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Airline Hub-Median Network Design by an Extended Two-Stage Stochastic Programming Method: A Case Study

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Abstract

The hub location decision is a long term investment and any changes in it take considerable time and money. In real situations, some parameters are uncertain hence, deterministic models cannot be more efficient. The ability of two-stage stochastic programming is to make a long-term decision by considering effects of it in short term decisions simultaneously. In the two-stage stochastic programming for hub location problems, the location is decided in the first stage and optimal flow allocations are determined in the second stage. In this paper, the two-stage stochastic programming is described and then a practical stochastic model is employed for determining hub locations in the Iranian aviation. Also, a survey of the model under fuel subsidy omission is conducted using extended two-stage stochastic programming. Demand and the cost of resources (fuel) are considered uncertain in this study. The results show Tehran, Mashhad, Isfahan, Shiraz and Yazd can be hubs in the air network of Iran.

Keywords: Two-Stage Stochastic Programming; Demand Uncertainty; Hub and Spoke Network; Iran Aviation.

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1. Introduction

Hub location problem is one of the important issues in the location theory which has some applications in air, ground and water transportation systems and telecommunication networks design. Over the past two decades, researchers are interested in hub location problems and their applications in various areas. Indeed, hubs as special facilities have some main applications in transshipment and switching points in many to many distribution systems. The difference of hub networks in comparison to the others like fully meshed network is concentrating flows in hubs for taking advantage of economies of scale instead of servicing directly. Hub location problems are basically concerned with locating hub facilities and allocating non-hub nodes to hubs. Therefore, the initiation flow from origin nodes is consolidated on hubs and then allocated to destinations. The main objective of these problems is to select some nodes of the network according to the amount of flow and situation of nodes and equipped these nodes to special facilities as hubs then allocates other nodes to hubs. Decreasing the transportation costs or time in most distribution systems like air transportation systems are some of the advantages of hub and spoke networks. Hub median location problem is one of the categories of location problems where the objective is to minimize the total transportation cost. According to the literature, O'Kelly presented a mathematical formula for a hub location problem and studied airline passenger networks for the first time (O'Kelly, 1987:394-404). In addition, he introduced the single allocation hub location problem with fixed costs. Campbell proposed the first linear integer formulation for p-hub median problem and uncapacitated hub location problems (Campbell, 1994:387-405). For more comprehensive review in hub location problems, the reader is referred to (Alumur and Kara, 2008:1-21; Gelareh & Nikel, 2011:1092-111). Louveax studied on modeling in stochastic location problem (Louveax, 1993:127-154). Synder presented stochastic and robust location models for stochastic location problem (Synder, 2006:537-554). Mariano and Serra proposed a stochastic model for hub location in airline network by using queuing theory and chance constraint approach (Mariano and Serra, 2003: 983-10033). Sim introduced a p-hub center problem and used a chance constrain formulation for minimizing the service level requirement (Sim, 2009:3166-3177). Yang proposed a two- stage stochastic model for air freight hub location with uncertain demand (Yang, 2009: 4424-4430). In addition, Yang employed the model for air passenger network and described it with a case study in Taiwan and China air passenger network (Yang, 2010: 187-213). Liu et al. proposed a scenario-based stochastic model for transportation network location (Liu et al., 2009:1582-1590). They modeled the network retrofit as a two-stage stochastic programming and then solved it with developed algorithm. Contreras et al. studied stochastic uncapacitated hub location problems with uncertainty in demand, dependent and independent transportation costs and to solve it, he described a Monte-Carlo simulation (Contreras et al., 2011:518-528). Hub and spoke networks play an important role in decreasing operational costs of airlines. Now, the cost of air flight is more expensive than the ground transportation systems in Iran. The most paths of the current network are direct.

According to some studies about the effect of hub networks in improving the distribution networks and transportation systems, it seems air transportation costs can be decreased by using hubs. In this study, an attempt was made to design a hub and spoke network and determine the location of hubs in air network of Iran. In real situations, hub location is a long term decision and any changes in it can be more expensive. Then deciding about hubs is strategic decision which influences tactical and operational activities of a company and it is important in improving the productivity. In the real world some parameters like costs, demands and travel times are not certain. Thus, deterministic models cannot be more compatible with environmental conditions. The air market usually has a seasonal demand variation. In other words, some months due to holidays, the demand of flight increases. Classical methods are used sensitivity analysis for considering uncertainty but these methods cannot generate robust solutions for the problem in uncertainty (Azar et al., 2011:2). Indeed, sensitivity analysis is suitable for analyzing the quality of solutions. Table 1 shows some applied studies about the hub location problem in various areas.

Table 1 Some Applications of Hub Location Problems.

Authors	Aspect	Stochastic / Deterministic	Application
O'Kelly	First mathematical model for hub location	deterministic	Airline passenger networks
Jaillet et al.	Designing capacitated network	deterministic	Airline with a fixed share of market
Marriano & Serra	Hub location model with stochastic number of customers at hub airlines by using the queuing theory	stochastic	Airline network
Eiselt & Mariano	Competitive hub location	deterministic	Air transportation
Takano & Arai	A proposed model for hub location	deterministic	Maritime transportation
Yang	A proposed model for designing hub network with uncertain demand	stochastic	Taiwan & Chine air network
Gelareh et al.	Competitive hub location problem	deterministic	Liner shipping industries
Bashiri & Karimi	Hub covering location	deterministic	Iran Aviation
Mohammadi et al.	A queue model hub covering location	stochastic	Cargo transportation system

In this study, first we employ a two-stage stochastic programming model that Yang proposed in the airline application for Iranian aviation and then extend the two-stage stochastic model for adding another uncertain parameter that is more compatible with real situations (Yang,2009: 4424- 4430; 2010, 187-213). To the best of authors' knowledge, it seems that this research is the first study on hub location problem in Iran aviation network using two-stage stochastic programming. In this study we propose an extended two-stage model in order to determine hub locations by considering important environmental changes, such as variations in the amount of customers or changes in price of fuel in a year period and use it for Iran aviation. It seems that the proposed model can be useful for improving the current air network of Iran. Undoubtedly, cost is one of the major parameters in operational optimization, customer attraction and their satisfaction. This paper is organized as following; at first general form of two-stage stochastic programming has been introduced. In Section 3, a two-stage stochastic model has been presented in details. In Section 4 we propose an extended two-stage stochastic model and then use it in Iran aviation network has been illustrated in Section 5. Section 6 concentrates to sensitivity analysis and finally the last section contains concluding remarks.

2. Two-Stage Stochastic Programming

Generally, the single-stage approaches provide the solutions with less information used and cannot consider all realistic conditions. So, the optimal solution of them is usually far from real optimality especially in real applications. The idea of two-stage stochastic programming with recourse is going back to the work of Beale and Dantzing in 1955 for the first time (Birge & Louveaux, 1997:184). The general form of two-stage stochastic programming is:

Stage 1:

$$\underset{x}{\text{Min}}\ c^T x + E[Q(x, \xi(\omega))] \tag{1}$$

s.t.

$$Ax = b \tag{2}$$

$$x \in X \tag{3}$$

Stage 2:

$$Q(x, \xi(\omega)) = \underset{y}{\text{Min}}\ q(\omega)^T y(\omega) \tag{4}$$

s.t.

$$T(\omega)x + W(\omega)y(\omega) = h(\omega) \tag{5}$$

$$y(\omega) \in Y \tag{6}$$

1. The objective function of Stage 1 is made of two terms, a deterministic term with decision variable *x* and expected value of optimal solution of Stage 2. The vector *x* is a long term decision that is independent of uncertainty. It is denoted by a $n_1 \times 1$ vector. In the first stage a decision on the basis available information should be made. In Stage 2, *ω* represent a random event or scenario and *Ω* is the set of all scenarios. *q(ω)* is the vector of probabilities of scenarios. Parameters *T(ω), W(ω)* and

h(ω) are the data of problem represented by matrices with sizes $m_2 \times n_1$, $m_2 \times n_2$ and $m_2 \times 1$ respectively. For a particular realization *ω*, in Stage 2 when the value of parameters *q (ω), T (ω), W(ω)* and *h(ω)* are known then the second stage variable, *y(ω)*, can be determined. *W* and *T* are called recourse and technology matrices respectively and if *W* is not random then the problem is named fixed recourse. When an optimization procedure is repeated many times with a same probability distribution of the data, then according to the law of large numbers this can give an optimal decision on average (Shapiro, 2008:183-220).

3. Two-Stage Stochastic Model (Yang's Model) for Hub-Median Network

Usually, studies in hub location problems consider three assumptions: 1) using a discount factor (*α*) in inter-hub connections for economies of scale and it is a number in unit interval. 2) The hub network is complete and all of the hubs are connected to each other. 3) Direct link between two non-hub nodes is not allowed. In this paper we use a discount factor (*β*) for hub and non-hub node connections and a discount factor (*α*) for inter hub ones. Also direct service is allowed here. In other words, two nodes i and j can be connected directly $(i \rightarrow j)$ or through one or more hub $(i \rightarrow k \rightarrow t \rightarrow j)$. In the first stage of model, x_k is the decision variable and it represents hub locations. The notation and variables used in the model are defined as follows:

It is the setup fixed cost of hub *k*.	f_k
It denotes the mathematical expectation operation.	E (.)
It represents a scenario (realization).	ω
It is a random vector that related to *ω* and consisting the uncertain parameters.	$\xi(\omega)$
It is the optimal value of Stage 2 according to all scenarios.	$Q(x,\xi(\omega))$
It denotes the demand level (flow) of every pairs for a selected scenario.	$D(\omega)$
It represents the values of probability of every scenario.	$p(\omega)$
It is a unit transportation costs for non-stop (direct) routes between *i* and *j*.	c_{ij}
It is the variable cost of hub connected links is denoted as that is equal to $\beta c_{ik} + \alpha c_{kt} + \beta c_{tj}$.	$c_{iktj}(\omega)$
It is a large positive number.	*V*
It is the first stage decision variable which represents the location of hubs. It is equal one if the node k is a hub and otherwise is zero.	x_k

It represents the decision variables of second stage. It is equal one if the two nodes i and j are connected directly.	$y_{ij}(\omega)$
It represents the decision variables of second stage for hub connected services. It is equal one if the two nodes i and j are connected by using hub nodes k or t.	$y_{iktj}(\omega)$

In the second stage, the objective function is to minimize the variable cost for all scenarios. The model is:

Stage 1:

$$\text{Min}_x \quad \sum\nolimits_{k\in N} f_k x_k + \text{E}[Q(x,\xi(\omega))] \tag{7}$$

s.t.

$$x_k \in \{0,1\} \tag{8}$$

Stage 2:

$$Q(x,\xi(\omega)) = \underset{y,x}{\text{Min}} \sum_{i\in N}\sum_{j\in N} D_{ij}(\omega) c_{ij} y_{ij}(\omega) + \sum_{i\in N}\sum_{j\in N}\sum_{K\in N}\sum_{t\in N} D_{ij}(\omega) c_{iktj}(\omega) y_{iktj}(\omega) \tag{9}$$

s.t.

$$y_{ij}(\omega) + \sum_{k\in N}\sum_{t\in N} y_{iktj}(\omega) = 1 \quad ; \quad \forall i,j : i \neq j \tag{10}$$

$$\sum_{i\in N} y_{ik}(\omega) + \sum_{i\in N} y_{ki}(\omega) \leq V(1-x_k) \quad ; \quad \forall k : i \neq k \tag{11}$$

$$\sum_{t\in N} y_{kkti}(\omega) + \sum_{i\in N} y_{itkk}(\omega) \geq 2x_k \quad ; \quad \forall i,k : i \neq k \tag{12}$$

$$y_{kktt}(\omega) \geq x_k + x_t - 1 \quad ; \quad \forall k,t \tag{13}$$

$$\sum_{i\in N}\sum_{j\in N}\sum_{t\in N} (y_{iktj}(\omega) + y_{itkj}(\omega)) - \sum_{i\in N}\sum_{j\in N} y_{ikkj}(\omega) \leq V x_k \quad \forall k : i \neq j \tag{14}$$

$$\sum_{i\in N}\sum_{j\neq i\in N}\sum_{t\in N} D_{ij}(\omega)\left(y_{iktj}(\omega) + y_{itkj}(\omega)\right) - \sum_{i\in N}\sum_{j\neq i\in N} D_{ij}(\omega) y_{ikkj}(\omega) \leq U_k x_k \ \forall k,\omega \tag{15}$$

$$\sum_{i\in N}\sum_{j\neq i\in N}\sum_{t\in N} D_{ij}(\omega)\left(y_{iktj}(\omega) + y_{itkj}(\omega)\right) - \sum_{i\in N}\sum_{j\neq i\in N} D_{ij}(\omega) y_{ikkj}(\omega) \leq L_k x_k \ \forall k,\omega \tag{16}$$

$$V\left(\sum_{i\in N}\sum_{j\in N}\sum_{\substack{t\in N\\ \neq k}}\left(y_{iktj}(\omega)+y_{itkj}(\omega)\right)\right)-\sum_{i\in N}\sum_{j\in N}y_{ikkj}(\omega)\geq x_k \quad ; \quad \forall k,\omega : i\neq j\ , i\neq k\ ,j \tag{17}$$

$$0\leq y_{ij}(\omega)\leq 1 \quad ; \quad \forall i,j : i\neq j \tag{18}$$

$$0\leq y_{iktj}(\omega)\leq 1 \quad ; \quad \forall i,k,t,j \tag{19}$$

Constraint (10) ensures that all demands from each origin are transported to its destinations. On the other hand, nodes of network are serviced directly (y_{ij}) or through one or more hub (y_{iktj}). Constraint (11) ensures that if node k is a hub, non-stop flight should not be represented as y_{tk} or y_{ki}. Constraint (12) ensures that only non-stop and one-hub-stop services are allowed if origin or destination is a hub. Constraint (13) requires only non-stop service is allowed when both origin and destination is a hub. Constraint (14) requires no hub-connected services transshipping at node k are allowed, if nodek is not a hub. In constraints (15) and (16) U_k and L_k are lower and upper bounds of every nodes. Constraint (17) denotes that at least one hub connected flight must transship at node k, if k is assigned to ahub.

The other constraints indicate that decision variables are binary.

4. Extended Two-Stage Stochastic Model

In the previous sections, we employed a traditional two-stage stochastic model as proposed by Yang (Yang, 2010:187-213) and surveyed the model by considering demand variations. Now we develop the model and study the effects of fuel cost changes and demand variation simultaneously. The new model is:

Initial stage:

$$\min_{x} \sum_{k \in N} f_k x_k + \mathrm{E}(Q_1(x, \xi(\omega')))] \quad (20)$$

Stage 1:

$$Q_1(x, \xi(\omega')) = \min_{x} \quad \mathrm{E}[Q_2(x, \xi(\omega))] \quad (21)$$

s.t.

Constraint (8)

Stage 2:

$$Q_2(x, \xi(\omega)) = \min_{y,x} \sum_{i \in N} \sum_{j \in N} D_{ij}(\omega) c_{ij} y_{ij}(\omega) + \sum_{i \in N} \sum_{j \in N} \sum_{K \in N} \sum_{t \in N} D_{ij}(\omega) c_{iktj}(\omega) y_{iktj}(\omega) \quad (22)$$

s.t.

Constraints (10) -(19)

The difference between this model and the previous one is in an initial stage. The objective of this stage is to minimize the expected value of the optimal value of Stage 1. ω' represents random events related two scenarios, with and without fuel subsidy with probability $p(\omega')$. In Stage 1, ω represents scenarios related to the demand variation (i.e., high, medium and low). Contreras et al. proved that stochastic uncapacitated hub location problems with uncertain dependent transportation costs (changing the price of resources such as fuel) are equivalent to the associated expected value problem (Contreras, 2011:518-520).

5. Numerical Results

In the real world, some parameters of the problem like demand are not certain. In air passenger market, the demand variation is seasonal and the demand level increases in some months. In this paper, we employ the two-stage stochastic programming and determine hubs locations in Iran aviation. Often the cost of air flight is more expensive than ground transportation systems in Iran. Therefore determining optimal routes and using hubs in air network play an important role in decreasing operating costs and improving the productivity. Operating costs usually include salary of crews, catering services and fuel. According to the reported data of

Iran air companies, the demand level increase in summer and Norooz holidays in comparison to average of demands in a year.

At first, in order to validate, we modeled air network between Taiwan and China similar to (Yang, 2010:187-213). There was a good agreement between the results and data reported in (Yang, 2010:187-213). Data on inside air passenger market between the 20 cities in Iran that are important routes were used in this study for estimating hub locations. We note that some data are not exact because of some restrictions. The model is solved by using the data of Table 2. According to the historical data, we consider three scenarios high, medium and low for variation of demand level with associated probabilistic assigned by a subjective judgment. Indeed, the demand level represents the ratio of the amount of demand for each scenario to the average of the amount of annual demand. Mostly in scenario high, the demand variation is less than doubled and in scenario low, it is more than half of the average of the amount of annual demand. We consider the values 1.3, 1 and 0.8 as the different demand levels for scenarios. The probability distribution of scenarios is discrete with the values of 0.33, 0.5 and 0.17 for high, medium and low demand respectively (for more details see (Yang, 2010:187-213)). The discount factors are difficult to acquire. Yang introduced an indicator factor for estimating discount factors (Yang, 2010:187-213). It depends on operating cost, maximum available seats and average of load factor for a type of flight. The discount factors for network are estimated by this indicator factors. There are different solution methods for two-stage stochastic models such as Benders' decomposition, L-shaped and deterministic equivalent (DE) method (Birge & Louveaux, 1997:184). DE is simple and good solution method for condition which there is only three scenarios, high, medium and low (Birge & Louveaux, 1997:184). Thus, the formulation of DE model is used in the solution procedure of the stochastic model. This model is solved with commercial software GAMS, and the MIP module of OSL solver. All tests were executed by personal computer with CPU 2GHz, 2 GB of RAM. The locations of hubs are the same in the three scenarios. Hub nodes are Tehran, Isfahan, Mashhad, Shiraz and Yazd. But the allocation of nodes can be different for each scenario. Table 3 shows some of the proposed routes of the stochastic model for all scenarios.

Table 2 Data Settings for 20 Nodes in Designing the Iran Aviation Hub Network.

Number of nodes	20
Unit cost ($/seat km)	0.08
Demand level (high, medium, low)	1.3, 1, 0.8
Probability (high, medium, low)	0.33, 0.5, 0.17
Discount factor α (high, medium, low)	0.75, 0.79, 0.83
Discount factor β (high, medium, low)	0.83, 0.87, 0.92

Table 3 Some Air Network Routes for 20 Cities based on Results of the Stochastic Model.

Hub Stop Routes	Direct Routes
Isfahan-Shiraz-Kish	Tabriz-Kermanshah
Shiraz-Tehran-Sari	Tabriz- Rasht
Shiraz-Isfahan-Hamedan	Abadan-Ahvaz
Ahvaz-Tehran-Mashhad	Kerman-Birjand
Bushehr-Isfahan-Tehran-Rasht	Zahedan-Chabahar
Kerman-Yazd-Mashhad	Gorgan-Sari
Kish-Shiraz-Mashhad	Bushehr- Ahvaz

The Tehran-Kish path is one of the crowded connections in the air network and in a year, in which there are about one thousand flights between them. Let if the cost of a direct flight between two cities, Tehran and Kish, is about 89.44$ per seat for each flight. Then, it seems that the hub stop path, Kish-Shiraz-Tehran will be economic. The cost of this path is about 48.44$ per seat for each flight. It means that using hubs in the air network is more cost effective and it saves the money about 41000 dollars in a year for only this route. If we consider a fully connected network for these 20 cities then the total cost is about USD ($) 304048350. Whereas, by using hubs and considering demand variations, the total cost is about USD ($) 242976102. In the present air network, most of the routes are direct and even some of the flights are made without using the maximum capacity of the planes. Therefore, the airlines suffer heavy costs. In additional, in some cases, the flight is cancelled, resulting in customer's dissatisfaction in a long period. Figure 1 shows some of the paths between 20 cities in three conditions, namely direct, one-hub stop and two-hub stop connections.

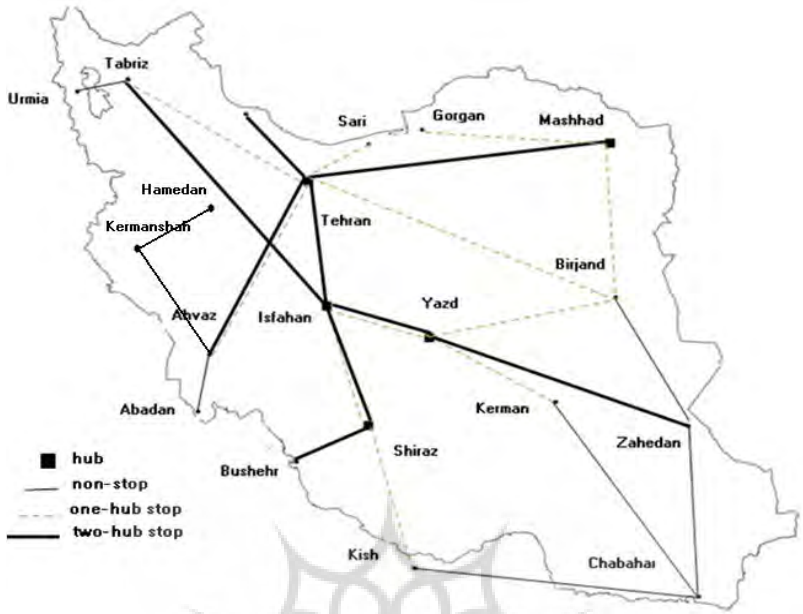

Fig 1 Some Routes based on Results of the Proposed Model in 3 Conditions.

Table 4: Numerical Results of Model for 20 Nodes in Iran Aviation Hub Network Design Case Study

	Stochastic Value	Deterministic Value	Expected Value
Total cost ($)	242976101.7	224153919.5	238480174.3
Hub location	Tehran, Mashhad, Isfahan, Shiraz, Yazd	Tehran, Mashhad, Isfahan, Shiraz, Yazd	Tehran, Mashhad, Isfahan Shiraz, Yazd
Non-stop (H, M, L)	(50, 54, 54)	54	54
Hub-stop (H, M, L)	(330, 326, 326)	326	326

According to the results of stochastic model, the proposed routes are different in three scenarios and the number of direct and hub stop routes are different in high, low and medium demand level. We explain it in next section in details. Now, we change the stochastic model to p-hub one. We solve the model with restriction in number of hubs (p). As Table 5 shows, for instance if we want to use only one hub in air network the model proposed Isfahan as a good candidate for it. So the total cost of this network is $269740135.

Table 5 Optimal Solutions of p-H ub Stochastic Model when there is only Demand Variation in 3 Scenarios.

p	Hubs	Total Cost
1	Isfahan	269740135
2	Tehran, Isfahan	249672369
4	Tehran, Mashhad, Isfahan, Shiraz	234732340
6	Tehran, Mashhad, Isfahan, Ahvaz, Shiraz, Yazd	229757561
10	Tehran, Mashhad, Isfahan, Ahvaz, Shiraz, Yazd, Birjand, Chabahar, Hamedan	227056221

In the real situation in addition to the demand level variation, transportation costs also can be uncertain because of changing the price of resource (fuel). At present, the government will decide to omit the subsidy of fuel in air transportation system. Here, we study the effects of changing the price of fuel. We considered two scenarios, with subsidy (i.e., Scenario1) and without subsidy (i.e., Scenario 2) and solved the p-hub two-stage stochastic model. Thus, we defined two scenarios with discrete probability function with value 0.4 and 0.6 respectively. Unit cost is different for 2 scenarios. In scenario 1, it is about 0.08 and in scenario 2 is 0.32. Table 6 shows the total cost and the hub locations by considering these scenarios.

Table 6. Optimal solution of problem when there is only cost variation in 2 scenarios.

p	Hubs	Total Cost
1	Isfahan	711254956
2	Tehran, Isfahan	661085128
3	Tehran, Isfahan, Shiraz	647103584

According to the data in Table 7 and the results of extended model, hub locations are Tehran, Mashhad, Shiraz, Isfahan and Yazd. The total cost is about 672583084.9. Allocations are the same in two scenarios, with and without fuel subsidy. Thus, any changes in the price of fuel can't affect the allocations in practice.

Table 7 Data Settings for 20 Nodes in Iran Aviation Hub Network Design Case Study

Number of Nodes	20
Unit cost ($/seat km) with and without subsidy	0.08, 0.32
Demand level (high, medium, low)	1.3, 1, 0.8
Probability with and without subsidy	0.4, 0.6
Probability (high, medium, low)	0.33, 0.5, 0.17
Discount factor α (high, medium, low)	0.75, 0.79, 0.83
Discount factor β (high, medium, low)	0.83, 0.87, 0.92

6. Sensitivity Analysis

In this section, we analyze stochastic model for different discount factors and fixed costs. For this, we employ the two-stage stochastic model for 7 nodes and survey effects of changes in α, β *and* f_k.

6.1 Fixed Cost and Unit Transportation Cost Analysis

The fixed cost (f_k) is the setup cost for hubs. In airline networks, fixed cost can be regarded something such as the rent of administrative office, the payment of crews and administrative staffs, passenger processing handling. As Figure 2 shows, the number of hubs increases in low fixed cost. Because of objective of problem that is to minimize the total transportation costs, when set up cost of a hub is cheap then the number of hubs increase. In fact amount of f_k play an important role in the number of hubs.

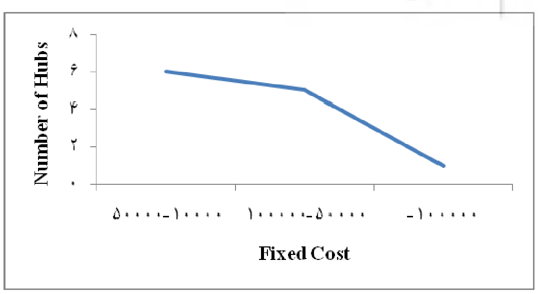

Fig 2 Effect of Amount of Fixed Cost on Number of Hubs

When the transportation unit cost increases, it can change in the number of hubs, such as the condition that the fixed cost decreases. The change in the number of hubs depends on the amount of the fixed cost; because, the fixed cost restrains the number of hubs in uncapacitated hub median location.

6.2 Discount Factors Analysis

Usually in most studies on hub and spoke networks, hub to hub arcs are discounted by a fixed discount factor (α). It is usually exogenously determined. By this simplification hub location models can be formulated as mixed-integer programming (MIP). Campbell showed that when α decreases, hubs tend to spread farther apart and the number of spokes decreases (Campbell, 1994:387-405). Therefore, a lower inter-hub rate factors allocation to nearest hub. Inversely, for large α hub interactions are expensive and hubs are down closer to gather to reduce inter-hub transportation costs. In classical models inter-hub discount factor is not dependent on the amount of flow. O'Kelly and Bryan showed that the assumption of flow-independent discount factor may effect on optimal hub location and allocation

(O'Kelly & Bryan, 1998:614-615). For more comprehensive review, the reader is referred to (Alumur&Kara, 2008:1-21; O'Kelly & Bryan, 1998:614-615; Horner & O'Kelly, 2001). Contreras et al. proved that stochastic uncapacitated hub location problems with uncertain demand are equivalent to the associated expected value problem (Contreras et al.,2011:518-528).

He considered flow-independent discounting factor for inter-hub connections. However, here there are two discount factors α and β for inter-hub and hub-non-hub connections respectively. Both of them are flow-dependent for closing to real condition. For analyzing discount factor we use the same fixed cost values for all nodes and different α and β for every scenario. With this realistic assumption, the expected value model and stochastic model are not equivalent (see Table 2). For the sensitivity analysis, we consider three cases. In case 1 α and β have smaller values at high demand and larger values at lower demand. Inversely, in case2 α and β have larger values at high demand and smaller values in low one and Main case is the evaluation criterion. Table 8 shows the effect of changes in discount factors on hub-stop and non-stop services in different scenarios for three cases.

Table 8 Discount Factors Analysis in 3 Cases.

	Case 1	**Main Case**	**Case 2**
	High, Medium, Low	High, Medium, Low	High, Medium, Low
Discount factor α	0.4, 0.6, 0.78	0.5, 0.6, 0.7	0.62, 0.6, 0.68
Discount factor β	0.5, 0.75, 0.975	0.625, 0.75, 0.875	0.775, 0.75, 0.85
Number of non-stop	12, 20, 24	16, 20, 22	20, 20, 22
Number of hub-stop	30, 22, 18	26, 22, 20	22, 22, 20
Hub node	1	1	1

There are two discount factors in this model; however, it seems that this model is similar to a classical one with a discount factor ($\frac{\alpha}{\beta}$). While the value of $\frac{\alpha}{\beta}$ is low, the number of hubs increase. Indeed, the value of $\frac{\alpha}{\beta}$ can play an important role in the number of hubs. Thus, the ratio of $\frac{\alpha}{\beta}$ should be the same for maintaining the level incentive for being a hub. It seems that using two discount factors for inter-hub and hub-non-hub connections maybe impact on the reliability of the presented model. As Table8 shown, hub node in all of three cases is node 1. In addition, we change the model with one discount factor which means β is omitted. The results of the sensitivity analysis show the number of hubs is same in 3 cases but the location of them are different. According to results of sensitivity analysis this model is more practical. Figures 3 and 4 depict the effects of the discount factor variation on the numbers of non-stop and hub-stop connections, respectively.

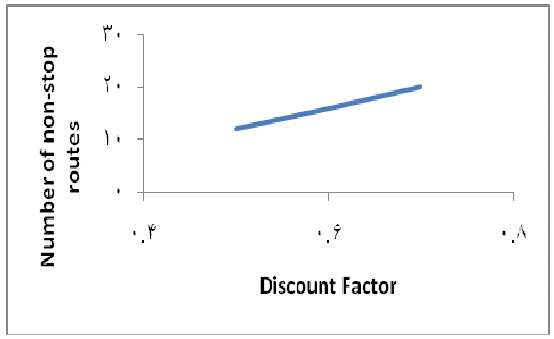

Fig 3 Effect of Discount Factor Variation on umber of Non-stop Connections.

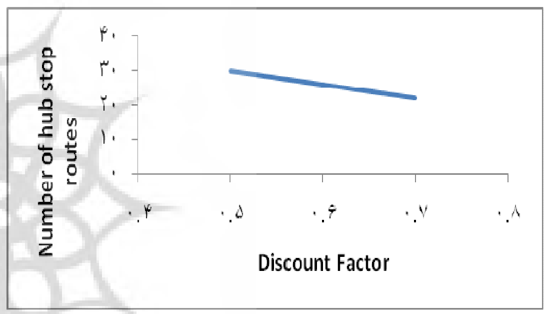

Fig 4 Effect of Discount Factor Variation on umber of Hub-stop Connections

7. Conclusion

In real-world situation, some parameters of the given problem are not certain. Most of studies about hub location in airline network are deterministic models. In the air passenger market, demand is uncertain because of seasonal variations. Then, deterministic models maybe cannot be efficient in determining optimized air routes. In this study, we extended traditional stochastic programming and use a practical stochastic model for estimating hub location in Iran air network. Now, this network is almost fully connected and it services demand points directly. Thus, changing the network structure to hub one could decrease operating costs and improve the productivity. We also change the model to p-hub stochastic model .The effect of variation of fuel price was also considered. We tested the model in different conditions and surveyed the effects of fixed cost and flow dependent discount factors. In fact, the discount factor should be flow-dependent. By considering this realistic assumption, the stochastic model with uncertain demand was not equivalent to the expected value one. Every country's aerial industry as a strategic industry has a vital role in its economic and social development. Today, only 2.5 percent of transportations provided in Iran are by air. One of the reasons of this low percentage is that the air transportation is more costly than the ground one. Consequently, determining the optimal routes and using hubs can have crucial roles in reducing the operating costs, hence causing an improvement in the industry. 26 percent of operational cost is related to fuel. The government intends to eliminate the fuel subsidy. This will cause a further increase in the costs. Thus, changing the present routes of air network is inevitable. Results of this present study indicated that the operating cost decreases by using hubs in air passenger transportation system this can augment customer satisfaction. On the other hand, hub stops can result in undesired time delay and lead to customer dissatisfaction. Investigation the customer satisfaction on hub and spoke networks is a complex problem which is a good idea for the future research.

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طراحی شبکه هاب میانه خطوط هوایی با استفاده از برنامه‌ریزی تصادفی دو مرحله ای توسعه یافته:مطالعه موردی

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تصمیم‌گیری درباره مکان هاب یک سرمایه‌گذاری بلندمدت محسوب می‌شود و هرگونه تغییر در آن زمان‌گیر و هزینه‌بر خواهد بود. در شرایط واقعی، برخی از پارامترها قطعی نبوده و مدل‌های قطعی نمی‌توانند زیاد کارا باشند. توانایی برنامه‌ریزی احتمالی دو مرحله‌ای در اتخاذ تصمیم بلندمدت با در نظر گرفتن همزمان اثرات آن بر روی تصمیمات کوتاه‌مدت است. در مسائل مکان‌یابی هاب با استفاده از برنامه‌ریزی احتمالی دو مرحله‌ای، مکان هاب در مرحله اول و چگونگی تخصیص بهینه جریان در مرحله دوم تعیین می‌گردد. در این مقاله، به تشریح برنامه‌ریزی احتمالی دو مرحله‌ای می‌پردازیم و یک مدل احتمالی عملی را جهت تعیین مکان هاب‌ها در هواپیمایی ایران به کار می‌گیریم. همچنین، با توسعه برنامه‌ریزی احتمالی دو مرحله‌ای، مدل را در شرایط حذف یارانه سوخت نیز مورد بررسی قرار می‌دهیم. در این مطالعه، میزان تقاضا و هزینه منابع (سوخت) غیر قطعی در نظر گرفته شده است. نتایج نشان می‌دهد که شهرهای تهران، مشهد، اصفهان، شیراز و یزد می‌تواند به عنوان هاب در شبکه هوایی ایران انتخاب شوند.

واژگان کلیدی: برنامه‌ریزی احتمالی دو مرحله‌ای، تقاضای غیر قطعی، شبکه‌های هاب، هواپیمایی ایران.

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