Advances in Mathematical Finance & Applications, 6(3), (2021), 631-651
DOI: 10.22034/AMFA.2020.1902265.1446

Published by IA University of Arak, Iran
Homepage: www.amfa.iau-arak.ac.ir

Oil Price Estimating Under Dynamic Economic Models Using Markov Chain Monte Carlo Simulation Approach

Kianoush Fathi Vajarga[a]*, Hossein Eslami Mofid Abadi[b], Ebrahim Abbasi[c]

[a]*Department of Statistics, North Branch, Islamic Azad University, Tehran, Iran*
[b]*Department of Accounting & Management, Shahryar Branch, Islamic Azad University, Shahryar,Iran*
[c]*Faculty of Social Sciences & Economics, Alzahra University, Tehran, Iran*

ARTICLE INFO

Article history:
Received 15 June 2020
Accepted 07 November 2020

Keywords:
Oil Prices
Stochastic Volatility
Jump-Diffusion Process
Markov chain Monte Carlo Simulation

ABSTRACT

This study, attempts to estimate and compare four different models of jump-diffusion class combined with stochastic volatility that are based on stochastic differential equations, and their parameters latent variables are estimated by Markov chain Monte Carlo (MCMC) methods. In the Stochastic Volatility with Correlated Jumps (SVCJ) model, volatilities are scholastic, and the term jump is added to both scholastic prices and volatilities. The results of this study showed that this model is more efficient than the others are, as it provides a significantly better fit to the data, and therefore, corrects the shortcomings of the previous models and that it is closer to the actual market prices. Therefore, our estimating model under the Monte Carlo simulation allows an analysis on oil prices during certain times in the periods of tension and shock in the oil market.

1 Introduction

Focus is on efficient estimation of a dynamic space-time panel data model that incorporates spatial dependence, temporal dependence, as well as space-time covariance and can be implemented in large N and T situations, where N is the number of spatial units and T the number of time periods. Quasi-maximum likelihood (QML) estimation in cases involving large N and T poses computational challenges because optimizing the (log) likelihood requires: 1) evaluating the log-determinant of an NT x NT matrix that appears in the likelihood, 2) imposing stability restrictions on parameters reflecting space-time dynamics, as well as 3) simulations to produce an empirical distribution of the partial derivatives used to interpret model estimates that require numerous inversions of large matrices [32]. Hence, Crude oil price as one of the major global economic metrics are of vital importance to policy-makers, producers, and consumers of financial markets and is constantly studied by them. Crude oil is not only a major input into the production sector, but it also is used as an asset base for the large and growing segments of the financial market.

It is also clear that oil prices do vary, and undergo severe shocks. Some periods when oil prices experienced sudden volatilities include Iraq's invasion of Kuwait and the Persian Gulf War in 1990-1991, the invasion of Iraq in 2003, the financial crisis in the Asia and Russian markets in 1997-1998 and the recent financial crisis in 2008. Therefore, in the analysis of financial issues and the prediction of economic time series, the dynamics of asset base are determined through using jump-diffusion stochastic

* Corresponding author. Tel.: +989124500627
E-mail address: *k_fathi@iau-tnb.ac.ir , fathi_kia10@yahoo.com*

differential equations. In jump-diffusion models, in addition to term “diffusion” which is a geometric Brownian diffusion model with a Wiener random process, the term “jump” is also included with a Poisson process which is dependent on the type of market and plays an important and effective role in modeling. Jump models react to and explain sudden changes in the market [1-4]. These models show high activities of small price changes using diffusion process that is of Brownian motion and persisting along the horizontal axis of behavior. It shows low activities of big price changes using jump process, and stated that the big price changes and volatilities such as sudden increase and decrease of prices are merely done through jumps. In this regard, we compare four different models based on stochastic differential equations of a certain jump-diffusion model with random volatilities. These models include pure jump-diffusion model (JD) [22], pure stochastic volatility model (SV) [21], stochastic volatility with jumps in prices (SVJ) [1], and stochastic volatility model with correlated jumps (SVCJ) where the jumps are supported in both random prices and volatilities and presented as our most common model compared against the other three jump-diffusion models.

Experimental results show that the random jumps and volatilities would not be enough to adequately show the oil prices, however, when combined to fit the data, they are significantly optimized. The only model that gives a satisfactory representation of the price series and jumps in volatilities is SVCJ model [15, 24, 25, 26]. In addition, as jump-diffusion stochastic differential equation usually has no explicit solution, we need to use numerical methods. Monte Carlo simulation approach is one of the most versatile and widely used numerical methods simulated based on random sampling for estimating parameters. Since the unknown parameters are estimated through random samples of population, obviously, according to the principle of consistency, a larger sample size will lead the estimator to a closer true value of the parameter [38, 40]. Therefore, this framework provides effective estimations on model parameters and the process of random volatilities, jump times and sizes. This method that is based on repeated calculations and random or pseudo-random numbers also helps build simulated economic models to predict changes in the price of the market.

In addition, in programming with MATLAB, listed in the appendix, directories simulation techniques that are another step of the Monte Carlo simulation algorithm, is applied and implemented for each geometric Brownian and jump-diffusion models introduced in Section 2. This provides a much better understating to the asset base dynamics through using such models. The article is therefore organized as below. Section 2, presents an introduction and description of the four models for the asset base designed to correct the shortcoming and improve the financial market model called Black-Scholes [7, 8], that is modeled under the dynamics of scholastics jumps and volatilities. Section 3 presents a numerical example in accordance with real financial markets, and discusses and analyzes oil prices during the period of intense market tensions, including the Persian Gulf War and the recent financial crisis. Then in Section 4, programming the aforementioned models based on Monte Carlo simulation in a certain period and to analyze the models, we examine and compare the simulated average prices to actual market prices. Conclusions and recommendations for further studies are presented at the end of the article.

2 Literature Review and Background

2.1 Financial Models

One of the most well-known models in the field of financial markets and financial simulations is the Black- Scholes [6, 8] model, in which the asset base price follows a geometric Brownian motion process where asset price volatility and mobility is assumed fixed, therefore the dynamic or random price changes cannot be predicted. An important issue in the study of the dynamics observed in the market is simulating the impact of events such wars, operation failures, and market crash that lead to uncertainty. They cannot be fully modeled by pure constant processes such as Brownian motion model, because the occurrence of any of such events could lead to a sudden jump in the financial markets. To solve this issue and to improve the Black-Scholes [6, 8] model, economic and financial modeling pays more attention to jump-diffusion stochastic differential equations, where in addition to the components of Brownian diffusion, the term "jump" is also included in the model for more consistency and a better explanation of the market and reaction to sudden market changes. This section of the paper introduces four different models developed from the basic Black-Scholes model, including Heston, Merton, Bates, and Stochastic volatility model with correlated jumps.

2.1.1 Stochastic Volatility Model (SVM)

Understanding the stochastic properties of spot volatility is of importance for risk management in the emissions market. While the effectiveness of stochastic volatility models (with or without jumps) has been advocated by academics and practitioners in the equities market, few attempts have been made to apply them to the newly-developed commodities market governing CO_2 emission allowances [20]. The Heston [21] model is a type of stochastic volatility model (SV) and describes the volatility of the asset as a stochastic process rather than constant. It is a certain case of a general model with jumps restricted to zero($dJ_t^s = dJ_t^v = 0$). In this process, asset prices (such as stocks, bonds, oil, etc.) follow the below diffusion process:

$$dS(t) = S(t)(\mu dt + \sqrt{v(t)}dW_s(t)) \tag{1}$$

This process is similar to the geometric Brownian process, with the exception that it has a stochastic volatility and its instantaneous variance applies in a CIR process:

$$dv(t) = k\big(\theta - v(t)\big)dt + \varepsilon\sqrt{v(t)}dW_v(t) \tag{2}$$

Where ε, θ, and κ are all positive and fixed, θ is the long run average price variance, κ is the rate at which v_t reverts to θ, and ε is time-dependent and determines the instantaneous variance W_v and W_v are the Brownian motion under the risk-neutral measure with the correlation of

$corr[dW_s(t), dW_v(t)] = \rho dt$,

where ρ is the constant correlation parameter belonging to the interval [-1,1] that can be considered the correlation parameter between the logarithm of the asset returns and volatility of the asset base. Experimental studies have shown that in some markets such as stock market, stock prices and volatility are inversely correlated. That is to say a negative correlation exists between stock prices and volatility in short $\rho<0$.

Typically, the correlation parameter ρ, seems negative and it suggests that the decline in prices usually correlates with rises in volatility, sometimes referred to as "reverse effect" [7]. Negative ρ implies that the conditional return distribution (on the initial stock price S_t and volatilityV_t) skews to the left. This process (Equation 2) was initially proposed by Cox, Ingersoll, and Ross [37], for modeling the short-term interest rate with non-negative property. In equation (2), $\kappa(\theta - v(t))$ is called the drift rate, therefore, if $\theta > v(t)$, the drift rate is positive and for $\theta < v(t)$, the drift rate of the variance is negative. In other words, a higher or lower variance from the long-term average θ is instantaneous and variance tends to return to its θ value. This characteristic of the average variance is called average rate of return.

2.1.2 Merton′s Jump-Diffusion Model (MJDM)

Unlike Brownian motion which is characterized by consistency of its sample directions, empirical evidence based on market data shows the existence of random, discrete jumps on the price movement. Given the skewness and elongation in the empirical distribution of oil price returns, Merton Jump-Diffusion model is an appropriate one for oil prices. In addition to the components of Brownian, jump-diffusion processes include processes with discrete movements. The process was first introduced by Merton [22]. In his paper, he pointed out that the empirical evidence, with the probability of one; do not support the models with continuous sample movement. He also found that jumps exist in asset prices. In addition, for more accurate pricing of derivatives, he proposed to model those prices as a jump-diffusion process rather than pure diffusion model (geometric Brownian motion process). the continuous process normally representing component of asset prices are called Wiener Process, and the component of jump are called Poisson Process by Merton. The input rate of assets data is described by a random Poisson distribution. It is assumed that the inputs are independent and distributed. Therefore, he introduced the jump-diffusion process as a combination of jump and diffusion processes. In fact, jump-diffusion process, is the Black-Scholes model plus the jump component. This process is generally shown as:

$$X_t = \mu t + \sigma W_t + \sum_{i=1}^{N_t} Y_i \tag{3}$$

Where, the first phrase represents the continuous part (drift and diffusion, respectively), and the third phrase shows discreteness (jump) in jump-diffusion process. Merton described the base asset price process $S_t$with a stochastic differential equation of the model as:

$$dS_t = S_{t^-}(\mu dt + \sigma dW_t + dY_t) \tag{4}$$

Where t≥0 and $S_0 \geq 0$. μ are the rate of assets return (when there is no jump), σ is the volatility parameter, both of which are considered fixed. Also S_{t^-} is the left limit of S_t at the time of t and describes the amount before the jump. And $Y = \{Y_t, t \geq 0\}$ is a compound Poisson process such that:

$$Y_t = \sum_{k=1}^{N_t} \varepsilon_k \tag{5}$$

Therefore:

$$dY_t = \varepsilon_{N_{t^-}+1} dN_t \tag{6}$$

Where $N = \{N_t, t \geq 0\}$is a Poisson process with intensity of λ> 0. Thus:

$$dN = \begin{cases} 1 \quad , & \lambda dt \text{ with the probability of} \\ 0\ , & (1-\lambda dt) \text{ with the probability of} \end{cases} \tag{7}$$

And $\varepsilon_1, \varepsilon_2, \ldots$ are random variables independent from N and W with the mean below:

$$\kappa = E(\varepsilon_i) < \infty \tag{8}$$

Suppose the time for jump 1k for the Poisson process N is shown by τ_κ, therefore, the size of the jump for the process Y is:

$$\Delta Y_{\tau_\kappa} = Y_{\tau_\kappa} - Y_{\tau_\kappa^-} = \varepsilon_k \tag{9}$$

Where τ_κ is $\kappa \in \{1,2,\ldots\}$. Where the κ growth is

$$\Delta S_{\tau_\kappa} = S_{\tau_\kappa} - S_{\tau_\kappa^-} = S_{\tau_\kappa^-}\Delta Y_{\tau_\kappa} = S_{\tau_\kappa^-}\varepsilon_\kappa \tag{10}$$

$$S_{\tau_\kappa} - S_{\tau_\kappa^-} = S_{\tau_\kappa^-}\varepsilon_\kappa S_{\tau_\kappa} = S_{\tau_\kappa^-}(1+\varepsilon_\kappa) \tag{11}$$

Therefore,

$$\frac{S_{\tau_\kappa}}{S_{\tau_\kappa^-}} = (1+\varepsilon_\kappa) \tag{12}$$

Where S is the jump rate at the time of t. In order to have a nonnegative S, we assume that $\varepsilon_\kappa \geq -1$. In average, we learn that the jump rate in nonnegative according to equations (7) and (8) for all $\kappa \in \{1,2,\ldots\}$ is non-negative, thus:

$$Y_t = \sum_{k=1}^{N_t} \varepsilon_k \tag{13}$$

Merton considered the distribution of jump rate κ as my log-normal. Then, according to equations (4), (6) and (11):

$$dS = S[\mu dt + \sigma dW_t + (Y-1)dN] \tag{14}$$

In some cases, assuming $\kappa = E(\varepsilon_i) = E(\Upsilon - 1)$ the mean returns is used as:

$$dS = S[\mu dt + \sigma dW_t + (Y-1)dN] \tag{15}$$

This means that if κ is larger than $\frac{\mu}{\lambda}$ negative coefficient for dt indicates a descending trend for the asset price and if κ less than $\frac{\mu}{\kappa}$ the asset base price moves above the average. If $dN = 0$, then $\lambda = 0$. Therefore, the Black-Scholes model (with no jumps) is achieved. The stochastic differential equation considered by Merton for the price of crude oil (16), S_t is the crude oil price, μ the instantaneous expected return, σ^2 is the instantaneous return variance, W_t standard Wiener process with distribution $dW_t \sim N(0, dt)$, the price discreteness is expressed by Poisson counting N_t. Shown as Y_t in the independent Poisson process equation (10), where dW is independent Brownian motion and dY Poisson process, λ is the intensity of the Poisson process that describes the average number of data per unit time and expressed as $\text{prob}[\Delta N_t = 1] = \lambda dt$ and $\text{prob}[\Delta N_t = 0] = 1 - \lambda dt$. When skewed data is inputted, jumps crude oil price moves from S_{t^-} toward $S_t = \Upsilon S_{t^-}0$ and $\kappa \equiv E(\Upsilon - 1)$ where $(\Upsilon - 1)$ is the random variable percentage change in oil prices in case of an Poisson and E is the operator of hope on the random variable Y. The size of the jump ε_t is dependent from W_t and N_t and it is assumed that contains $\varepsilon_t \sim N(\beta, \sigma^2)$. The σdW is the instantaneous return of the normal price volatilities and dY describes the irregular price volatilities. If $\lambda = 0$ then the dynamic performance is the same as that of Black-Scholes and Merton [35]:

$$\begin{cases} {dS_t}/{S_t} = (\mu - \lambda\kappa)dt + \sigma dW_t & \text{No Poisson entry} \\ {dS_t}/{S_t} = (\mu - \lambda\kappa)dt + \sigma dW_t + (Y-1)dN_t & \text{In case of Poissoin entry} \end{cases} \tag{16}$$

2.1.3. The Bates stochastic volatility jump-diffusion model (SVJ)

For the dynamics of the asset base, we assume that the stochastic differential equation for S follows the integration of the Merton [31] jump-diffusion process and Heston [18] stochastic volatility process. The dynamics of S under Q Martingale size follows the stochastic differential equation below:

$$dS = (\mu - \lambda\kappa)Sdt + \sqrt{\nu}SdW_1 + (\Upsilon - 1)SdN \tag{17}$$

$$d\nu = \kappa_\nu(\theta - \nu)dt + \sigma\sqrt{\nu}dW_2 \tag{18}$$

Where;

μ the instantaneous return of the asset base at the unit of time,

ν the instantaneous volatility of the asset base at the unit of time,

W_1 Wiener process under Q. Also, the entry of Poisson jump is defined as:

$$dN = \begin{cases} 1, & \lambda dt \text{ with the probability of} \\ 0, & (1-\lambda dt) \text{ with the probability of} \end{cases}$$

$$\kappa = E_Q[(\Upsilon - 1)] = \int_0^\infty (\Upsilon - 1)G(\Upsilon)d\Upsilon \tag{19}$$

Where $G(\Upsilon)$ is the continuous probability density function for multi-step jump measurements, and Y generated by Q. In the long-term average θ for ν ν ' κ_ν, average variance rate, σ is the instantaneous variance at time unit, W_2 is the standard continuous Wiener process with W_1 under Q Martingale size where $E[dW_1 dW_2] = \rho dt$. It should be noted that dN 'Υ' $dW_1$and dW_2 are discrete. We assume that r is the interest rate of neutral risk and the share of a compounded continuous return is at the rate of Y, where r and Y are fixed. It can be easily generalized that these functions are determined by time. As Heston, we assume that the risk of market price volatility is proportional to $\sqrt{v}$ and expressed as $\lambda_v\sqrt{v}$ where λ_v is fixed [2].

$$\Upsilon - 1 = Z_t^s \sim N(\mu_y, \sigma_y^2) \tag{20}$$

Stochastic volatility model with SVJ [2], is an extension to the model SV [18], in which stochastic jumps in prices occur. Overall, J_t^s (or ε_t) is a pure Poisson process and$\forall t$ $J_t^v = 0$. It is assumed that a distributed jump sizes are as below:

$$\Upsilon - 1 = Z_t^s \sim N\left(\mu_y, \sigma_y^2\right) \tag{21}$$

This is a typical model, which includes integrated component in the distribution of returns. This component adds mass to the tails of return distributions. When negative μ_y implies relatively less mass in the right tail, an increase of σ_y adds mass to both tails, and vice versa [18].

2.1.3 Stochastic Volatility Model with Correlated Jumps (SVCJ)

In Stochastic Volatility Model with Correlated Jumps (SVCJ) model expressed as stochastic volatility model with correlated jumps, jumps simultaneously affect both price and volatility. Stochastic volatility model with jumps correlated Stochastic Volatility Model with Correlated Jumps (SVCJ) that arises in this model, both price and volatility jumps at the same time it affects. In this research, as well as continuous time models for the logarithm of the spot price is as follows:

$$dv(t) = k\big(\theta - v(t)\big)dt + \sigma\sqrt{v(t)}dw^2(t) + Z_t{}^v dN_t \tag{22}$$

Its general form is expressed by the following stochastic differential equation as:

$$dS(t) = S(t)(\mu dt + \sqrt{v(t)}dw^1(t) + Z_t{}^y dN_t) \tag{23}$$
$$dv(t) = k\big(\theta - v(t)\big)dt + \sigma\sqrt{v(t)}dw^2(t) + Z_t{}^v dN_t$$

Where S is the spot price, and V is random variance. Continuous dynamics is applied by two correlated Brownian motions of w^1 و $w^2$with correlation ρ where $E(dw_t^s dw_t^y) = \rho dt$. In this model, volatiles undergo jumps. Jump in volatilities and prices is used by the same Poisson process with a constant intensity λ. In the case of a normal Poisson process $N_t^y = N_t^v$. $Z_t{}^y$ and $Z_t{}^v$ are the random jump sizes and since the jumps in Poisson process are constant in both price and volatility, jump sizes can be correlated. Where jumps in volatility has exponential distribution as

$$Z^Y|Z^V \sim N\left(\mu_Y + \rho_J Z^V, \sigma_Y{}^2\right); \qquad Z^V \sim EXP(\mu_V) \tag{24}$$

Where σ_V is the volatility parameter, σ_V is the average long-term volatility, κ controls mean reversion speed, μmeasures the expected return logarithm if there is no price jump. It should be noted that the reverse effect created for the SV base model does not apply to SVCJ as only small price changes due Brownian shocks affect the volatilities. On the other hand, big price changes caused by jumps have no effect on the volatilities in the SVJ model. The SVCJ specifications correct this shortcoming in the SVJ model. Whenever ρ_j is negative, volatilities in market crash increase. Furthermore, in this model, small changes in prices may not have a noticeable effect on volatility, while for large changes price jumps do occur (for example, $\rho = 0$ and $|\rho_J| > 0$. It is also probable that this model attributes large market jumps entirely to the increases in volatility by setting the parameters in the price jump distribution, μ_y , ρ_J and σ_y to zero.

2.2 Background of Research

Li and et al. [31], investigated the efficient of Bayesian estimation for GARCH-type models via Sequential Monte Carlo. He said that, the advantages of sequential Monte Carlo (SMC) are exploited to develop parameter estimation and model selection methods for GARCH (Generalized Auto Regressive Conditional Heteroskedasticity) style models. It provides an alternative method for quantifying estimation uncertainty relative to classical inference. Also, he said that, even with long time series, it is demonstrated that the posterior distribution of model parameters are non-normal, highlighting the need for a Bayesian approach and an efficient posterior sampling method. Moreover, he said that, efficient approaches for both constructing the sequence of distributions in SMC, and leave-one-out cross-validation, for long time series data are also proposed. Finally, he used of an unbiased estimator of the likelihood is developed for the Bad Environment-Good Environment model, a complex GARCH-type model, which permits exact Bayesian inference not previously available in the literature. Ma et al. [35], investigated the of the dual control Monte- Carlo method for tight bounds of value function under Heston stochastic volatility model. The aim of this paper is to study the fast computation of the lower and upper bounds on the value function for utility maximization under the Heston stochastic volatility model with general utility functions.

It is well known there is a closed form solution to the HJB equation for power utility due to its homothetic property. It is not possible to get closed form solution for general utilities and there is little literature on the numerical scheme to solve the HJB equation for the Heston model. In this paper we propose an efficient dual control Monte-Carlo method for computing tight lower and upper bounds of the value function. His identify a particular form of the dual control which leads to the closed form upper bound for a class of utility functions, including power, non-HARA and Yaari utilities. Finally, in this research perform some numerical tests to see the efficiency, accuracy, and robustness of the method. Moreover, the numerical results of this research support strongly proposed scheme. Le Sage et al. [32], investigated of the Markov Chain Monte Carlo estimation of spatial dynamic panel models for large samples. Theirs, used of the set forth a Markov Chain Monte Carlo (MCMC), estimation procedure capable of handling large problems, which we illustrate using a sample of T=487 daily fuel prices for N=12, 435 German gas stations, resulting in N x T over 6 million. The procedure produces estimate equivalent to those from QML and has the additional advantage of producing a Monte Carlo integrated estimate of the log-marginal likelihood, useful for purposes of model comparison. their MCMC estimation procedure uses:1) a Taylor series approximation to the log determinant based on traces of matrix products calculated prior to MCMC sampling, 2) block sampling of the spatiotemporal parameters, which allows imposition of the stability restrictions, and 3) a Metropolis-Hastings guided Monte Carlo integration of the log marginal likelihood. Also, there are provide an efficient approach to simulations needed to produce the empirical distribution of the partial derivatives for model interpretation.

Hong et al. [22], investigated the Leverage effect on stochastic volatility for option pricing in Hong Kong: A simulation and empirical study. This paper explores the importance of incorporating the financial leverage effect in the stochastic volatility models when pricing options. For the illustrative purpose, his first conduct the simulation experiment by using the Markov Chain Monte Carlo (MCMC) sampling method. Their then make an empirical analysis by applying the volatility models to the real return data of the Hang Seng index during the period from January 1, 2013 to December 31, 2017. Our results highlight the accuracy of the stochastic volatility models with leverage in option pricing when leverage is high. In addition, the leverage effect becomes more significant as the maturity of options increases. Moreover, leverage affects the pricing of in-the-money options more than that of at-the-money and out-of-money options. Our study is therefore useful for both asset pricing and portfolio investment in the Hong Kong market where volatility is an inherent nature of the economy. Tsiliyannis [41], investigated the survey Markov chain modeling and forecasting of product returns in remanufacturing based on stock mean-age. He said that, Markov-chain Monte-Carlo simulation enables assessment of the efficacy of the forecasting method.

Exploiting reliable, current information, the method may provide improved estimates of product returns compared to linear models that relate returns to past levels of sales and/or returns, and utilize conventional regression, recursive least squares, or adaptive identification methods. Forecasting efficiency is higher as measured by mean or integral absolute error, and particularly so, regarding peaks and lows of the return flow. The results may be useful for enhanced acquisition of returns with reduced stock inventories and efficient planning of remanufacturing operations. Lux [35], investigated the Estimation of agent-based models using sequential Monte Carlo methods. Here his resort to Sequential Monte Carlo (SMC) estimation based on a particle filter. This approach is used here to numerically approximate the conditional densities that enter into the likelihood function of the problem. in this research, his with approximation his simultaneously obtain parameter estimates and filtered state probabilities for the unobservable variable (s) that drive(s) the dynamics of the observable time series. In this research examples, the observable series will be asset returns (or prices) while the unobservable variables will be some measure of agents' aggregate sentiment. his apply Sequential Monte Carlo (SMC) to two selected agent-based models of speculative dynamics with somewhat different flavor. The empirical application to a selection of financial data includes an explicit comparison of the goodness-of-fit of both models. Shao and et al. [39], investigated the survey of the Pricing and simulating catastrophe risk bonds in a Markov-dependent environment. He said that, at present, insurance companies are seeking more adequate liquidity funds to cover the insured property losses related to natural and manmade disasters. Past experience shows that the losses caused by catastrophic events, such as earthquakes, tsunamis, floods, or hurricanes, are extremely high. An alternative method for covering these extreme losses is to transfer part of the risk to the financial markets by issuing catastrophe-linked bonds.

In this paper, they propose a contingent claim model for pricing catastrophe risk bonds (CAT bonds). First, using a two-dimensional semi-Markov process, their derive analytical bond pricing formulae in a stochastic interest rate environment with aggregate claims that follow compound forms, where the claim inter-arrival times are dependent on the claim sizes. Furthermore, their obtain explicit CAT bond prices formulae in terms of four different payoff functions. Next, their estimate and calibrate the parameters of the pricing models using catastrophe loss data provided by Property Claim Services from 1985 to 2013. Finally, their use Monte Carlo simulations to analyze the numerical results obtained with the CAT bond pricing formulae. Kim et al. [29], investigated the stochastic volatility of the futures prices of emission allow-ances with A Bayesian approach. he said that understanding the stochastic nature of the spot volatility of emission allowances is crucial for risk management in emissions markets. hence, in this study, by adopting a stochastic volatility model with or without jumps to represent the dynamics of European Union Allowances (EUA) futures prices, estimate the daily volatilities and model parameters by using the Markov Chain Monte Carlo method for stochastic volatility (SV), stochastic volatility with return jumps (SVJ) and stochastic volatility with correlated jumps (SVCJ) models. the empirical results of this research, reveal three important features of emissions markets.

First, the data presented herein suggest that EUA futures prices exhibit significant stochastic volatility. Second, the leverage effect is noticeable regardless of whether or not jumps are included. Third, the inclusion of jumps has a significant impact on the estimation of the volatility dynamics. Finally, the results shown that market becomes very volatile and large jumps occur at the beginning of a new phase. moreover, he said that these findings are important for policy makers and regulators. Lian et al. [30], investigated the State-dependent jump risks for American gold futures option pricing. In this study, they investigate the valuation of American-style options when the underlying gold futures price follows a pure diffusion structure with state-dependent jump dynamics.

Under such dynamics, the jump events are described as a compound Poisson process with a log-normal jump amplitude, and the regime-switching arrival intensity is captured by a hidden Markov chain whose states represent the economic states. Considering the different jump risk assumptions, their use the Merton measure and Esscher transform to derive risk-neutral gold futures price dynamics under an incomplete market setting. To achieve a desired accuracy level, the least-squares Monte Carlo method is used to approximate the values of American gold futures options. Our empirical and numerical results based on actual market data are provided to illustrate the importance of incorporating state-dependent jump risks when pricing American put options on gold futures. Estevez et al. [17], investigated the estimation of general equilibrium model in dynamic economies using Markov Chain Monte Carlo methods. This paper describes a general procedure to do Bayesian inference based on the likelihood evaluation of the stochastic general equilibrium models (MEGE) through Markov Chain Monte Carlo methods (MCMC). The proposed methodology involves log linearizing the model, transformed into state space form, then use the Kalman filter to evaluate the likelihood function and finally apply the Metropolis Hastings algorithm to estimate the posterior distribution parameters.

Technique is illustrated using the stochastic growth of basic model, considering quarterly data on the Venezuelan economy between the first quarter of (1984), through the third quarter of (2004). The empirical analysis made allows us to conclude that the algorithms used to estimate the model parameters work efficiently and low computational cost, the estimates obtained are consistent, that is, estimates of the predictions adequately reflect the behavior of the product, employment, consumption and investment per capita in the country. The graphs of the estimated histograms show bimodal and skewed distributions. García [19], investigated Convergence and Biases of Monte Carlo estimates of American option prices using a parametric exercise rule. This paper presents an algorithm for pricing American options using Monte Carlo simulation. The method is based on using a parametric representation of the early exercise decision. It is shown that, as long as this parametric representation subsumes all relevant stopping-times, error bounds can be constructed using two different estimates, one which is biased low and one which is biased high. Both are consistent and asymptotically unbiased estimators of the true option value. Results for high-dimensional American options confirm the viability of the numerical procedure.

The convergence results of the paper shed light into the biases present in other algorithms proposed in the literature. Longstaff and Schwartz [33], propose an algorithm for pricing American options called least-squares Monte Carlo (LSM) approach. This technique proceeds by simulating forward paths using the Monte Carlo simulation, and then performs backward iterations by applying least-squares approximation of the continuation function over a collection of basic functions. This algorithm is simple to implement within existing Monte Carlo frameworks, and has the additional advantages that the continuation functions are constructed explicitly and it is easy to calibrate to existing market prices. Based on the previously mentioned advantages, their then adopt this approach to approximate the American option prices [30]. Chib and Greenberg [14], investigated presented several Markov chain

Monte Carlo simulation methods that have been widely used in recent years in econometrics and statistics. Among these is the Gibbs sampler, which has been of particular interest to econometricians. Although the paper summarizes some of the relevant theoretical literature, its emphasis is on the presentation and explanation of applications to important models that are studied in econometrics. This research includes a discussion of some implementation issues, the use of the methods in connection with the EM algorithm, and how the methods can be helpful in model specification questions. Many of the applications of these methods are of particular interest to Bayesians, but also point out ways in which frequentist statisticians may find the techniques useful.

3 Research Methodology

3.1 Monte Carlo Simulation and Stochastic Differential Equation

In this simulation, we present the expected value $E[g(X(T))]$ for a solution, X, of a known stochastic differential equation with a known function of g. In general, bipartite approximation error contains two parts: random error, and time discretization error. Statistical error estimate is based on the central limit theorem. Error estimation for the time - discretization error of the Euler method directly measures with one remained phrase the accuracy of $\frac{1}{2}$ robust approximation. Consider the following stochastic differential equation:

$$dX(t) = a\big(t, X(t)\big) + b\big(t, X(t)\big)dW(t) \tag{25}$$

How can the value of $E[g(X(T))]$ be calculated on $t_0 \le t \le T$? Monte Carlo method is based on the approximation of

$$E\big[g\big(X(T)\big)\big] \cong \sum_{j=1}^{N} \frac{g\left(\bar{X}(T;\omega_j)\right)}{N}, \tag{26}$$

Where $\bar{X}$ is an approximation of X, according to Euler method, the error in the Monte Carlo is:

$$E\big[g\big(X(T)\big)\big] \sum_{j=1}^{N} \frac{g(\bar{X}(T;\omega_j))}{N}, \tag{27}$$

$$= E\big[g\big(X(T)\big) - g\big(\bar{X}(T)\big)\big] - \sum_{j=1}^{N} \frac{g\left(\bar{X}(T;\omega_j)\right) - E[g(\bar{X}(T))]}{N}. \tag{28}$$

3.2 Monte Carlo Estimation and Central Limit Theorem

Assume the vector (U_1 … … U_N) and $U_i \sim u\big([0,1]^d\big)$ for i = 1, ... , N; the standard Monte Carlo estimation I is defined as follows:

$$\bar{I}_f = \bar{I}_N = \frac{1}{N}\sum_{i=1}^{N} f(U_i) \tag{29}$$

Regarding the law for large numbers:

$$P\left(\lim_{N\to\infty} \bar{I}_N = I_f\right) = 1 \tag{30}$$

The Variance of $f(U)$ for the square integrable f is as follows:

$$\sigma_f^2 = var[f(U)] = \int_0^1{}_d \big(f(x) - I_f\big)^2 \, dx \tag{31}$$

Regarding the central limit theorem:

$$\bar{\mathrm{I}}_{\mathrm{N}} - \mathrm{I}_{\mathrm{f}} \rightarrow N\left(0, \frac{\sigma_f^2}{N}\right) \tag{32}$$

3.3 Oil Market and Sample Data

Oil market modeling methods are categorized into three groups: structural, computational and reduced form or financial models. Although sometimes a hybrid framework encompasses more than one approach, most models usually emphasize only one methodology. The intention of these categories is not to create a formal taxonomy of oil price models, but instead to build some terminology that aids in their comparison [20]. In other word, Understanding the evolution of the oil price is important to consumers, firms and policymakers because oil price fluctuations affect economic decisions across all segments of the global economy [3, 4, 20, 27, 28]. Oil price fluctuations, however, are difficult to anticipate due to unexpected shifts in supply and demand [10].

In practice, the oil price will only be as predictable as its determinants, implying that the better one can identify and understand the determinants of past oil price fluctuations, the more realistic interpretations and better forecasts one can make [1, 11, 12, 13]. The unexpected component of a change in the price of oil is referred to as an oil price shock, defined as the difference between the expected price of oil and its eventual realization [16]. Hence, in this research, variability and dynamics of oil prices during the two critical periods of oil shocks and pressure on oil market including the Persian Gulf War and the 2008 financial crisis are studied to fit an appropriate model for explanation and greater compatibility with these types of markets and forecasting prices. For this purpose, a twenty-year period of daily prices of West Texas Intermediate crude oil (WTI) in spot market presented in Cushing (Oklahoma) from1989 to 2009 will be used. Because WTI crude oil prices is one of the most important international standards for oil and one of the most influential oil prices in America. In addition, it is considered the benchmark for most of the derivatives on the New York Mercantile Exchange (NYMEX).

In this research, with the review of literature review and background, the hypothesis of research, writhed as below:

" Price estimating under dynamic economic models (jump diffusion with stochastic volatility) using Markov chain Monte Carlo simulation, provides a significantly better fit to the data and have more power to price estimating under dynamic economic models."

4 Analysis

4.1 Describe Analysis

In this research, variability and dynamics of oil prices during the two critical periods of oil shocks and pressure on oil market including the Persian Gulf War and the 2008 financial crisis are studied to fit an appropriate model for explanation and greater compatibility with these types of markets and forecasting prices. For this purpose, a twenty-year period of daily prices of West Texas Intermediate crude oil (WTI) in spot market presented in Cushing (Oklahoma) from 25.05.1989 to 05.25.2009 will be used. Because WTI crude oil prices is one of the most important international standards for oil and one of the most influential oil prices in America. In addition, it is considered the benchmark for most of the derivatives on the New York Mercantile Exchange (NYMEX). According to the sample period, spot prices are as follows:

Fig.1: Spot Price Charts
Source: finding of research

According to the figure, oil prices has experienced two major crises during our period sample and has caused changes in financial time-series process, and it shows two big jumps in prices. The first jump occurred during the 1990-1991 Iraqi invasion of Kuwait following the Persian Gulf War, and the second during the recent financial crisis that peaked in 2008. It also illustrates some of the jumps and volatilities during the Asian and Russian financial crises in 1997 and 1998. In other words, perhaps the most uncommon feature of the data is the recent boom in oil prices from 2002 to the end of 2008, when from around $20 per barrel, the prices increased to over $145. In the preliminary analysis, Table 1 demonstrates the descriptive statistics based on the sample data.
Average of oil time series when compared to its standard deviation is large. A comparison on the standard deviation of the mean shows that during the investigation, this variable has undergone large swings in oil prices and shows large volatilities in oil price returns. Significant negative skewness and elongation can be observed with high peaks and thicker tails and cannot be properly represented by a normal distribution. Such elongation and skewness is the common feature of asset returns. Time series demonstrates outstandingly high volatility over the sample period, especially during the Persian Gulf War and the financial crisis has. (Logarithm of the daily returns of oil prices is calculated using the formula $Y_t = \ln({}^{S_t}/_{S_{t-1}})$).

Table 1: Statistical description of log data daily oil output in Period1989-2009

Elongation	Skewness	SD	Min data	Max data	Mean
28.4200	-1.1302	2.6803	-40.2039	26.9804	0.0219

Source: finding of research

4.2 Results of estimating

4.2.1 Estimation of parameters

Estimating the distribution parameters of the diffusion process models with latent variables such as volatility and random jumps, the Markov chain Monte Carlo numerical methods is applied. If parameters and latent variables in the SVCJ model are $\theta = \{\mu, \mu_Y, \sigma_Y, \lambda, \alpha, \beta, \sigma_V, \rho, \rho_J, \mu_V\}$ and $X_t = \{V_t, Z_t{}^y, Z_t{}^v, J_t\}$, all components of θ and X are assumed as random variables by MCMC method, Joint distribution of parameters and latent variables conditional on the data referred to as the posterior distribution is:

$$P(\theta, X|Y) = P(Y|\theta, X)P(X|\theta)P(\theta) \tag{33}$$

$$\mu_Y \sim \mathrm{N}(0,100)\ , \sigma_V{}^2 \sim \mathrm{IG}(2.5,0.1)\ , (\alpha,, \beta) \sim \mathrm{N}(0_{2*1}, I_{2*2})\ \ , \mu \sim N(0,25) \tag{34}$$

$$\sigma_Y{}^2 \sim \mathrm{IG}(10,40)\ \ , \rho \sim U(-1,1)\ , \rho_J \sim N(0,0.5)\ , \mu_V \sim IG(10,20)\ , \qquad \lambda \sim Be(2,40) \tag{35}$$

Posterior distribution can be decomposed into three factors; $P(Y|\theta, X)$ data likelihood probability, $P(X|\theta)$ prior distribution of latent variables conditional on the parameters, and $P(\theta)$ the prior distribution of the parameters, where for the prior distributions parameters:

$$\mu_Y \sim \mathrm{N}(0,100)\ \ , \sigma_V{}^2 \sim \mathrm{IG}(2.5,0.1)\ \ , (\alpha,, \beta) \sim \mathrm{N}(0_{2*1}, I_{2*2})\ \ , \mu \sim N(0,25) \tag{36}$$

$$\sigma_Y{}^2 \sim \mathrm{IG}(10,40)\ \ , \rho \sim U(-1,1)\ , \rho_J \sim N(0,0.5)\ , \mu_V \sim IG(10,20)\ , \qquad \lambda \sim Be(2,40) \tag{37}$$

Results from MCMC numerical methods using MATLAB, for each of the models are summarized in the table below. The columns mean and standard deviation are the mean and posterior standard deviation obtained from the estimation.

Table 2: Estimation of Parameters

Parameter	Heston model (SV)		Merton model (JD)		Bates model (SVJ)		Stochastic volatility correlated jumps (SVCJ)	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
μ	0.0461	0.0261	0.0796	0.0300	0.05336	0.0261	0.0481	0.0244
μ_Y	-	-	-0.5300	0.2991	1.5166	0.8700	-1.3333	1.4594
σ_Y	-	-	5.9998	1.9339	6.9609	1.2886	7.8986	1.2817
λ	-	-	0.1073	0.0121	0.0245	0.0077	0.0183	0.0049
K	0.0171	0.0034	-	-	0.0106	0.0027	0.0194	0.0065
θ	6.6543	0.8394	-	-	5.5287	0.8930	2.7422	0.5288
$\sigma_V{}^2$	0.1296	0.0186	-	-	0.0576	0.0108	0.0374	0.0102
ρ	-0.399	0.0600	-	-	-0.1083	0.0781	-00798	0.0901
μ_V	-	-	-	-	-	-	3.0957	1.4469
ρ_J	-	-	-	-	-	-	-0.1242	0.3870
$\sigma_c{}^2$	-	-	3.2581	0.1288	-	-	-	-

Source: Finding of Research

4.2.2 Comparison of Models

One way to identify the best model obtained by the remnants is through the Euler differencing model where the residuals for each model are:

$$\varepsilon_{t+1}^{Y,SVCJ} = \frac{Y_{t+1} - \mu - Z_{t+1}^{Y} J_{t+1}}{\sqrt{V_t}} \sim N(0,1)$$

$$\hat{\varepsilon}_{t+1}^{Y,SVJ} = \frac{Y_{t+1} - \hat{\mu} - \hat{Z}_{t+1}^{Y} \hat{J}_{t+1}}{\sqrt{\hat{V}_t}}\ ;\ \hat{\varepsilon}_{t+1}^{Y,SV} = \frac{Y_{t+1} - \hat{\mu}}{\sqrt{\hat{V}_t}}\ ;\ \hat{\varepsilon}_{t+1}^{Y,JD} = \frac{Y_{t+1} - \hat{\mu} - \hat{Z}_{t+1}^{Y} \hat{J}_{t+1}}{\hat{\sigma}_c} \tag{38}$$

Residues is estimated to have approximately a normal distribution $N(0,1)$. Figure 2 shows the normal probability plots for the four models. Normal probability plots is used for assessing whether the data has a normal distribution. Many statistical methods based on the assumption that the distribution of normal is created. Therefore, the normal probability plots can provide assurance that this assumption is acceptable, or that it refutes the assumption. Plus, sign in the charts expresses empirical probability against the data value for each point in the data. In the normal probability plot, if all data points fall near the line, the assumption of normality is reasonable. Otherwise, if the curve points are away from the line, the normality assumption is not justified. Analysis of normality is typically a mixture of normal probability plots with a test of normality.

The Jarque-Bera test is a test to check the normality of time series. The null hypothesis of the test is that the data are normally distributed. In other words, the null hypothesis is a joint hypothesis of zero skewness and zero stretch. This test has an asymptotic distribution of (χ^2 with the DOF 2. The test statistic is $JB = \frac{n}{6}(s^2 + \frac{(k-3)^2}{4})$ where n is the sample size, s is the sample skewness, and k is sample elongation. Great statistics indicates that the data do not follow a normal distribution. The test applies a table of critical values calculated using Monte Carlo simulation for sample sizes of less than 2000 and the significant level α between 0.001 and 0.5. According to the figure, it can be seen that the SV and JD models, their corresponding residues significantly deviate from normality. The charts for SVCJ and SVJ models have improved significantly. Residuals for the model SVCJ shows no strong sign of abnormalities. Also, using the Jarque-Bera test, we check and examine the assumption of normality of the residues. The p-value of the test for SVCJ model, is 0.088 indicating that the test cannot reject the null hypothesis of normality at any standard significance level (0.05, 0.001) and have a normal distribution. Null hypothesis is rejected for the SVJ models and the p-value of SV and JD are both less than10^{-10}, which strongly indicates a rejection of the assumption of normality of residual distribution for all significant levels.

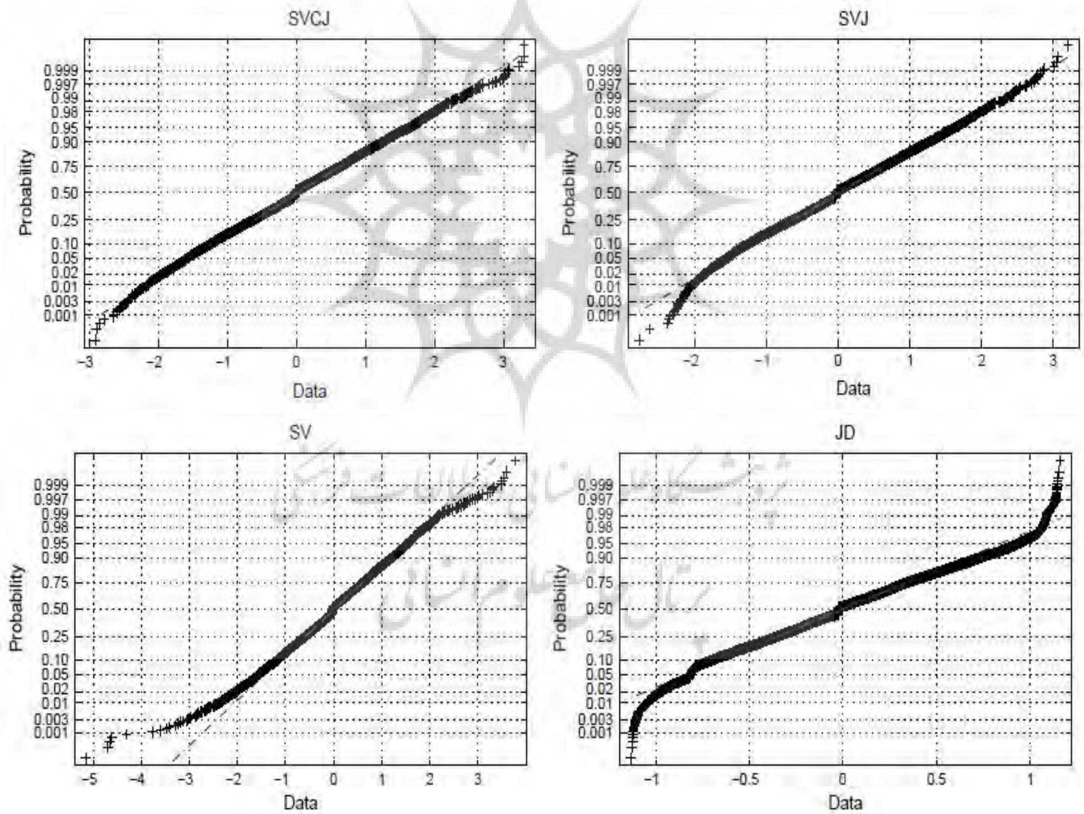

Fig. 2: Normal Probability Plots for Models SVCJ, SVJ, SV and JD

Source: Finding of Research

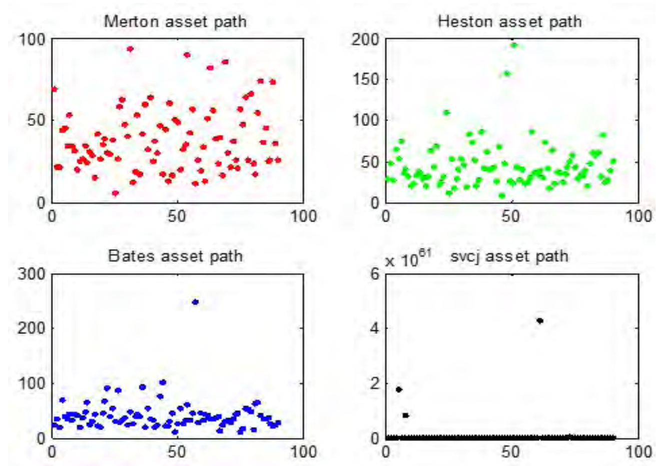

Fig. 3: Scatter Plots for Models SVCJ, SVJ, SV and JD

Estimating the parameters and checking the normality assumption for the estimated residuals of the models, we compare models and select the best model to fit the sample data set for oil price, under the Monte Carlo simulation technique in a specific time, and compare the average simulated prices with the actual market price. Models with Euler differencing functions are implemented in MATLAB and letting the parameter estimates (Table 2) in them, the output will be similar to the Figures 3, 4 and 5.

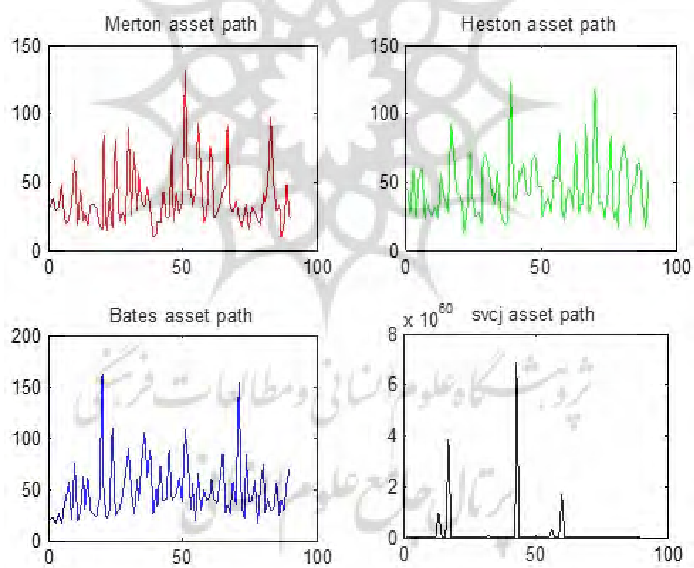

Fig. 4: Plots for Models SVCJ, SVJ, SV and JD

Two of the best oil price paths simulated by Monte Carlo for the next three months (the day 90th) are presented above. Each graph is plotted according to the characteristics and features available in the structure of the functions of each model, such as jumps and stochastic volatility in the price paths, which have a decisive role in the behavior of oil prices. The table below shows the average oil price obtained through simulating price paths for each of the models.

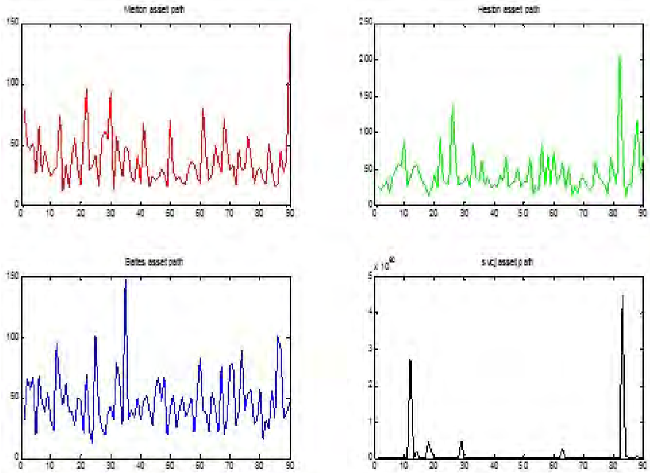

Fig. 5: Oil Price Paths Charts by Monte Carlo Simulation for SVCJ, SVJ, SV, and JD Models
Source: Finding of Research

Table 3: The Average Oil Prices Calculated by Simulating Models Price Paths

Stock price mean	Asset Paths Merton	Asset Paths Heston	Asset Paths Bates	Asset Paths SVCJ
	39.9928	44.31801	42.34361	11.02702

Source: Finding of Research

The price per barrel of WTI crude oil spot market on the 90th day on May 9, 1986 was about $15.83. As it can be seen the closest average price to the actual market price is obtained through simulating the price paths of SVCJ model.

6 Conclusion

In this research, variability and dynamics of oil prices selecting a sample of WTI oil spot market during the two critical periods of oil shocks and pressure on oil market including the Persian Gulf War and the 2008 financial crisis are studied. Plotting the WTI spot price charts and tables describing the statistical data of daily oil output log data, it is clearly seen that the distribution of the logarithm of asset prices (crude oil) is not normal and the distributions has a taller central peak, thicker tails and negative skewness and elongation compared to a normal distribution. In addition, across the sample and particularly during some of the most critical periods of the oil shock, the price volatility is high and outstanding.

According to the experimental results, we seek a model which combining the jumps and stochastic volatilities better fit to the data and have a more acceptable representation of the oil prices dynamics. In this context, estimating the parameter under the represented models in Section II, using one of the methods for diagnosing an appropriate model, is to check the assumption of normality of the residues estimated from the discrete functions of the models. By checking this assumption for all models, we showed that only the Jarque-Bera test approves the normality assumption of the estimated residuals for SVCJ model at all significant levels of the standard normal distribution and has a normal distribution. We simulate oil prices path for all the models with Monte Carlo simulation method at a certain time for our sample period. By separately comparing the average prices for each model, we find that models with actual market prices, the SVCJ model is closer to the actual price of the market and it is the best model fit to the original data. Price paths simulation Graphs also indicate that the jumps in both price and scholastic volatility provide a significantly better fit to the actual values of the market, and can modify the price paths to the original data, and even further promote jumps in the volatility model. Also, observing the scatter plots, Merton [36] and Heston [21] models have too many outliers, but the Bates model with correlated scholastic volatility (SVCJ) has less outliers than the price assumed in the simulation, and if there are outliers, its amount will not increase. Finally, we showed that jumps in both prices and volatility are vital and influential factors. We suggest they be used in future studies on modeling the oil price.

Moreover, in this section, for the more conclusion, the result of this research compared with some of the research that pay to oil price estimating under dynamic economic models using Markov Chain and Monte Carlo simulation approach. Where, we referred and compared the results of this research with research of the Li et al. [31], Ma et al. [35], Le Sage et al. [32], Hong et al. [22], Tsiliyannis [41], Lux [35], Shao et al. [39], Kim et al. [29], Lian et al. [30], Estevez et al. [17], García [19], Longstaff and Schwartz [33], Chib and Greenberg [14]. But, one of the important researches that compared with this research, was Kim et al. [12], research. Where, Kim et al. [12], In theirs study, by adopting a stochastic volatility model with or without jumps to represent the dynamics of European Union Allowances (EUA) futures prices, theirs estimate, the daily volatilities and model parameters by using the Markov Chain Monte Carlo method for stochastic volatility (SV), stochastic volatility with return jumps (SVJ) and stochastic volatility with correlated jumps (SVCJ) models. Hence, the empirical results of this research, reveal three important features of emissions markets. First, the data presented herein suggest that European Union Allowances (EUA) futures prices exhibit significant stochastic volatility. Second, the leverage effect is noticeable regardless of whether or not jumps are included. Third, the inclusion of jumps has a significant impact on the estimation of the volatility dynamics. Finally, the market becomes very volatile and large jumps occur at the beginning of a new phase. Moreover, theirs said that these findings are important for policy makers and regulators. where, also, the results of this study showed that this model is more efficient than the others are, as it provides a significantly better fit to the data, and therefore, corrects the shortcomings of the previous models and that it is closer to the actual market prices. Therefore, our estimating model under the Monte Carlo simulation allows an analysis on oil prices during certain times in the periods of tension and shock in the oil market and it can be said that these results are consistent with the results of research by Kim et al. [29].

References

[1] Alquist, R., Kilian, L., Vigfusson, R.J., *Forecasting the Price of Oil. In G. Elliot and A. Timmersmann (ed.)*, Handbook of Economic Forecasting, Amsterdam: North Holland, 2013, **2**, P. 427-507.

[2] Agah, M., Malekpoor, H., Bagheri, A., *Investigating the Effect of Financial Constraints and Different Levels of Agency Cost on Investment Efficiency*, Advances in Mathematical Finance and Applications, 2017, **2**(4), P. 31-47. Doi: 10.22034/amfa.2017.536264

[3] Ahmadi, R., Kordloei, H., *The Effect of Financial Distress on the Investment Behavior of Companies Listed on Tehran Stock Exchange*, Advances in Mathematical Finance and Applications, 2018, **3**(4), P. 17-28.
Doi: 10.22034/amfa.2019.565459.1108

[4] Bates, D.S., *Jumps and stochastic volatility: exchange rate processes implicit in Deutsche mark options*, Review of Financial Studies,1996, **9**, P. 69-107. Doi: 10.3386/w4596.

[5] Barsky, R.B., Kilian, L., *Do We Really Know that Oil Caused the Great Stagflation? A Monetary Alternative. In B.S. Bernanke and K. Rogoff (ed.)* NBER Macroeconomics Annual, Cambridge: MIT Press., 2001, **16**, P. 137-183.

[6] Barsky, R.B., Kilian, L., *Oil and the Macroeconomy since the 1970s*, Journal of Economic Perspectives, 2004, **18**(4), P. 115-134.

[7] Black, F., *Studies of Stock Price Volatility Changes, Proceedings of the 1976 Meetings of the American Statistical Association*, Business and Economics Statistics Section, 1976, P. 177-181.

[8] Black, F., Scholes, M., *The Pricing of Options and Corporate Liabilities, Journal of Political Economy*, Published By: The University of Chicago Press, 1973, **81**(3), P. 637-654, https://www.jstor.org/stable/1831029

[9] Kilian, L., *The Economic Effects of Energy Price Shocks*, Journal of Economic Literature, 2008, **46**(4), P. 871-909.

[10] Baumeister, C., Kilian, L., *Fourty Years of Oil Price Fluctuations: Why the Price of Oil May Still Surprise Us*, Journal of Economic Perspectives, 2016, **30**(1), P. 139-160.

[11] Baumeister, C., Kilian, L., *Understanding the Decline in the Price of Oil since June 2014*, Journal of the Association of Environmental and Resource Economics, 2016, **3**(1), P. 131-158.

[12] Baumeister, C., Kilian, L., *Real-Time Forecasts of the Real Price of Oil*, Journal of Business and Economic Statistics, 2012, **30**, P. 326-336.

[13] Baumeister, C., Kilian, L., *Real-Time Analysis of Oil Price Risks using Forecast Scenarios*, IMF Economic Review, 2014, **62**, P. 119-145.

[14] Chib, S., Greenberg, E., *Markov Chain Monte Carlo Simulation Methods in Econometrics*, Econometric Theory,1996, **12**(3), P. 409-431. Doi: 10.1017/S0266466600006794.

[15] Dibachi, H., Behzadi, M.H., Izadikhah, M., *Stochastic Modified MAJ Model for Measuring the Efficiency and Ranking of DMUs*, Indian Journal of Science and Technology, 2015, **8**(8), P.1-7, Doi: 10.17485/ijst/2015/v8iS8/71505

[16] Economou, A., Agnolucci, P., Fattouh, B., De Lipis, V., *A Structural Model of the World Oil Market: The Role of Investment Dynamics and Capacity Constraints in Explaining the Evolution of the Real Price of Oil Andreas*, OIES PAPER: WPM 75, Oxford Institute for Energy Studies, Registered Charity, 2017, **286084**, P. 1-40.

[17] Estévez, G., Infante, S., Sáez, F., *Estimation of general equilibrium model in dynamic economies using Markov Chain Monte Carlo methods*, Rev. Mate. Teor. Aplic, 2012, **19**(1), P. 7–36.
Doi: 10.15517/rmta.v19i1.2102.

[18] Eraker, B., *Do stock prices and volatility jump? Reconciling evidence from spot and option prices*, The Journal of Finance, 2004, **59** (3), P. 1367–1403.

[19] García, D., *Convergence and Biases of Monte Carlo estimates of American option prices using a parametric exercise rule*, Journal of Economic Dynamics and Control, 2003, **27**(10), P. 1855-1879.
Doi: 10.1016/S0165-1889(02)00086-6.

[20] Hamilton, J. D., *Understanding Crude Oil Prices,* The Energy Journal, 2009, **30**(2), P. 179-206.

[21] Heston, S.L., *A closed-form solution for options with stochastic volatility with applications to bond and currency options.* Review of Financial Studies, 1993, **6**, P. 327–343. Doi: 10.1093/rfs/6.2.327

[22] Hong, H., Bian, Z., Chen, N., *Leverage effect on stochastic volatility for option pricing in Hong Kong: A simulation and empirical study*, The North American Journal of Economics and Finance, 2019, 100925.

[23] Huntington, H., Al-Fattah, S., Huang, Z., Gucwa, M., Nouri, A., *Oil Markets and Price Movements: A Survey of Models*, USAEE Working Paper No. 13-129, 2013, Doi: 10.2139/ssrn.2277330.

[24] Izadikhah, M., Khoshroo, A., *Energy management in crop production using a novel fuzzy data envelopment analysis model*, RAIRO-Oper. Res., 2018, **52** (2), P. 595-617, Doi: 10.1051/ro/2017082

[25] Izadikhah, M., Farzipoor Saen, R., *Ranking sustainable suppliers by context-dependent data envelopment analysis,* Ann Oper Res, 2020, **293**, P. 607–637, Doi: 10.1007/s10479-019-03370-4

[26] Karbasi Yazdi, H., Mohammadian, M., *Effect of Profitability Indices on the Capital Structure of Listed Companies in Tehran Stock Exchange*, Advances in Mathematical Finance and Applications, 2017, **2**(3), P. 1-11. Doi: 10.22034/amfa.2017.533085

[27] Kilian, L., *The Economic Effects of Energy Price Shocks,* Journal of Economic Literature, 2008, **46**(4), P. 871-909.

[28] Kilian, L., *Oil Price Shocks: Causes and Consequences*, Annual Review of Resource Economics, 2014, **6**(2014), P. 133-154.

[29] Kim, J., Park, Y.J., Ryu, D., *Stochastic volatility of the futures prices of emission allowances: A Bayesian approach*, Physica A, 2017, **465** (2017), P. 714–724.

[30] Lian., L., Szu-Lang., C., Jun, H., *State-dependent jump risks for American gold futures option pricing,* The North American Journal of Economics and Finance, 2015, **1**(1), 33, P. 115-133. Doi: 10.1016/j.najef.2015.04.001.

[31] Li, D., Clements, A., Drovandi, C., *Efficient Bayesian estimation for GARCH-type models via Sequential Monte Carlo*, Econometrics and Statistics, 2020, Doi: 10.1016/j.ecosta.2020.02.002.

[32] Le Sage, J.P., Chih, Y-Y., Vance, C., *Markov chain Monte Carlo estimation of spatial dynamic panel models for large samples*, Computational Statistics & Data Analysis, 2019, **138**, P. 107-125. Doi: 10.1016/j.csda.2019.04.003.

[33] Longstaff, F.A., Schwartz, E.S., *Valuing American options by simulation: A simple least-squares approach*, Review of Financial Studies, 2001, **14**, P. 113–147. Doi: 10.1093/rfs/14.1.113.

[34] Lux, T., *Estimation of agent-based models using sequential Monte Carlo methods,* Journal of Economic Dynamics and Control, 2018, **91**, P. 391-408. Doi: 10.1016/j.jedc.2018.01.021

[35] Ma, J., Li, W., Zheng, H., *Dual control Monte-Carlo method for tight bounds of value function under Heston stochastic volatility model*, European Journal of Operational Research, 2020, **280**(2), P. 428-440. Doi: 10.1016/j.ejor.2019.07.041.

[36] Merton, R.C., *Option pricing when underlying stock returns are discontinuous*. Journal of Financial Economics, 1976, **3**, P. 124–144. Doi: 10.1016/0304-405X(76)90022-2.

[37] Cox, J.C., Ingersoll, Jr. J.E., Ross, S., *A theory of the term structure of interest rates*, Econometrica, 1985, **53**(2), P. 385-407, Doi: 10.2307/1911242, https://www.jstor.org/stable/1911242

[38] Rezaei, N., Elmi, Z., *Behavioral Finance Models and Behavioral Biases in Stock Price Forecasting*, Advances in Mathematical Finance and Applications, 2018, **3**(4), P. 67-82. Doi: 10.22034/amfa.2019.576127.1118

[39] Shao, J., Papaioannou, D.A., Pantelousbc , A.A., *Pricing and simulating catastrophe risk bonds in a Markov-dependent environment*. Applied Mathematics and Computation, 2017, **309**(15), P. 68-84. Doi: 10.1016/j.amc.2017.03.041

[40] Tone, K., Toloo, M., Izadikhah, M., *A modified slacks-based measure of efficiency in data envelopment analysis*, European Journal of Operational Research, 2020, **287** (2), P. 560-571. Doi: 10.1016/j.ejor.2020.04.019.

[41] Tsiliyannis, C.A., *Markov chain modeling and forecasting of product returns in remanufacturing based on stock mean-age*, European Journal of Operational Research, 2018, **271**(2), P. 474-489. Doi: 10.1016/j.ejor.2018.05.026.

Appendix

```
% Set up the parameters.
lam = 2;
n = 1000;
% Generate the random varia-
bles.
uni = rand(1,n);
X = -log(uni)/lam;
% Get the values to draw the
theoretical curve.
x = 0:.1:5;
% This is a function in the
Statistics Toolbox.
y = exppdf(x,1/2);
% Get the information for the
histogram.
[N,h] = hist(X,10);
% Change bar heights to make
it correspond to
% the theoretical density -
see Chapter 5.
N = N/(h(2)-h(1))/n;
% Do the plots.
bar(h,N,1,'w')
hold on
plot(x,y)
hold off
xlabel('X')
ylabel('f(x) - Exponential')
function f()
disp('For simulation of
f_distribution')
a=input('Please inter a  : ');
b=input('Please inter b  : ');
for j=1:1000
  sum=0;
  for i=1:a
      sum=sum+randn^2;
  end;
  x(j)=sum;
  sum=0;
  for i=1:b
      sum=sum+randn^2;
  end;
  y(j)=sum;
end
ff=x./y;
mean(ff)
var(ff)
hist(ff)
function f()
a=input('Please inter a  : ');
b=input('Please inter b  : ');
for j=1:10000
  sum=0;
  for i=1:b
      sum=sum+(-a)*log(rand);
  end
  x(j)=sum;
end
mean_error=mean(x)-a*b
var_error=var(x)-a^2*b
hist(x)
clear;
unction f()
n=input('Please inter free
degree for simulation of
k_square distribution: ');
for j=1:1000
  sum=0;
  for i=1:n
      sum=sum+randn^2;
  end;
  x(j)=sum;
end
mean(x)
var(x)
hist(x)
function f()
n=input('Please inter free
degree for simulation of
t_distribution: ');
for j=1:1000
  sum=0;
  for i=1:n
      sum=sum+randn^2;
  end;
  x(j)=rand/sqrt(sum/n);
end
m_error=mean(x)
var_error=var(x)-n/(n-2)
hist(x)
clear;
clc;
n=input('Please inter free
degree for simulation of
t_distribution: ');
 for j=1:1000
  sum=0;
  for i=1:n+1
      z(i)=randn;
  end;

x(j)=mean(z)/(var(z)/sqrt(n+1)
);
end
m_error=mean(x)
var_error=var(x)-n/(n-2)
hist(x)
% Set up the parameters.
lam = 2;
n = 1000;
% Generate the random varia-
bles.
uni = rand(1,n);
X = -log(uni)/lam;
% Get the values to draw the
theoretical curve.
x = 0:.1:5;
% This is a function in the
Statistics Toolbox.
y = exppdf(x,1/2);
```

```
% Get the information for the histogram.
[N,h] = hist(X,10);
% the theoretical density - see Chapter 5.
N = N/(h(2)-h(1))/n;
% Do the plots.
bar(h,N)
hold on
plot(x,y,'r')
hold off
xlabel('X')
ylabel('f(x) - Exponential')
n = 1000;
t = 3;
lam = 2;
% Generate the uniforms needed. Each column
% contains the t uniforms for a realization of a
% gamma random variable.
U = rand(t,n);
% Transform according to Equation 4.13.
% See Example 4.8 for an illustration of Equation 4.14.
logU = -log(U)/lam;
X = sum(logU);
[N,h] = hist(X,10)
N = N/(h(2)-h(1))/n;
x = 0:.1:6;
y = gampdf(x,t,1/lam);
bar(h,N)
hold on
plot(x,y,'r')
p=0.5
 n=6
 N=100
 X=zeros(1,N)
 U=rand(N,n)
for i=1:N
ind=find(U(i,:)<=p);
X(i)=length(ind)
X=sum(rand(n,N)<=p);
end
hist(X)
lam = 0.5;
n = 500; % Sample size
x = zeros(1,n);
j = 1;
while j <= n
flag = 1;
% initialize quantities
u = rand(1)
i = 0;
p = exp(-lam);
F = p;
while flag % generate the variate needed
if u <= F % then accept
x(j) = i;
flag = 0;
j = j+1;
else % move to next probability
p = lam*p/(i+1);
i = i+1;
F = F + p;
end
end
end
edges = 0:max(x);
f = histc(x,edges);
bar(edges,f/N,1)
for i=1:10000
while(1)
u1=rand
u2=rand
v1=2*u1-1.0
v2=2*u2-1.0
s=v1*v1+v2*v2
if(s>=1.0 | s<=0.0)continue
end
b=sqrt(-2.0*log(s)/s)
x1=b*v1
x2=b*v2
break
end
end
scatter(x1,x2)
p=0.3;
Y = zeros(10000,1);
for i=1:10000
n = 0;
Y(i) = 200;
for j=1:200
if (rand() < p)
n = n + 1;
if (n==4) Y(i)=j;
end
end
end
end
f = zeros(200,1);
for i=1:200
f(i) = sum(Y == i);
end
% pdf
bar(f/10000)
```