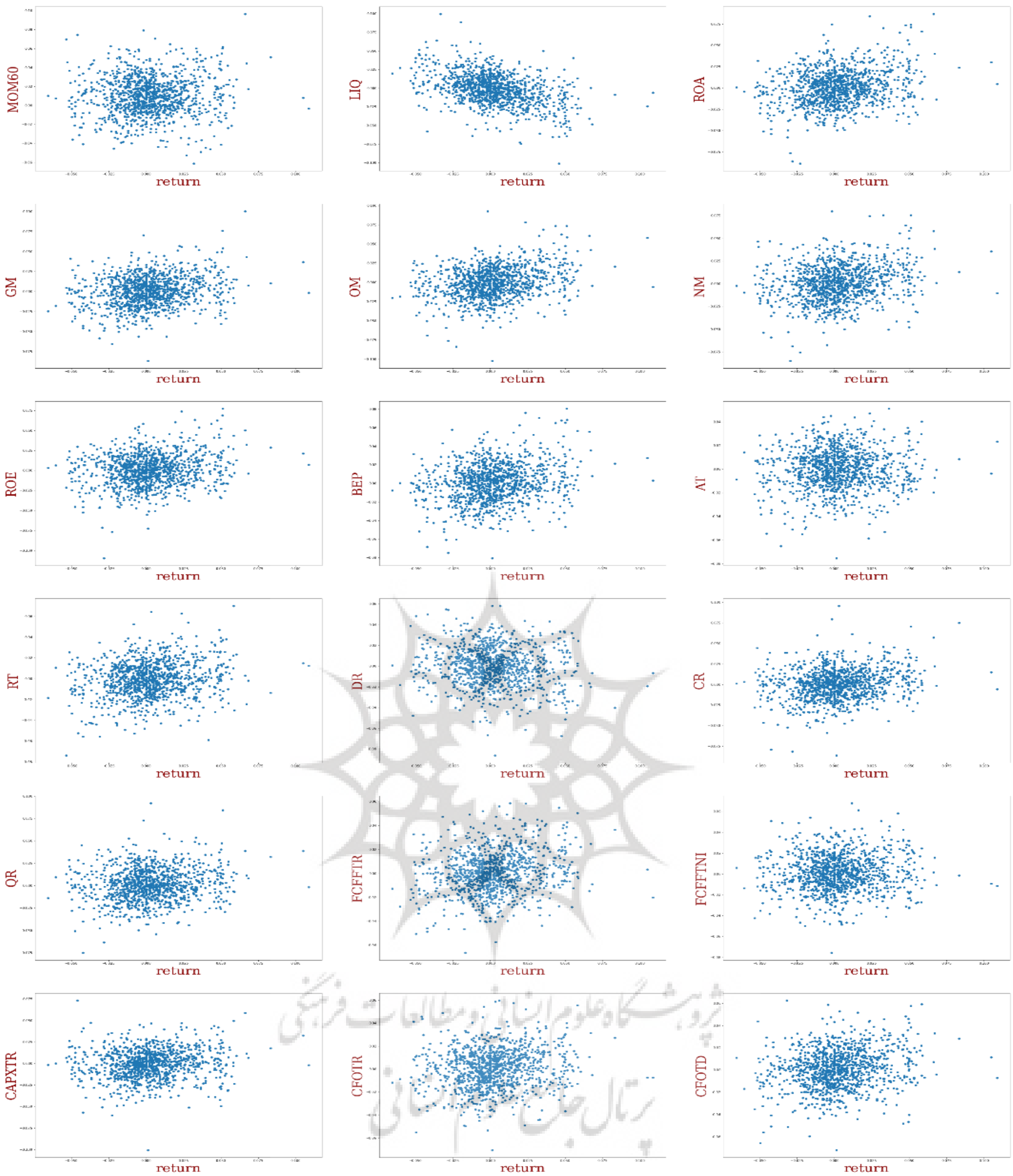

Fig. 6: The scatter plot of Folad share return and the risk factors

Find weight matrix $M \epsilon \{\beta_1, \beta_2, \ldots, \beta_n\}$, which

$$\text{Minimize } F(M) = \sum_{i=1}^{s\_Num} b_1\, mean(e_i) + b_2\, std(e_i) + b_3\, len(S) \tag{17}$$

where S_Num is the number of split scenarios training and test data and b_i is the parameters of the fitness function. Also, $e_i$is the error between the portfolio model of the estimated models and the secondary training data in the scenario i. The following figure can be drawn schematically to better understand the process of research.

3.2 Review of model and results

Isfahan Mobarakeh Steel's share(Folad) has been used to realize the model, and implement and execute the model steps. Mobarakeh Steel has a relatively strong relationship with the overall index(TEDPIX) due to this issue, which is considered an index-building stock. In the next two pages, the relationship between the return of this share and each of the risk factors is drawn. In between, the linear relationship between the asset return and total index, liquidity, company size, coefficient of price to book value, and equal-weighted index is quite evident. The important point is that since steel's share is one of the largest stocks in the market, its relationship with the overall index(TEDPIX), which is weighted according to the size of the companies, is quite predictable. Also, the relationship between liquidity and company size was quite predictable because Steel's share is one of the largest companies and one of the most liquid companies in the market, which shows the impact of the risk of these parameters on the performance of share return. But the most interesting point is the relationship between folad stock returns and market psychological (technical) risk factors. The absolute value between the price gap and moving averages (the risk factor indicates the stock that has moved and the stock that is trading in their equilibrium range) or the RSI and CCI indicates the impact of this share of market volatility. . In other words, since this share is fundamentally known to many market participants, it is heavily influenced by market emotional factors such as moving as a market leader, or stagnation due to the rest of the market, or limited volatile movements. As a result, it can be seen that the technical elements strongly represent the behavior of this share. The profitability and performance parameters of the share, such as the ratio of ROA, ROE, and the margin of operating and net earnings of the share, also seem to have some significant relationship with the return on the share. The following is a scatter chart of returns and each risk factor. It is necessary to place the share in the structure of the proposed model to extract the features. In this algorithm, which is based on a neural network with two hidden layers with 7 and 3 neurons, which is set based on trial and error, there is a gray wolf optimization algorithm with a fitness function as follows:

$$F(S) = \sum_{i=1}^{s\_Num} a_1\, mean(e_i) + 0.5\, std(e_i) + 0.005\, len(S) \tag{18}$$

After discovering the best features and building their optimal neural networks to reduce the risk of error, it is used as a portfolio of several neural network models with a penalty element for the number of models. The output of these two steps is table number 2 for the Folad share. As can be seen, the best network is a network in which both total and equal-weighted indices are used as input to the neural network. This is while in the optimal portfolio model of neural network models, the weight of the best network is zero. In other words, the elements that have the most weight in the portfolio model are two models with risk parameters of size, liquidity, total index, the absolute value of the distance from the short-term average, and the second model with parameters of the total index, liquidity, size, price-book value multiple, and technical parameters like Momentum and 20-day CCI, 14-day RSI. Which is well visible in Table 2. It can be seen that the optimized portfolio consists of a weighted average of 6 improved neural network models and is expected to be able to reduce the risk of model error.

Mobarakeh Steel has a relatively strong relationship with the overall index due to this issue, which is considered an index-building stock. In between, the linear relationship between yield and total index, liquidity, company size, price coefficient to book value, and weighted index is quite evident. It is important to note that since the share of steel is one of the largest stocks in the market, its relationship with the overall index, which is weighted according to the size of the companies, is quite predictable. Also, the relationship between liquidity and company size was quite predictable because Folad's share is one of the largest companies and one of the most liquid companies in the market, which shows the impact of the risk of these parameters on share return performance. But the most interesting point is the relationship between steel stock returns and market psychological (technical) risk factors. The absolute value between the price gap and moving averages (risk factor indicates the stock that has moved and the stock that is trading in its equilibrium range) or RSI and CCI indicates the impact of this share of market volatility. In other words, since this share is fundamentally known to many market participants, it is heavily influenced by market emotional factors such as moving as a market leader, stagnation due to the rest of the market, or limited oscillating movements. As a result, it can be seen that the technical elements strongly represent the behavior of this share. The profitability and performance parameters of the share, such as the ratio of ROA, ROE, and the margin of operating and net earnings of the share, also seem to have some significant relationship with the return on the share.

Table 2: The extracted factors and optimized weights in the neural network portfolio model

Weight in portfolio model	Model fitness	Factors
0	0.023155	TEDPIX, Total equal weighted Index
0.233434	0.023526	Market value, liquidity,TEDPIX, 5 day moving average Gap
0	0.02402	CCI, TEDPIX, Total equal weighted Index
0	0.024036	Market value, TEDPIX, Absolut 20 day moving average Gap, Absolut 5 day moving average Gap
0	0.024055	PE, Market value, TEDPIX
0.141955	0.024171	TEDPIX, Total equal weighted Index, PB
0	0.024171	TEDPIX, PE, Total equal weighted Index
0	0.024274	TEDPIX, Total equal weighted Index, Market value
0.132359	0.024281	Market value, Momentum 60 day, 20 day moving average Gap, liquidity, TEDPIX
0.168953	0.02429	Market value, PB, Absolut 60 day moving average Gap, PS, liquidity, TEDPIX
0.218521	0.024299	Market value, PB, Momentum 20 day, CCI, liquidity, TEDPIX
0	0.024341	TEDPIX, 5 day moving average Gap, Market value, , 5 day moving average Gap
0.104778	0.024345	Market value, PB, Momentum 20 day, RSI, liquidity, TEDPIX
0	0.024413	PB, ROE, Market value, TEDPIX, RSI, Momentum 60 day
0	0.024413	TEDPIX, Total equal weighted Index, PB, Market value

4 Test the hypotheses

This research has two hypotheses:
Hypothesis 1 in this model development is that creating a nonlinear and optimized neural network model based on machine learning techniques to improve model extraction can significantly improve the average efficiency and reduce model error risk (reduce model error variance) compared to the CAPM model and three factors Fama and French model
Hypothesis 2 in this article is whether creating an optimized portfolio of several nonlinear and optimized neural network models based on machine learning techniques to improve model extraction can significantly improve the average efficiency and reduce model error risk (reduce model error variance), Compared to the improved neural network model, CAPM, and the Fama and French three factors model. To test the above two hypotheses to make sure that the answer obtained is independent of time and independent of the order of data. Data in 10 different scenarios are divided into three random categories of the test, primary training dataset, and secondary training dataset. In each scenario, primary training for neural network training, secondary training for optimizing neural network input parameters, and selection of optimal portfolio from pricing models are determined, and test data are used to test hypotheses. We test the hypotheses in two stages, first, we test it just for a stock share(folad) then we test the hypotheses for all sample datasets.

4.1 Test the hypotheses for one sample stock

Finally, in table 3 below, we can see the error statistics in the two tests of the mean (t-test) and Leven test (test of uniformity of variance). As can be seen in table 3 The second hypothesis is that the portfolio model is better than the improved neural network models both in terms of less error and less risk of error at the level below 1% compared to the model. The neural network, Fama and French, and CAPM are verifiable, as can be seen in Table 3. But the first hypothesis, that the nonlinear model of the improved neural network is better, has slightly different conditions. This model has a confidence level below 1% compared to the CAPM model in the field of average absolute error, but the amount of error risk of this model cannot be confirmed at a confidence level better than 10%. The model is not much different from the three-factor Fama and French, and a 5% confidence level for the average error and a 10% confidence level can be considered for the error risk.

Table 3: Test the hypotheses for folad share as a sample

	The portfolio model (p-model)	The best NN model (NN_model)	Fama & french 3factor	Capm
Count of observation	1017	1.02E+03	1.02E+03	1.02E+03
Mean (MAE)	0.012589	1.40E-02	1.51E-02	1.57E-02
Std of absolute error	0.00956	1.18E-02	1.20E-02	1.24E-02
min	0.000154	1.60E-04	4.88E-05	5.78E-04
25%	0.004901	4.57E-03	5.16E-03	5.80E-03
50%	0.009915	1.10E-02	1.29E-02	1.37E-02
75%	0.017123	2.00E-02	2.26E-02	2.14E-02
max	0.045226	5.54E-02	5.37E-02	5.48E-02
portfolio_model t-stat	NaN	-2.91E+00	-5.28E+00	6.34E+00

Table 3: Test the hypotheses for folad share as a sample

	The portfolio model (p-model)	The best NN model (NN_model)	Fama & french 3factor	Capm
portfolio_model p-value	NaN	3.70E-03	1.44E-07	2.78E-10
portfolio_model levene-stat	NaN	2.94E+01	5.48E+01	5.09E+01
portfolio_model leven-p-value	NaN	6.70E-08	1.92E-13	1.32E-12
NN_model t-stat	NaN	NaN	-2.20E+00	3.24E+00
NN_model p-value	NaN	NaN	2.78E-02	1.22E-03
NN_model levene-stat	NaN	NaN	2.63E+00	2.57E+00
NN_model leven-p-value	NaN	NaN	1.05E-01	1.09E-01
Fama_model t-stat	NaN	NaN	NaN	1.06E+00
Fama_model p-value	NaN	NaN	NaN	2.87E-01
Fama_model levene-stat	NaN	NaN	NaN	1.69E-03
Fama_model leven-p-value	NaN	NaN	NaN	9.67E-01

4.2 Test the hypotheses for all stochastic samples

The stochastic population of the study encompassed companies accepted in the Tehran Stock Exchange and the stochastic sample is 30 companies that were selected randomly, during a 5-year period up to 2021. In order to homogenize the companies and to measure the study variables, the following inclusion conditions were applied to determine the selected companies: A) The company's fiscal year ended on March 20 and there was no change of fiscal year over the considered period; B) The company had been a member of the Exchange Stock Market prior to the period under study and its membership is not terminated; C) The company's data required to measure variables, especially daily stock price changes, are available; D) Their shares are traded during the study period and there are no more than three months of trade termination; and E) The company does not belong to financial and investment intermediary companies and banks. Accordingly, 30 companies were selected. In order to implement the simulation process, a combination of three companies in each industry was considered, and the stock prices and changes for each company were calculated daily. Then the collected information for each stock portfolio was processed using the python programming language. Based on all the data studied, for every 30 shares in the statistical sample, there were 28,701 observations, which can show the appropriate statistics for the proper functioning of the developed model. In the proposed models, functional superiority is considered as less in the absolute mean of the errors and also less deviation on the absolute of the error. the mean of errors will test with a t-test and the variances have been examined with the Leven test. the result has been stated in table--- below. Based on the results, the hypotheses can be examined as follows:

Hypothesis 1: The neural network model has a functional advantage over the benchmark models:

As can be seen, the amount of P-value generated in the tests is very low compared to the Fama Farang model and also compared to the CAPM model, and this indicates the rejection of the null hypothesis and the confirmation of the hypothesis. In other words, it can be said that the results of comparing the means will have good statistical validity at the level of 5% confidence. Due to the fact that the average absolute of the error is in the range of 10% better than the classic models. It can also be seen that the standard deviation of the model has a relatively good improvement compared to traditional models, and this means a reduction in error and also a significant error risk reduction of the developed neural network model, which can confirm the first hypothesis. In other words, it can be concluded that, as it seems logical, the formation of nonlinear models tailored to the characteristics of each share can improve the performance of pricing models.

Hypothesis 2: The neural network portfolio model has a functional advantage over the benchmark models:

Regarding the second hypothesis, like the first hypothesis, according to the P-values created in the mean and standard deviation tests, it seems that the difference between these cases is statistically confirmed, and you can see the average absolute value of error in the portfolio model. The neural network has an improvement of more than 20% compared to traditional models and an improvement in the range of 10% compared to the neural network model. The improvement of this model can be confirmed. On the other hand, it can be seen that model accuracy has been greatly increased and model error risk has also been greatly reduced. In other words, the model developed by a portfolio of neural networks has been able to significantly reduce the risk of error and also increase the accuracy of the model. As a result, the second hypothesis is confirmed

Table 4: Test the hypotheses for all stochastic samples

	The portfolio model (p-model)	The best NN model (NN_model)	Fama & french 3factor	Capm
Count of observation	28701	28701	28701	28701
Mean(MAE)	0.018161291	0.020431407	0.022093146	0.02295675
Std of absolute error	0.015261559	0.016956857	0.018399215	0.019179123
min	3.80828E-06	4.33562E-06	1.03393E-05	1.63201E-05
25%	0.006352379	0.007435531	0.007663122	0.007853076
50%	0.014001681	0.016211216	0.016850502	0.017672942
75%	0.026193655	0.028926509	0.032971455	0.034197999
max	0.095411139	0.11798486	0.118018445	0.132802935
portfolio_model t-stat	NaN	-16.85803132	-27.86495184	-33.14594269
portfolio_model p-value	NaN	1.30372E-63	9.6439E-170	1.1751E-238
portfolio_model levene-stat	NaN	170.9323102	667.9856713	1017.632207
portfolio_model leven-p-value	NaN	5.26435E-39	1.8954E-146	2.2987E-221
NN_model t-stat	NaN	NaN	-11.2512667	-16.71184805
NN_model p-value	NaN	NaN	2.45036E-29	1.51124E-62
NN_model levene-stat	NaN	NaN	164.0686342	355.71528
NN_model leven-p-value	NaN	NaN	1.64515E-37	4.19065E-79
Fama_model t-stat	NaN	NaN	NaN	-5.50486958
Fama_model p-value	NaN	NaN	NaN	3.71018E-08
Fama_model levene-stat	NaN	NaN	NaN	36.42748536
Fama_model leven-p-value	NaN	NaN	NaN	1.59421E-09

6 Conclusion

In this article, an attempt has been made to reach a coherent structure for discovering and extracting optimal pricing models in the Iranian capital market. This adaptive-robust approach tries to arrive at nonlinear and optimized models based on the parameters of each asset. Also, by being inspired by portfolio optimization and forming an optimal portfolio of extracted pricing models, an attempt has been made to reduce the risk of model error as much as possible. This approach can also cause the created model to be more stable over time and the model to make a significant error in fewer conditions. The introduction of the neural network portfolio model was able to reduce the average error of the CAPM and Fama and French three-factor models by nearly 20%, as well as reduce the maximum error by more than 25% and significantly reduce the risk of error. The core idea of this article was to use the machine learning technique to extract the best set of risk factors for the pricing of assets. In this article, the researcher offers to use portfolio-making logic to reduce the risk of errors in the models. In other words, this article proposed that in the factor pricing problem you can delaminate some of the errors by diversifying your risk factors. On the other hand; this research proposed a robust-adaptive approach to generate a factor pricing model that can be an optimized model based on specific features of every stock in the domestic capital market. This research has done its job due to the purpose of the research, which is to provide an efficient model for improving pricing models, and as a sample, the better performance of the proposed model for a share has been examined. However, it is suggested that in future research, the test space of the model be expanded and a bigger look can be taken at the performance of the portfolio model of the improved neural network models. It is also suggested that in future research, the proposed model be used to form a stock portfolio, as well as to calculate the performance of the stock portfolio and its risk. It is expected that the development and application of the model recommended in this paper can effectively improve the performance of risk assessment and portfolio optimization. From another perspective, the development of algorithmic market-making models based on the algorithm developed in this paper is another interesting area that can continue. The model presented in this paper is based on the concept of portfolio optimization of models, but can be expected by introducing concepts such as multi-agent modeling and using optimization algorithms inspired by how shareholders trade. In the capital market, it can create a more adaptable and robust model than the current model.

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