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Analysis of the Relationship between the Business Cycle and Inflation Gap in Time-Frequency Domain

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Controlling the business cycle and minimizing the inflation gap are considered as two major goals for monetary policy. Hence, the policymaker will be able to make more decisive decisions with an awareness of the dynamic relationship and causal relationship between these two variables. Accordingly, the present study uses a discrete and continuous wavelet transform to provide a new understanding of the relationship between these two variables in Iran's economy during the years 1990:2 – 2017:1. According to the results of the research, in the short-run (less than one year), the causal relationship has been bidirectional and procyclical. In the medium run (1 to 4 years), the causal relationship is countercyclical and from the inflation gap to the business cycle. In the long run (4 to 8 years), the business cycle is leading, and the two variables are in phase. Besides, the relationship between variables is highly unstable over time and depends on different scales. Therefore, inflation in Iran's economy is not merely a monetary phenomenon, and in the medium-term is affected by changes in the real sector. According to the results of the research, for the output and the inflation to be stable, it is recommended that the policymaker take both goals simultaneously.

Keywords: Business Cycle, Inflation Gap, Wavelet Transform, Monetary Policy.
JEL Classification: C22, E31, E32.

1 Introduction

The analysis of the relationship between the business cycle and inflation gap, defined as the difference between inflation and trend of inflation, is one of the important issues of the macroeconomics. This relationship has a fundamental role in monetary policy and its related objectives. Therefore, the study of the

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relationship between the business cycle and inflation gap has been the subject of various studies.

The priority of putting output smoothing or inflation is still a major concern for economies such as Iran. Because monetary policy is based on discretion and does not follow a specific monetary rule. Empirical studies (e.g., Taghinezhadomran and Bahman, 2012) also show that monetary policymakers in Iran's economy did not consider both the goals of controlling business cycles and minimizing the inflation gap at the same time. Hence, knowing the dynamic relationship between these two variables helps the monetary policymaker to select the target variable correctly. Despite the widespread literature on this subject, the relation between these two important variables is controversial and does not reveal a single analysis. Since related studies have used traditional econometric methods, there seems to be a lack of consensus on disability due to conventional econometric methods. More precisely, previous studies have had a serious constraint by ignoring the relationship between variables in different scales and changing it over time. In this regard, the present study attempts to investigate the relationship between the business cycle and inflation gap in the frequency domain and the time-frequency domain. For this purpose, the seasonal data during the years 1990 - 2017 and the discrete and continuous wavelet transform tool have been used. Investigating the causal dynamic relationship and its changes over time in terms of intensity and direction, as well as short-term, short-run, medium, and long-run analysis of variables distinguishes the present study from similar studies. The research is organized as follows:

In the second part, theoretical foundations related to research and previous studies are reviewed. At the beginning of the third section, the foundations of wavelet theory are reviewed. Then, the difference in wavelet analysis with the conventional econometric tool is described. Finally, continuous wavelet transforms, and discrete wavelet transformation tools are described. Analysis of results using this tool is the fourth part of the paper. Conclusions and policy proposals presented in the final section.

2 Literature Review

As previously stated, the purpose of this study is to investigate the dynamic relationship between the business cycle and inflation gap in Iran's economy. In this regard, the present section is dedicated to reviewing related theories and studies. First, the theoretical history of the relationship of variables is expressed. Then, a review of related empirical studies is presented.

2.1 Theoretical Foundations

Given the theoretical support and empirical techniques employed, the interpretation of the relationship between output gap and inflation (as representatives of the activities of the real sector and a nominal variable) has evolved. In the economics literature, the first analysis of these variables is attributed to Phillips (1958). He showed, with a simple statistical relation known as the Phillips curve, that nominal wages and unemployment rates in the United Kingdom in the short run have a stable and inverse relationship. This curve could explain a wide range of theories of inflation (such as demand and cost-push). The convincing logic of Philips curves on the impossibility of zero unemployment and price stability has added to the attractiveness of economists and policymakers. The Phillips curve, with changes from unemployment - wage changes to unemployment - changes in prices were used for policy purposes. The unemployment gap is used to illustrate the view that economic fluctuations are the result of supply and demand shocks. Mainly, according to Okun's law (1962), instead of the unemployment gap, the output gap is used as the index of demand pressure in the Phillips curve analysis (Claus, 2000). The relationship and the stable trade-off between the real sector and the inflation based on the Philips curve collapsed in the late 1960s and early 1970s, and the revealed facts did not support it. One of the most significant empirical evidence in this regard is the phenomenon of the US stagflation caused by 1974-1973 and 1979-1978 oil shocks. From a theoretical point of view, the Philips curve was also criticized. Also, from a theoretical point of view, the Philips curve was criticized. In particular, Phelps (1967) and Friedman (1968) argued that the idea that nominal variables (such as inflation) lead to permanent changes in real variables (such as output) is irrational. According to the theory of adaptive expectations, the trade-off between output and inflation is temporary, and the behavior of true variables, in the long run, is determined by the real variables (Romer, 2012).

Following the revolution of rational expectations in macroeconomic theories, the relationship between the real sector and inflation was theoretically evolved.

Indeed, Lucas's (1976) famous critique discredited the trade-off between output and inflation in the short term. Because, in his view, the expectations of economic agents about future inflation are formed correctly.

Structural models were subsequently developed, and the New Keynesian Phillips Curve (NKPC) became the main foundation for inflation–output gap analysis. These models are micro-founded and consider sticky prices and purely forward-looking inflation expectations. Variants of the NKPC depend on the choice of price-setting models and the measure of real marginal costs. For example, Roberts (1995) considered that the aggregate real marginal cost is proportional to the output gap measured using detrending techniques (Tiwari et al. 2014).

In addition to what is mentioned, should be considered that the deviation of inflation from its long-run trend affects the state of the output. On the one hand, rising prices positively affect the supply of goods and services. So, it is expected that output will increase in the aftermath of rising prices. On the other hand, since the continual change in the inflation gap is one of the main sources of instability in macroeconomics, it is accompanied by a dampening of the investment climate by reducing the volume of economic activity and recession.

On the other hand, since the continual change in the inflation gap is one of the main sources of macroeconomic instability, it can lead to a decline in the volume of economic activity and a recession by distorting the investment outlook. The importance of business cycles and the fluctuation of inflation from a policy perspective can also be considered.

The importance of business cycles and the inflation gap can be seen from a policy perspective. Because reducing these two variables, according to the monetary rule of Taylor, is a fundamental goal for monetary policy. In other words, in monetary literature, it has been claimed that macroeconomic stability could be achieved if output and inflation were stabilized around their long-run trend. Nevertheless, in economies where monetary policy is mainly driven by discretion, these two goals are not pursued at the same time. For example, empirical studies have shown that monetary authorities in Iran's economy only care about output deviations. If there is an in-phase causal relationship from the business cycle to the inflation gap, one can expect that inflation will also be controlled after stabilizing output. But if this relationship does not stabilize or change the phase of causality over time, achieving macroeconomic stability will be challenging.

2.2 Empirical Studies

In examining the relationship between the business cycle and inflation gap, the first issue is to separate the trend from the time component of the time series to calculate the two variables. This process is possible using different filters in econometrics. Mojab and Barakchian (2014) show that the identification of expansion and recession is not sensitive to the statistical method. Nevertheless, they state that the depth of the distance between the cyclical component and the trend is sensitive to the statistical method. Hence, the requirements related to the separation of time series and choose the appropriate method is particularly important. In the present study, the Hodrick – Prescott filter has been used to follow related studies (e.g., Özer and Özata, 2016; Tiwari et al., 2014; Taghinezhadomran and Bahman, 2012).

To estimate the relationship between the business cycle and inflation gap in the form of a Phillips curve, in the related research mainly the generalized method of moments has been used. For example, in the research of Jondeau and Bihan (2005), the generalized method of moments (GMM) approach has been used to estimate the Philips curve for some European countries and the United States. They say that considering three lag, the dynamics between the output gap and inflation can be better explained. In the studies of the Iranian economy, the generalized method of moments approach has been used. For example, Erfani et al. (2016) estimated the Hybrid New Keynesian Phillips Curve for Economy of Iran during 1971-2008 using the generalized method of moments approach. The results show that the output gap has a positive and significant effect on inflation. Besides, the coefficients of the expected inflation and lagged inflation variables are statistically significant, which indicate that firms look forward and backward in setting prices, but the coefficient of expected inflation variable is higher than that of lagged inflation, means that firms pay more attention to the expected inflation in setting current prices. The evaluation tests indicate the accuracy and reliability of models.

The generalized method of moments approach has limitations despite its widespread use. The dependence of the results on the instruments and the stationary of the variables are the most important of them. Besides, it is possible to create an endogeneity problem by estimating a single equation. The Vector Autoregressive (VAR) method has also been used in previous studies. For example, Özer and Özata (2016) concluded that there was an inverse causal relationship in the Turkish economy from the output gap to inflation. Mehnatfar Mikaeelee (2013) have shown that in Iran's economy, the inflation shock in the short run negatively affects the business cycle. In the long run, the shock of inflation has a positive effect.

Using spectral methods and analyzing the causal relationship in the frequency domain with related instruments (such as wavelet coherence) can solve the common problems of estimating single-equation methods[1]. Hence, these methods are more appropriate to examine the dynamics between macroeconomic variables. Recently, these methods have been used in studies that relate to the business cycle and the output gap. For example,

[1] For more details see Ramsey et al. (2010).

Assenmacher-Wesche and Gerlach (2007) show that the output - inflation gap is only related to high frequencies. In another study, Tiwari et al. (2014) have used the wavelet transform to analyze the relationship between output – inflation gap. The results of this study have shown that in the short and medium run, there is an inverse causal relationship between the output gap and inflation. By reviewing previous studies, the method used in this study and the resulting implications of it is introduced as a contribution for studies of Iran's economy.

3 Methodology

This paper proposes a novel wavelet analysis to revisit the relationship between output – inflation gap in Iran. Wavelet analysis is greatly distinctive from most conventional mathematical methods such as time-domain methods (correlation analysis and Granger causality, etc.), which cannot identify short-run and long-run relationships between time series, and frequency domain methods (Fourier analysis, etc.), which cannot reveal how such relationships change over time. It allows us to expand time series into a time-frequency space in which the local correlation and the casual relationship can be read off in a highly intuitive way. Therefore, it is very suitable for assessing simultaneously whether the relationship varies across frequencies and evolves. Also, wavelet analysis has a significant advantage over the well-known Fourier analysis, especially when the time series under study are non-stationary or locally stationary (Jiang et al., 2015).

3.1 Continuous Wavelet Transform

As in previous studies (Torrence and Compo, 1998 and Grinsted et al., 2004), our wavelet method involves feature extraction and multi-resolution analysis whereby wavelets are defined as

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right) \quad (1)$$

Wavelets are assumed to be the square-integrable function, i.e., $\psi(.) \in L^2(\mathbb{R})$. In Eq. (1), $1/\sqrt{s}$ refers to the normalization factor ensuring the unit variance of the wavelet, $\left\|\psi_{u,s}\right\|^2 = 1$. u is the location parameter providing the exact position of the wavelet. s is the scale dilatation parameter of the wavelet and defines how the wavelet is stretched. Accordingly, the larger scale implies more stretched wavelet, which is appropriate for the detection of lower frequencies.

The Morlet wavelet is a complex or analytic wavelet within a Gaussian envelope with good time-frequency localization. Formally, the Morlet' wavelet is given by

$$\psi^{M}(t)=\frac{1}{\pi^{1/4}}e^{i\omega_0 t}e^{-t^2/2} \tag{2}$$

Where ω_0 is the central frequency of the wavelet. Following Grinsted et al. (2004), Rua and Nunes (2009), and Baruník et al. (2011), we also set $\omega_0 = 6$. This choice of value for ω_0 enables a good balance between time and frequency localization. The Morlet wavelet is centered at the point $(0, w_0/2\pi)$ in the time-frequency domain (Aguiar-Conraria et al., 2008).

According to Aguiar-Conraria et al. (2008), one particularity of the applications of wavelets to economics is the almost exclusive uses of the discrete wavelet transform. This obsession is difficult to understand because sometimes the same type of analysis could be done more easily and straightforwardly using the continuous wavelet transform. Following, Rua and Nunes (2009) the continuous wavelet transform is given by

$$W_x(u,s)=\int_{-\infty}^{\infty}x(t)\frac{1}{\sqrt{s}}\overline{\psi\left(\frac{t-u}{s}\right)}dt \tag{3}$$

Specifically, $W_x(u,s)$is obtained by projecting the specific wavelet $\psi(.)$ on the selected time series. The major merit of the continuous wavelet transform is the aptitude to decompose and then consequently reconstruct the function $x(t)\in L^2(\mathbb{R})$ such as

$$x(t)=\frac{1}{C_\psi}\int_0^{\infty}\left[\int_{-\infty}^{\infty}W_x(u,s)\psi_{u,s}(t)du\right]\frac{ds}{s^2}, \quad s>0 \tag{4}$$

It is worth noting that the main feature of the wavelet transform is the energy preservation of the selected time series. This property is employed here for the power spectrum analysis, which specifies the variance as follows (Hathroubi and Aloui, 2016):

$$\|x\|^2=\frac{1}{C_\psi}\int_0^{\infty}\left[\int_{-\infty}^{\infty}|W_x(u,s)^2|du\right]\frac{ds}{s^2} \tag{5}$$

3.1.1 The Wavelet Coherence

According to the Fourier spectral approaches, the wavelet coherence (WTC) can be defined as the ratio of the cross-spectrum to the product of the spectrum of each series and can be treated as the local correlation both in time and frequency between two time series. At the same time, the wavelet coherency can be defined as the ratio of the cross-spectrum to the product of the spectrum of each series (Aguiar-Conraria et al., 2008). Following Torrence and Webster (1998), we define the WTC of two time series as:

$$R_t^2(s) = \frac{\left|S(s^{-1}W_{xy}(u,s))\right|^2}{S(s^{-1}|W_x(u,s)|^2)S(s^{-1}\left|W_y(u,s)\right|^2)} \tag{6}$$

Where S is a smoothing operator. Based on the work of Aguiar-Conraria and Soares (2011a), we focus on the wavelet coherency instead of the wavelet cross-spectrum because the wavelet coherency presents the advantage of normalization by the power spectrum of the two time series (Tiwari et al., 2014).

3.1.2 The Cross Wavelet Phase Angle

Since the wavelet squared coherence is between zero and one, negative and positive correlations cannot be recognized. To solve this problem, the phase difference is used. The phase difference provides useful information on the causal relationship by lead-lag interactions. This value for the two time series x and y is:

$$\phi_{x,y} = tan^{-1}\left(\frac{\Im\left\{W_n^{xy}\right\}}{\Re\left\{W_n^{xy}\right\}}\right), with \phi_{x,y} \in [-\pi, \pi] \tag{7}$$

where $\Im$ and $\Re$ are the imaginary and real parts of the smoothed cross-wavelet transform, respectively. In the present study, the values $\phi_{x,y}$ are interpreted in terms of angular arrows. The phase difference analysis is depicted in Fig. 1.

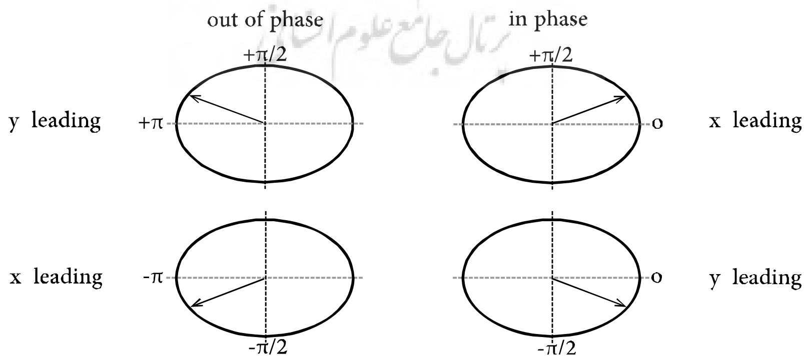

Figure 1. Phase-Differences and Their Interpretation.

Source: Rösch and Schmidbauer (2016)

A phase-difference of zero indicates that the time series move together at the specified time-frequency; if $\phi_{x,y} \in \left(0, \frac{\pi}{2}\right)$, then the series move in phase, but the time series x leads y; if $\phi_{x,y} \in \left(-\frac{\pi}{2}, 0\right)$, then it is y that is leading; a phase-difference of π (or -π) indicates an anti-phase relation; if $\phi_{x,y} \in \left(\frac{\pi}{2}, \pi\right)$, then y is leading. Time series x is leading if $\phi_{x,y} \in \left(-\pi, -\frac{\pi}{2}\right)$ (Aguiar-Conraria and Soares, 2011b).

3.2 Discrete Wavelet Transform

Time series decomposition can be adjusted from a continuous wavelet transform with location and time parameters to conversion with a finer number of time scales with a different number of wavelet coefficients in each scale. It is a discrete wavelet transform.

There are two types of wavelets defined on different normalization rules: father wavelets φ and mother wavelets ψ . The father wavelet integrates to 1, and the mother wavelet integrates to 0:

$$\int \varphi(t)dt = 1 \tag{8}$$

$$\int \psi(t)dt = 0 \tag{9}$$

Roughly speaking, the father wavelets are good at representing the smooth and low-frequency parts of a signal, and the mother wavelets are useful in describing the detail and high-frequency components. Thus, they are used in pairs within a family of wavelet functions, with father wavelets used for the trend components and the mother wavelets for all the deviations from the trend.

Any function *f(x)* in $L^2(R)$ to be represented by a wavelet analysis can be built up as a sequence of projections onto father and mother wavelets generated from φ and ψ through scaling and translation as follows:

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - k) \tag{10}$$

$$\phi_{j,k}(t) = 2^{-j/2}\phi(2^{-j}t - k) \tag{11}$$

The wavelet representation of the signal or function *f(x)* in $L^2(R)$ is given as:

$$f(x) = \sum_{k=1}^{\infty} A_{j,k}\phi_{j,k}(x) + \sum_{k=1}^{\infty} d_{j,k}\psi_{j,k}(x) + \sum_{k=1}^{\infty} d_{j-1,k}\psi_{j-1,k}(x) + \cdots + \sum_{k=1}^{\infty} d_{1,k}\psi_{1,k}(x) \tag{12}$$

Where *j* is the number of multi-resolution components, and *k* ranges from 1 to the number of coefficients in the specified component. The coefficients nts given by the wavelet transform coefficie are $d_{1,k}, \ldots, d_{j-1,k}$, $d_{j,k}$, $A_{j,k}$ the projections

$$A_{j,k} = \int \phi_{j,k}(x) f(x) dx \tag{13}$$

$$d_{j,k} = \int \psi_{j,k}(x) f(x) dx \quad , \text{ for j=1, 2,…, J.} \tag{14}$$

The functions are called the smooth signal and the detail signals, respectively, which constitute a decomposition of a signal into orthogonal components at different scales. Similarly to the wavelet representation of a signal in $L^2(R)$, a signal *f(x)* can now be expressed in terms of these signals:

$$f(x) = A_J(x) + D_J(x) + D_{J-1}(x) + \ldots + D_1(x) \tag{15}$$

As each term in equation (15) represent components of the signal *f(x)* at different resolutions, it is called a multi-resolution decomposition.

The coarsest scale signal $A_J(x)$ represents a coarse-scale smooth approximation to the signal. Adding the detail signal $D_J(x)$ gives a scale 2^{J-1} approximation to the signal, $A_{J-1}(x)$, which is a refinement of the coarsest approximation $A_J(x)$. Further refinement can sequentially be obtained as:

$$A_{j-1}(x) = A_j(x) + D_{j-1}(x) = A_J(x) + D_J(x) + D_{J-1}(x) + \ldots + D_j(x) \quad (16)$$

The collection $\{A_J, A_{J-1}, A_{J-2}, \ldots, A_1\}$ provides a set of multi-resolution approximations of *f(x)* (Atkins and Sum, 2003).

4 Results

4.1 Data

In this study, to obtain the business cycle, the cycle component of GDP[1] was separated using the Hodrick-Prescott filter. Since the inflation targeting in Iran's economy is not officially implemented by the Central Bank (Komijani et al., 2014), It is assumed that the monetary policy intends to stabilize inflation over its long-run trend. Therefore, Hodrick-Prescott's filter has been used to estimate the inflation gap.

In the following, the relationship between variables is analyzed using wavelet covariance, wavelet correlation, wavelet cross-correlation, and wavelet coherence.

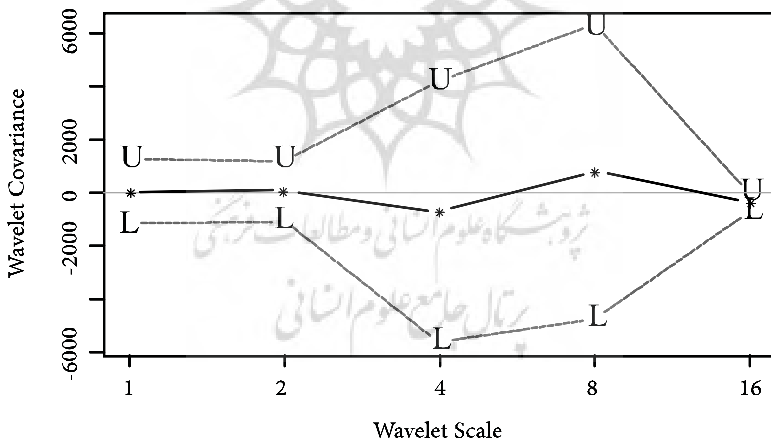

Figure 2. Wavelet Covariance Business Cycle and Inflation Gap.

[1] Without oil and seasonally adjusted; 1383=100

4.2 Discrete Wavelet Analysis Results

Based on the methodology section, business cycles and inflation gap have been decomposed to 5 levels using maximal overlap discrete wavelet transform (MODWT) and D4 filter (which is a family of Daubechies wavelets). The first to fifth levels are related to time scales of 2 to 4 seasons, 4 to 8 seasons, 8 to 16 seasons, 16 to 32 seasons, and more than 32 seasons. The first level shows the short-run scale. The medium run scale has been achieved by the second and third levels of decomposition. The fourth and fifth levels show long run and very long run scales.

Figures (2) and (3) show wavelet covariance and wavelet correlation between business cycles and inflation gap.

The wavelet covariance shows how the two variables are related to each other. According to the coefficient obtained for wavelet covariance, there is no relationship between the business cycle and the inflation gap in the short run. But, in the medium run and very long run, they are inversely related. In the long run, these two variables are positively related. Since covariance does not provide information about the intensity of the relationship between variables, in Fig. 3, the wavelet correlation coefficient is presented to examine the magnitude of the association of each series.

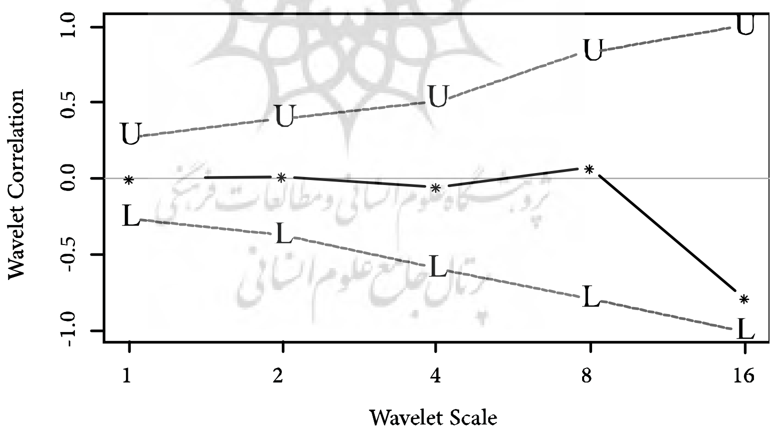

Figure 3. Wavelet Correlation between the Business Cycle and Inflation Gap

The correlation coefficient on different scales indicates the variability of the intensity of the relationship between variables in different time scales. In the short run, the business cycle and the inflation gap do not correlate. By increasing the time scale, the variables are associated with each other. So, in the very long run scale, the correlation is intense and negative.

Using wavelet cross-correlation in positive and negative lags, we can judge the causal relationship by understanding the lead-lag analysis. With this introduction, for each time scale, the correlation between the business cycle and inflation gap with 36 positive lag (half right in each level) and 36 lag (half-left in each level) are presented in Fig. 4.

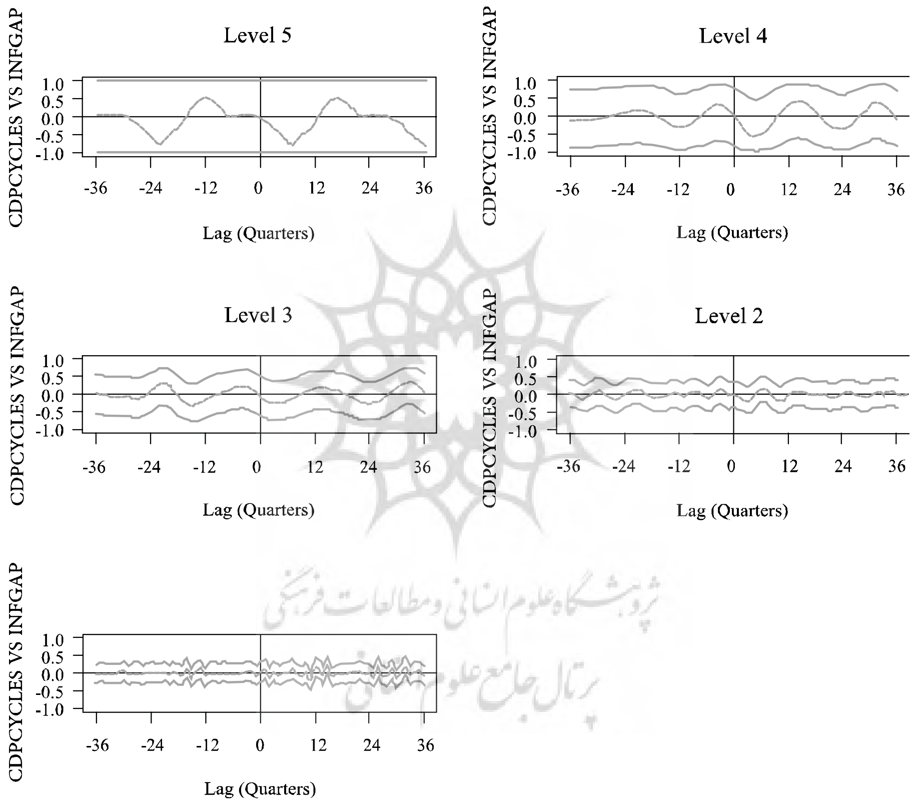

Figure 4. Wavelet Cross-Correlation between the Business Cycle and Inflation Gap.

For each level, if the correlation of the positive (negative) lags of the business cycle with the current values of the inflation gap has a significant difference with zero, the figure is skewed to the right (left). In this case, the business cycle (inflation gap) is considered to be the leading variable, and the inflation gap (business cycle) is lagging. In other words, there is bidirectional causality from the business cycle (inflation gap) to the inflation gap (business cycle). If in both sides, the wavelet cross-correlation has a significant difference with zero, there are two-directional causalities between the variables.

According to Fig. 4, at first to third levels, the cross-correlation coefficient is low in all lags. Therefore, on scales of less than 16 chapters, cannot detect the causality relationship between business cycles and the inflation gap. At the fourth level of decomposition, the figure is drawn to the right, and there is a significant correlation between some business cycle lags and the inflation gap. At the fifth to the sixth level of decomposition, the business cycle is leading. At the very long run scale, there is a two-directional causality between the business cycle and the inflation gap. By the way, it should be noted that the sign and intensity of the causal relationship are very unsteady, and there is no stable cyclic movement between the variables.

4.3 Continues Wavelet Analysis Results

Regarding the widespread variation in correlation coefficients in Figures (2) to (4), the continuous wavelet transform has been used for analysis in the time-frequency domain. For this purpose, the wavelet correlation coefficient has been used. For this purpose, cross wavelet coherence has been used, and causal relationship and co-movement between the variables is analyzed. Also, the continuous wavelet power spectrum is used to analyze their fluctuations in the time-frequency domain.

Figures (5) and (6) show the wavelet power spectrum for the variables used in the research. As previously stated, the wavelet power spectrum provides useful information on the local variance of variables and, consequently, their fluctuations over time. In these figures, the horizontal axis represents time. The right vertical axis is the time scale (in terms of the season). The left vertical axis shows the wavelet power (indicating volatility). By increasing the time scale, the analysis is carried out over a long run and, conversely, by reducing it, analyzes the short-term fluctuations. We classify the frequency on the y-axis into three bands: 0~1-year time scale, 1- to 4-year time scales, and 4- to 8-year time scales, corresponding to short-run, medium-run, and long-run relationships between the business cycle and inflation gap. As the vertical axis on the right shows, moving from bottom to top, dark colors represent intense fluctuations and bright colors reflecting small fluctuations in the time series[1]. The thick black contours designate the 5% significance level estimated

[1] In Fig. 7, these numbers represent the intensity of the wavelet coherence.

from Monte Carlo simulations using a phase randomized surrogate series. The thick black contours designate the 5% significance level estimated from Monte Carlo simulations using a phase randomized[1].

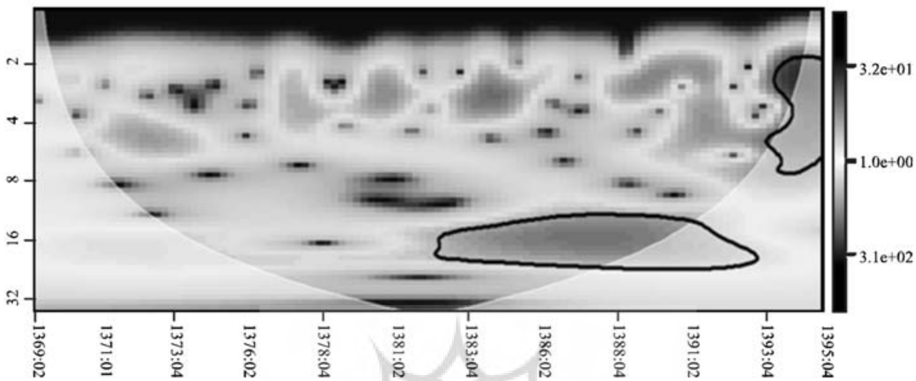

Figure 5. The wavelet power spectrums of the business cycle. The horizontal axis represents time. The vertical axis on the left represents scales (measured by years), while the vertical axis on the right refers to wavelet power (measured as volatility). The thin white line is used to specify the edge effect. The thick black contour represents the 5% significance level against the red noise. Only sections that can be interpreted are located in a significant area and surrounded by a parabola (to prevent edge effect). It should be noted that in all areas where fluctuations are small, the wavelet power has not been significant.

Figure 5 shows that the fluctuations of the business cycle in the short run and the scales below two years have been low. In these scales, the recession of 2013 and 2014, which led to a sharp fall in output from its long-run trend, is obvious. On scales above 12 seasons, the wavelet power is intense and is within the interpretable area. By analyzing in the time domain, Figure (5)

[1] Following Torrence and Compo (1998) and Grinsted et al. (2004), when transforming finite-length time series into wavelets, errors probably occur at both the ends of wavelet power spectrum and wavelet coherency, as the CWT assumes that the series are cyclical. The method to avoid the errors is to pad the ends of the series with zeros. However, zero padding will cause discontinuities at the endpoints, especially as one moves to larger scales (lower frequencies) and declines in the amplitude (variance) near the edges as more zeroes enter the transform. As a consequence, due to this type of edge effect, we cannot distinguish inside the COI whether the decline in lower frequencies is a true decline or an artifact of the zero padding. Therefore, it must take special attention not to misread the results in the COI.

indicates the strong fluctuation of output due to oil revenues and subsequent recession in the period of 2005 - 2014.

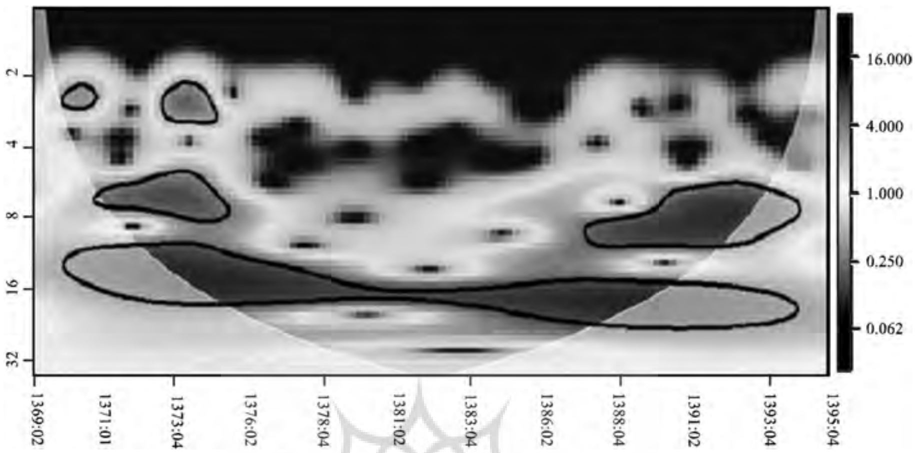

Figure 6. The wavelet power spectrums of the inflation gap. The horizontal axis represents time. The vertical axis on the left represents scales (measured by seasons), while the vertical axis on the right refers to wavelet correlation. The thin white line is used to specify the edge effect. The thick black contour represents the 5% significance level against the red noise. Only sections that can be interpreted are located in a significant area and surrounded by a parabola (to prevent edge effect). It should be noted that in all areas where fluctuations are small, the wavelet power has not been significant.

Figure (6) illustrates one of the most important problems in Iran's economy. According to Figure 6, the inflation gap has been strongly fluctuating at all scales. The increasing inflation until mid-1996 has caused the wavelet power to be intense and significant at almost all frequencies. From 2006 to the end of the period that can be interpreted, the wavelet's power for inflation is very high in the medium and long term. It can be attributed to widespread monetary expansion in the ninth and tenth governments that reflected in inflation. It can be attributed to widespread monetary expansion in the ninth and tenth governments, which led to the volatility of inflation.

Comparing Figures (5) and (6), it was found that business cycles and inflation fluctuations experienced intense fluctuations at all scales. However, the fluctuation of the inflation gap was much wider.

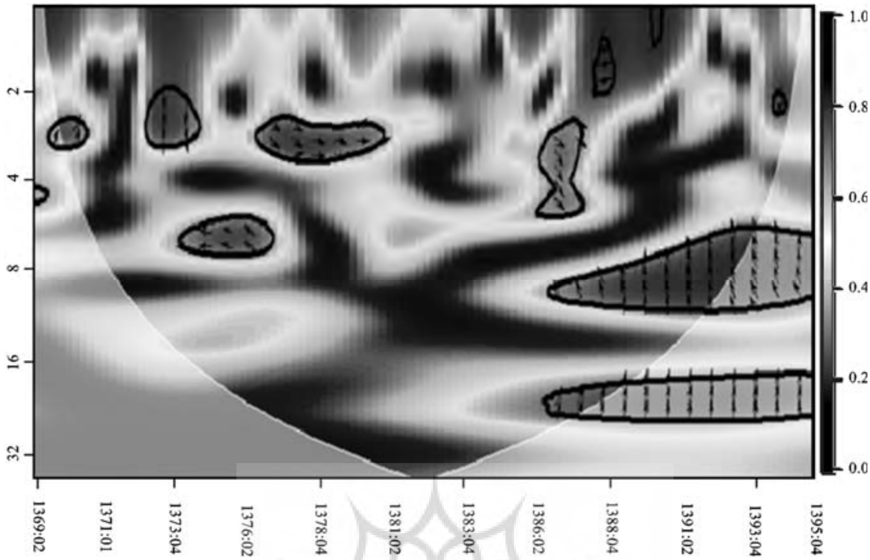

Figure 7. The wavelet coherence and phase difference between the business cycle and inflation gap. The horizontal axis represents time. The vertical axis on the left represents scales (measured by years), while the vertical axis on the right refers to wavelet power (measured as volatility). The thin white line is used to specify the edge effect. The thick black contour represents the 5% significance level against the red noise. Only sections that can be interpreted are located in a significant area and surrounded by a parabola (to prevent edge effect). It should be noted that in all areas where fluctuations are small, the wavelet power has not been significant. Arrows pointing to other directions mean lags or leads between them. For example, arrows pointing straight up mean that business cycle leads inflation gap by one-quarter of the corresponding scale or lags behind inflation gap by three-quarters of the corresponding scale. It is noteworthy that phase differences can also be suggestive of causality between the business cycle and inflation gap.

A wavelet power spectrum is a suitable tool for analyzing time series fluctuations but does not provide any information about co-movement and causal relationship. Accordingly, using wavelet coherence and phase differences, the dynamics of the relationship between the business cycle and the inflation gap have been analyzed. In Fig. 7, the left and right vertical axes represent the frequency (in seasons) and the wavelet correlation coefficient, respectively. The dark (bright) color expresses the maximum (minimum) correlation coefficient, i.e., one (zero). The direction of the angled arrows in

Fig. 7 represents the leading variable and are interpreted as in Fig. 11. In the present study, to convert time series and analyzes to be possible in the time-frequency domain, the Morlet wavelet is used at constant frequency 6.

In the 0 to 1-year time scale (short run), a strong correlation is evident in Fig. 7. However, the leading variable, the intensity of correlation, and the co-movement of two time series vary in this interval. According to the figure, at the scales less than two seasons, the business cycle is a leading variable. But, at the 2 to 4 seasons, the business cycle is only leading in the intervals 1991:4 – 1992:4 and 2015: 1 - 2015: 2. In Other significant areas, inflation gap is leading. Therefore, there is no anti- phase co-movement between variables at the short-run scale.

At the medium run, the inflation gap is leading. In the intervals, 2007 - 2009, the variables are in phases and the intervals between 1995 - 1999 and 2007 - 2014, tare anti-phase. The effect of the inflation gap in the second period was more intense and delayed.

At the more than four years scales, there is a stable, direct, and severe relationship between the business cycle and the inflation gap during 2007-2010. In this period, the business cycle is leading. So, in the long run, with the increase (decrease) in the output gap, the inflation gap increases (decreases).

At the more than four years scales, continuous wavelet transform does not show a specific relationship. This result could be due to a shortage of data. However, the discrete wavelet transform showed two directional causalities between the business cycles and the inflation gap with a numerous change in the coefficient.

According to the results, at the scales less than two and between 20 and 26 seasons, business cycles directly and significantly affect the inflation gap. At the 6 to 10 scales, the inflation gap is leading, and the time series are anti-phase.

The information in Fig. Seven is summarized in Table (1).

[1] Business cycles and inflation fluctuations are considered as the time series x and the time series y.

Table 1
The dynamics of the causal relationship between the business cycle and the inflation gap

Countercyclical Causality		Procyclical Causality		Causality / Time Scale	
Inflation gap is leading	The business cycle is leading	Inflation gap is leading	The business cycle is leading		
-	-	-	2004:4 – 2011:2 (correlation= 0.95)	0 to 2 seasons	Short run (less than 1 year)
-	-	1997:2 – 2000:1 (correlation= 0.85) 2007:2 – 2009:2 (correlation= 0.75)	1992:1 – 1992:3 (correlation= 0.8) 2015:2 – 2015:3 (correlation= 0.85)	2 to 4 seasons	
-	-	2007:2 – 2010:1 (correlation= 0.8)	-	4 to 6 seasons	Medium run (1 to 4 years)
1994:3 – 1997:4 2007:3 – 2015:1 (correlation= 0.9)	-	-	-	6 to 16 seasons	
-	-	-	2007:4 – 2011:2 (correlation= 0.8)	Long run (1 to 4 years)	

Source: Research findings

5 Conclusions and Policy Implications

The business cycle and the inflation gap are two important variables in macroeconomics their smoothing of which is an important target for monetary policy. Empirical studies relied on traditional methods. It was not possible to examine the relationship between the variables in different scales and their changes over time. Besides, the results of the studies are different. To overcome this problem, the present paper studies the wavelet transform tool to investigate the dynamics of the relationship between the business cycle and the inflation gap in Iran's economy during 1990:2 – 2016:1. The wavelet cross-correlation showed that in the long run (scale 4 to 8), the business cycle is a leading variable. At the scale greater than eight-year scale, there is a two-directional causal relationship between the variables. An analysis of the wavelet coherence space indicated that there was a strong correlation between the business cycle and the inflation gap in the short-run (less than one year). At this scale, the casual relationship is highly unstable. In the medium run, although the inflation gap is leading, phase difference and the intensity of the relationship is stable. The discrete and continuous wavelet transform showed that in the long run, the business cycle is leading and directly affects the inflation gap. This result is consistent with the study of Erfani et al. (2016). But it does not endorse the results of Mehnatfar and and Mikaeelee (2013). Using different tools and considering the two-directional causal relationships between variables are the most important causes of this difference. Also, to provide an applied policy implication, in the present study, the inflation gap has been used instead of inflation. Compared to the research by Tiwari et al. (2014), the results are similar, except for the long run. With attention to the results, the following policy is suggested:

- The monetary policymaker can forecast the inflation gap in the short-run (less than six seasons) and long-run by using business cycle changes and take the appropriate actions. The result shows that at these two scales, inflation is not merely a monetary phenomenon, and changes in the real sector affect it. Therefore, to control inflation in the long run, it is not desirable to give excessive weight to monetary policy.
- If monetary policy only seeks to minimize the inflation gap, in the medium run output will be diverted from its potential trend and instability in the real sector will occur. As discrete wavelet analysis showed, there is a two-directional and very unstable causal relationship at the more than eight years; it is not expected that the inflation gap would be reduced by minimizing output gaps. Therefore, for the stability of output and inflation, it is recommended that the policymaker take both goals simultaneously.

6 References

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