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Fuzzy Data Envelopment Analysis Approach for Ranking of Stocks with an Application to Tehran Stock Exchange

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ABSTRACT

The main goal of this paper is to propose a new approach for efficiency measurement and ranking of stocks. Data envelopment analysis (DEA) is one of the popular and applicable techniques that can be used to reach this goal. However, there are always concerns about negative data and uncertainty in financial markets. Since the classical DEA models cannot deal with negative and imprecise values, in this paper, possibilistic range directional measure (PRDM) model is proposed to measure the efficiencies of stocks in the presence of negative data and uncertainty with input/output parameters. Using the data from insurance industry, this model is also implemented for a real case study of Tehran stock exchange (TSE) in order to analyse the performance of the proposed method.

1 Introduction

Data envelopment analysis (DEA) is a non-parametric technique for performance assessment and ranking the homogeneous decision making units (DMUs). This methodology as proposed by Charnes et al. [2] is a linear programming (LP) technique in performance measurement that incorporates multiple inputs and outputs. Charnes et al. [2] extended the definition of efficiency and Farrell's [7] idea. DEA is a practical and applicable approach and it is widely employed in various fields and real-life problems such as financial markets including insurance, banking, and stock exchange (e.g. [6, 11, 13, 22]). One of the most important applications of DEA in financial markets is stocks evaluation and their rankings. It should be noted that in measuring the stocks efficiency using the DEA, two important points should be considered in financial markets: negative data and uncertainty.

Since classic DEA models cannot deal with negative values in efficiency measurement, these models exclude DMUs which has a negative input and/or output, so their efficiencies and rankings are unknown. For eliminating this problem, a researcher must utilize models which are usable in the presence of negative data, because presence of negative data is inevitable in real-world applications, especially in financial markets such as stock exchange. Therefore, different viewpoints and studies are introduced for dealing with negative data in DEA. In continuous, several researches that are presented in this filed which can handle negative values without translation of original data are presented.

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Portela et al. [20] proposed a range directional measure (RDM) model based on directional distance approach. Another model is the modified slacks-based measure (MSBM) which is proposed by Sharp et al. [23] in order to measure the efficiency of decision making units in the presence of negative values in inputs and outputs. Emrouznejad et al. [7] presented a semi-oriented radial measure (SORM) model by using of partitioning approach for dealing with negative values in DEA. Cheng et al. [3] proposed a variant of radial measure (VRM) models for performance assessment of DMUs where negative data are present. Matin et al. [17] proposed a modified semi-oriented radial measure model for improving some issues in target setting with SORM model.

After discussing about the importance of dealing with negative data, the necessity of considering the uncertainty will be discussed. In real cases, most of the times, the values of inputs and/or outputs of DEA models are tainted by uncertainty. The integration of DEA models and fuzzy set theory is one of the approaches that is regarded by researchers [12]. There are considerable researches about fuzzy data envelopment analysis (FDEA) models that are categorized by Hatami-Marbini et al. [9] and Emrouznejad et al. [4] expanding this classification into six groups, the new classification is presented in Fig. 1:

Fig. 1: The Classification of FDEA Field

As can be seen in Figure 1, the possibility approach is one of the popular methods for proposing the FDEA models. It should be noted that the fundamental principles of the possibility theory at first was introduced by Zadeh [24] and then extended by many researchers. In this approach, according to the attitude of decision makers (DMs), three measures of possibility, necessity, and credibility are presented in order to measure the chances of occurrence of fuzzy constraints and the transform of fuzzy chance constraints to their equivalent crisp ones in one special confidence level [19]. Guo et al.

[8] were the pioneer researchers in the possibility approach and their study was based on possibility and necessity measures. Lertworasirikul et al. [14] used the concept of chance constrained programming (CCP) that was proposed by Charnes and Cooper [1] and a possibility measure for transforming the FDEA model into a possibility linear programming problem. Lertworasirikul et al. [15] developed the fuzzy DEA model of the BCC model instead of CCR model, based on possibility and credibility approaches. Lertworasirikul et al. [16] presented a credibility approach as an alternative way for solving FDEA models. Hossainzadeh et al. [10] proposed a fuzzy DEA with fuzzy chance constraint multi objective programming method by credibility measure. Payan and Sharifi [18] introduced a method for measuring the fuzzy malmquist productivity index (MPI) by using of credibility theory that is implemented in social security organizations. Ruiz and Sirvent [21] extended a fuzzy cross-efficiency evaluation to FDEA based on the possibility approach presented by Lertworasirikul et al. [14]. Peykani et al. [19] presented robust possibilistic data envelopment analysis (RPDEA) models based on three measures of possibility, necessity and credibility. Since the application of this study is performance measurement of stocks, the presence of negative data and uncertainty in some of the input/output data is inevitable. Thus, the FDEA model is proposed for measuring the efficiency of stocks in the presence of negative and imprecise values. The FDEA model for dealing with negative data are proposed based on RDM models with envelopment and multiplier forms by applying the possibility approach. Also, FDEA model of this paper will be implemented in a real case study of Tehran stock exchange (TSE). It should be noted that the research's case study is Insurance industry of TSE. The rest of this paper is organized as follows. Preliminaries of this study, will be introduced in Section 2. Then, fuzzy data envelopment analysis modeling that is based on possibility approach will be presented in Section 3. The proposed FDEA models are implemented for a case study of TSE and the results will be evaluated in Section 4. Finally, the conclusions and some directions for future researches are given in Section 5.

2 Preliminaries

The modeling and formulations of range directional measure model in envelopment and multiplier forms in addition to some basic definitions of fuzzy concepts and possibility measure will be explained in the following sections.

2.1 Range Directional Measure

Portela et al. [20] proposed range directional measure model for dealing with negative data. RDM with a structure based on directional distance function that is capable of evaluating a DMU, in the presence of negative inputs and/or outputs. Suppose that there are n homogenous decision making units $\text{DMU}_j \left(j = 1,...,n \right)$ that convert m inputs $x_{ij} \left(i = 1,...,m \right)$ into s outputs $y_{rj} \left(r = 1,...,s \right)$ and where DMU_p is an under evaluation DMU. For all of the DMUs, it is assumed that there is an ideal point $\left(x_I , y_I \right)$ in the form of Equation (1):

$$Ideal\ Point\left(x_I\ ,\ y_I\right)=\left(\underset{j}{Min}\left\{x_{ij}\ ,\ i=1,...,m\right\}\ ,\ \underset{j}{Max}\left\{y_{rj}\ ,\ r=1,...,s\right\}\right) \tag{1}$$

This Ideal point can be one of DMUs, but this point usually is out of feasibility domain. R_{ip}^{-} and R_{rp}^{+} vectors are the range of possible improvements for the decision making unit under evaluation, they are defined as Equations (2) and (3), respectively:

$$R_{ip}^{-}=x_{ip}-\underset{j}{Min}\left\{x_{ij}\right\},\qquad i=1,...,m \tag{2}$$

$$R_{rp}^{+}=\underset{j}{Max}\left\{y_{rj}\right\}-y_{rp}\ ,\qquad r=1,...,s \tag{3}$$

Based on the notion of the range of possible improvements, Portela et al. [20] defined the range directional model as follows:

$$\begin{aligned} &\text{Max}\quad \beta_p \\ &\text{S.t.}\quad \sum_{j=1}^{n}\lambda_j\ x_{ij}\le x_{ip}-\beta_p\ R_{ip}^{-}\ ,\ \ \forall i \\ &\qquad\ \sum_{j=1}^{n}\lambda_j\ y_{rj}\ge y_{rp}+\beta_p\ R_{rp}^{+}\ ,\ \ \forall r \\ &\qquad\ \sum_{j=1}^{n}\lambda_j=1 \\ &\qquad\ \lambda_j\ge 0\ ,\ \ \forall j\ ,\qquad \beta_p\ge 0 \end{aligned} \tag{4}$$

Model (4) is the envelopment form of RDM model under variable return to scale (VRS) assumption. With respect to this notion that the range of possible improvement is always non-negative, RDM model can be used in the presence of negative values. In the RDM model, β_p is inefficiency value for an under evaluation DMU. By applying the $v_i\left(i=1,...,m\right)$ and $u_r\left(r=1,...,s\right)$ as dual variables for constraints of inputs and outputs, respectively, the dual form of RDM is as follows:

$$\begin{aligned} &\text{Min}\quad \sum_{i=1}^{m}v_i x_{ip}-\sum_{r=1}^{s}u_r y_{rp}+w_p \\ &\text{S.t.}\quad \sum_{i=1}^{m}v_i x_{ij}-\sum_{r=1}^{s}u_r y_{rj}+w_p\ge 0\ ,\ \ \forall j \\ &\qquad\ \sum_{i=1}^{m}v_i R_{ip}^{-}+\sum_{r=1}^{s}u_r R_{rp}^{+}\ge 1 \end{aligned} \tag{5}$$

$u_r, v_i \geq 0, \quad \forall r, i$

Model (5) is the multiplier form of RDM model under variable return to scale assumption.

2.2 Possibility Measure

Let the triple $(\Lambda, P(\Lambda), Pos)$ be a possibility space where a universe set Λ is a non-empty set, containing of all possible events and $P(\Lambda)$ is the power set of Λ. For each $F, G \in P(\Lambda)$, there are non-negative numbers, $\text{Pos}\{F\}$ and $\text{Pos}\{G\}$, the so-called possibility measure have the following properties:

- $\text{Pos}\{\varnothing\} = 0$
- $\text{Pos}\{\Lambda\} = 1$
- $If\ F \in P(\Lambda) \Rightarrow 0 \leq \text{Pos}\{F\} \leq 1$
- $\text{Pos}\{\bigcup_i F_i\} = Sup_i(\text{Pos}\{F_i\})$
- $If\ F, G \in P(\Lambda)\ and\ F \subseteq G \Rightarrow \text{Pos}\{F\} \leq \text{Pos}\{G\}$
- $If\ F, G \in P(\Lambda) \Rightarrow \text{Pos}\{F \bigcup G\} + \text{Pos}\{F \bigcap G\} \leq \text{Pos}\{F\} + \text{Pos}\{G\}$

Note that, the possibility measure is the optimistic measure. If the decision maker prefers a pessimistic viewpoint in order to an allude risk, can use the necessity measure instead of possibility measure [19].

3 The Possibilistic RDM Model

In this section, the fuzzy RDM model in envelopment and multiplier forms are proposed based on possibility approach. The envelopment RDM and multiplier RDM models considering uncertainty on inputs and outputs, are as Models (6) and (7), respectively:

$$\begin{aligned}
\text{Max}\quad & \beta_p \\
\text{S.t.}\quad & \sum_{j=1}^{n} \lambda_j\, \tilde{x}_{ij} \leq \tilde{x}_{ip} - \beta_p\, \tilde{R}_{ip}^{-}, \quad \forall i \\
& \sum_{j=1}^{n} \lambda_j\, \tilde{y}_{rj} \geq \tilde{y}_{rp} + \beta_p\, \tilde{R}_{rp}^{+}, \quad \forall r \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0, \quad \forall j, \qquad \beta_p \geq 0
\end{aligned} \tag{6}$$

$$\text{Min} \quad \sum_{i=1}^{m} v_i \tilde{x}_{ip} - \sum_{r=1}^{s} u_r \tilde{y}_{rp} + w_p \tag{7}$$

$$\text{S.t.} \quad \sum_{i=1}^{m} v_i \tilde{x}_{ij} - \sum_{r=1}^{s} u_r \tilde{y}_{rj} + w_p \geq 0, \quad \forall j$$

$$\sum_{i=1}^{m} v_i \tilde{R}_{ip}^{-} + \sum_{r=1}^{s} u_r \tilde{R}_{rp}^{+} \geq 1$$

$$u_r, v_i \geq 0, \quad \forall r, i$$

Note that the inputs and outputs have a trapezoidal distribution $\tilde{x}(x^1, x^2, x^3, x^4)$ and $\tilde{y}(y^1, y^2, y^3, y^4)$ with condition of $x^1 < x^2 < x^3 < x^4$ and $y^1 < y^2 < y^3 < y^4$, respectively.

In order to familiar with possibilistic programming, suppose that $\tilde{\xi}$ and Φ are fuzzy and crisp numbers, respectively. Additionally $\tilde{\xi}$ is a fuzzy number with trapezoidal distribution that is determined by $\tilde{\xi}(\xi^1, \xi^2, \xi^3, \xi^4)$ with $\xi^1 < \xi^2 < \xi^3 < \xi^4$. According to possibility measure, converting of fuzzy chance constraints $\{\tilde{\xi} \leq \Phi\}$ and $\{\tilde{\xi} \geq \Phi\}$ into their equivalent crisp ones in one special confidence level (α) is equal to Equations (8) and (9), respectively:

$$\text{Pos}\{\tilde{\xi} \leq \Phi\} \geq \alpha \Leftrightarrow (1-\alpha)\,\xi^1 + \alpha\,\xi^2 \leq \Phi \tag{8}$$

$$\text{Pos}\{\tilde{\xi} \geq \Phi\} \geq \alpha \Leftrightarrow \alpha\,\xi^3 + (1-\alpha)\,\xi^4 \geq \Phi \tag{9}$$

Now, in order to dealing with uncertainties in fuzzy chance constraints in fuzzy RDM models and converting them to their equivalent crisp ones, possibility measure is used. According to measures of possibility, envelopment and multiplier forms of FDEA model are defined as follows:

$$\text{Max} \quad \beta_p \tag{10}$$

$$\text{S.t.} \quad \text{Pos}\left\{ \sum_{j=1}^{n} \lambda_j \, \tilde{x}_{ij} \leq \tilde{x}_{ip} - \beta_p \, \tilde{R}_{ip}^{-} \right\} \geq \alpha_i, \quad \forall i$$

$$\text{Pos}\left\{ \sum_{j=1}^{n} \lambda_j \, \tilde{y}_{rj} \geq \tilde{y}_{rp} + \beta_p \, \tilde{R}_{rp}^{+} \right\} \geq \alpha_{m+r}, \quad \forall r$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0, \ \forall j, \quad \beta_p \geq 0$$

$$\text{Min} \quad \Psi \tag{11}$$

$$\text{S.t.} \quad \text{Pos}\left\{\sum_{i=1}^{m} v_i \tilde{x}_{ip} - \sum_{r=1}^{s} u_r \tilde{y}_{rp} + w_p \leq \Psi\right\} \geq \alpha_0$$

$$\text{Pos}\left\{\sum_{i=1}^{m} v_i \tilde{x}_{ij} - \sum_{r=1}^{s} u_r \tilde{y}_{rj} + w_p \geq 0\right\} \geq \alpha_j, \quad \forall j$$

$$\text{Pos}\left\{\sum_{i=1}^{m} v_i \tilde{R}_{ip}^{-} + \sum_{r=1}^{s} u_r \tilde{R}_{rp}^{+} \geq 1\right\} \geq \alpha_{n+1}$$

$$u_r, v_i \geq 0, \quad \forall r, i$$

It should be noted that α is confidence level for satisfying the objective function and constraints. Finally, by applying Equations (8) and (9), an equivalent crisp of fuzzy chance constraint according to one specific confidence level with using of possibility measure is as follows:

$$\text{Max} \quad \beta_p \tag{12}$$

$$\text{S.t.} \quad \sum_{j=1}^{n}\left(\left(1-\alpha_i\right)x_{ij}^{1} + \left(\alpha_i\right)x_{ij}^{2}\right)\lambda_j + \left(\left(1-\alpha_i\right)R_{ip}^{1-} + \left(\alpha_i\right)R_{ip}^{2-}\right)\beta_p \leq$$

$$\left(\left(\alpha_i\right)x_{ip}^{3} + \left(1-\alpha_i\right)x_{ip}^{4}\right), \quad \forall i$$

$$\sum_{j=1}^{n}\left(\left(\alpha_{m+r}\right)y_{rj}^{3} + \left(1-\alpha_{m+r}\right)y_{rj}^{4}\right)\lambda_j - \left(\left(1-\alpha_{m+r}\right)R_{rp}^{1+} + \left(\alpha_{m+r}\right)R_{rp}^{2+}\right)\beta_p \geq$$

$$\left(\left(1-\alpha_{m+r}\right)y_{rp}^{1} + \left(\alpha_{m+r}\right)y_{rp}^{2}\right), \quad \forall r$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0, \ \forall j, \quad \beta_p \geq 0$$

$$\text{Min} \quad \Psi \tag{13}$$

$$\text{S.t.} \quad \sum_{i=1}^{m}\left(\left(1-\alpha_0\right)x_{ip}^{1} + \left(\alpha_0\right)x_{ip}^{2}\right)v_i - \sum_{r=1}^{s}\left(\left(\alpha_0\right)y_{rp}^{3} + (1-\alpha_0)y_{rp}^{4}\right)u_r + w_p \leq \Psi$$

$$\sum_{i=1}^{m}\left(\left(\alpha_j\right)x_{ij}^{3} + \left(1-\alpha_j\right)x_{ij}^{4}\right)v_i - \sum_{r=1}^{s}\left(\left(1-\alpha_j\right)y_{rj}^{1} + (\alpha_j)y_{rj}^{2}\right)u_r + w_p \geq 0, \quad \forall j$$

$$\sum_{i=1}^{m}\left(\left(\alpha_{n+1}\right)R_{ip}^{3-}+\left(1-\alpha_{n+1}\right)R_{ip}^{4-}\right)v_i+\sum_{r=1}^{s}\left(\left(\alpha_{n+1}\right)R_{ip}^{3+}+(1-\alpha_{n+1})R_{ip}^{4+}\right)u_r\geq 1$$

$$u_r, v_i \geq 0,\quad \forall r, i$$

Models (12) and (13) are fuzzy RDMs in envelopment and multiplier forms, respectively and they are based on possibility approach.

4 A Case Study: Insurance Industry

Now, the implementation of possibilistic DEA model is presented for a real world case study from Tehran stock exchange. The Insurance industry of TSE is selected for this research and the goal of this section is measuring the efficiency of stocks in Insurance industry.

To achieve this goal, at first, inputs and outputs of DEA models are chosen for efficiency measurement of stocks from different and important viewpoints of valuation, liquidity, leverage, return and growth ratios. Inputs and outputs of DEA models are shown in Table 1:

Table 1: The Inputs and Outputs of DEA Models

	Financial Criteria	**Symbol**	**Description**
Inputs	Price to Earnings Ratio (P/E)	I (1)	Stock price divided by net income per share
	Quick Ratio	I (2)	Total current assets minus inventory divided by total current liabilities
	Solvency Ratio-II	I (3)	Total liability divided by shareholders equity
Outputs	Rate of Return (ROR)	O (1)	Net income minus dividends divided by common shares
	Liquidity	O (2)	Degree which presents stock ability to be bought or sold in the market quickly
	Earnings per Share (EPS) Growth Rate	O (3)	Current quarters EPS divided by the previous quarters EPS minus one

The Insurance industry involving 18 stocks is selected, financial data are extracted from March 2016 to March 2017 and based on expert opinion the triangular fuzzy distribution is considered. Summary of real-world data from Insurance industry of Tehran stock exchange that are used in this research are as shown in Table 2:

Table 2: Summary of Real-World Data from Insurance Industry of TSE

Stocks	Inputs			Outputs		
	I (1)	I (2)	I (3)	O (1)	O (2)	O (3)
Stock 01	(5.026 , 1.005)	(0.998 , 0.200)	(4.311 , 0.862)	(-7.187 , 1.437)	(55.866 , 11.173)	(41.776 , 8.355)
Stock 02	(9.783 , 1.957)	(1.178 , 0.236)	(1.728 , 0.346)	(61.841 , 12.368)	(65.359 , 13.072)	(0.508 , 0.102)
Stock 03	(10.158 , 2.032)	(0.974 , 0.195)	(3.418 , 0.684)	(27.539 , 5.508)	(434.783 , 86.957)	(5.303 , 1.061)
Stock 04	(5.324 , 1.065)	(0.975 , 0.195)	(4.307 , 0.861)	(8.546 , 1.709)	(50.505 , 10.101)	(-2.933 , 0.587)
Stock 05	(7.573 , 1.515)	(0.809 , 0.162)	(3.014 , 0.603)	(59.776 , 11.955)	(20.040 , 4.008)	(11.163 , 2.233)
Stock 06	(5.005 , 1.001)	(1.537 , 0.307)	(5.049 , 1.010)	(-4.825 , 0.965)	(64.516 , 12.903)	(27.215 , 5.443)
Stock 07	(7.015 , 1.403)	(1.183 , 0.237)	(2.616 , 0.523)	(15.884 , 3.177)	(22.936 , 4.587)	(11.815 , 2.363)
Stock 08	(7.126 , 1.425)	(1.040 , 0.208)	(5.291 , 1.058)	(-12.418 , 2.484)	(20.325 , 4.065)	(19.515 , 3.903)
Stock 09	(10.503 , 2.101)	(0.724 , 0.145)	(9.234 , 1.847)	(22.868 , 4.574)	(526.316 , 105.263)	(37.168 , 7.434)
Stock 10	(7.016 , 1.403)	(0.810 , 0.162)	(7.739 , 1.548)	(-5.280 , 1.056)	(30.769 , 6.154)	(-7.394 , 1.479)
Stock 11	(6.468 , 1.294)	(2.507 , 0.501)	(0.586 , 0.117)	(5.597 , 1.119)	(52.632 , 10.526)	(-19.763 , 3.953)
Stock 12	(7.564 , 1.513)	(2.510 , 0.502)	(0.399 , 0.080)	(0.723 , 0.145)	(33.670 , 6.734)	(4.975 , 0.995)
Stock 13	(7.615 , 1.523)	(1.189 , 0.238)	(2.193 , 0.439)	(-29.451 , 5.890)	(22.124 , 4.425)	(3.747 , 0.749)
Stock 14	(9.022 , 1.804)	(0.783 , 0.157)	(4.734 , 0.947)	(0.808 , 0.162)	(33.784 , 6.757)	(2.612 , 0.522)
Stock 15	(7.902 , 1.580)	(1.674 , 0.335)	(2.275 , 0.455)	(-12.123 , 2.425)	(23.041 , 4.608)	(-2.107 , 0.421)
Stock 16	(8.951 , 1.790)	(0.703 , 0.141)	(5.507 , 1.101)	(77.234 , 15.447)	(26.882 , 5.376)	(-37.283 , 7.457)
Stock 17	(7.653 , 1.531)	(1.568 , 0.314)	(1.128 , 0.226)	(11.789 , 2.358)	(20.790 , 4.158)	(23.928 , 4.786)
Stock 18	(8.632 , 1.726)	(0.842 , 0.168)	(4.623 , 0.925)	(-5.233 , 1.047)	(30.395 , 6.079)	(-40.810 , 8.162)
Max	(10.503 , 2.101)	(2.510 , 0.502)	(9.234 , 1.847)	(77.234 , 15.447)	(526.316 , 105.263)	(41.776 , 8.355)
Min	(5.005 , 1.001)	(0.703 , 0.141)	(0.399 , 0.080)	(-29.451 , 5.890)	(20.040 , 4.008)	(-40.810 , 8.162)

It should be noted that the presented data in Table 2 are symmetrical triangular membership function that the first and second values in parentheses are central and perturbation of this fuzzy number. Now, after collecting data, all of the FRDM models that are proposed in this study will be run for different confidence levels α. The results of PRDM-E and PRDM-M models is presented in Tables 3 and 4, respectively:

Table 3: The Results of Possibilistic RDM Model in Envelopment Form (PRDM-E)

Stocks	Confidence Levels (α)				
	$\alpha = 0$	**$\alpha = 0.25$**	**$\alpha = 0.50$**	**$\alpha = 0.75$**	**$\alpha = 1$**
Stock 01	0.31903	0.57406	0.76673	0.90799	1
Stock 02	0.63792	0.76751	0.86496	0.94035	1
Stock 03	0.45407	0.65290	0.80029	0.91257	1
Stock 04	0.32807	0.51967	0.76349	0.90749	1
Stock 05	0.49539	0.67723	0.81421	0.91862	1
Stock 06	0.28015	0.53363	0.76902	0.90926	1
Stock 07	0.48090	0.59226	0.68764	0.83028	0.97954
Stock 08	0.24929	0.38519	0.51413	0.63311	0.75659
Stock 09	0.02743	0.30080	0.65111	0.86054	1
Stock 10	0.15604	0.26976	0.47975	0.64461	1
Stock 11	0.83942	0.89771	0.94120	0.97434	1
Stock 12	0.90717	0.94030	0.96540	0.98479	1
Stock 13	0.41027	0.50451	0.60969	0.82643	0.98462
Stock 14	0.32233	0.42259	0.51536	0.65895	0.92000
Stock 15	0.41157	0.48782	0.57136	0.69102	0.84136
Stock 16	0.04583	0.25051	0.60664	0.83849	1
Stock 17	0.73335	0.82833	0.90039	0.95617	1
Stock 18	0.14913	0.24267	0.38679	0.60279	0.82872

Table 4: The Results of Possibilistic RDM Model in Multiplier Form (PRDM-M)

Stocks	Confidence Levels (α)				
	$\alpha = 0$	**$\alpha = 0.25$**	**$\alpha = 0.50$**	**$\alpha = 0.75$**	**$\alpha = 1$**
Stock 01	1	1	1	1	1
Stock 02	1	1	1	1	1
Stock 03	1	1	1	1	1
Stock 04	1	1	1	1	1
Stock 05	1	1	1	1	1
Stock 06	1	1	1	1	1
Stock 07	1	1	1	1	0.97954
Stock 08	1	1	1	0.92416	0.75659
Stock 09	1	1	1	1	1
Stock 10	1	1	1	1	1
Stock 11	1	1	1	1	1
Stock 12	1	1	1	1	1
Stock 13	1	1	1	1	0.98462
Stock 14	1	1	1	1	0.92000
Stock 15	1	1	1	0.95818	0.84136
Stock 16	1	1	1	1	1
Stock 17	1	1	1	1	1
Stock 18	1	1	1	1	0.82872

According to Table 3, the results of fuzzy envelopment RDM models indicate that as the confidence level α increases from 0 to 1 for satisfying the objective function and constraints, the objective function is also increased. Furthermore, with respect to Table 4, the results of fuzzy multiplier RDM models show that by increasing the confidence level α from 0 to 1 for satisfying the objective function and constraints, the objective function is decreased. Finally, for ranking all stocks, the average efficiency of stocks based on envelopment and multiplier forms of fuzzy RDM model under 5 confidence levels are calculated. The ranking of all stocks based on possibilistic RDM model are given in Fig. 2:

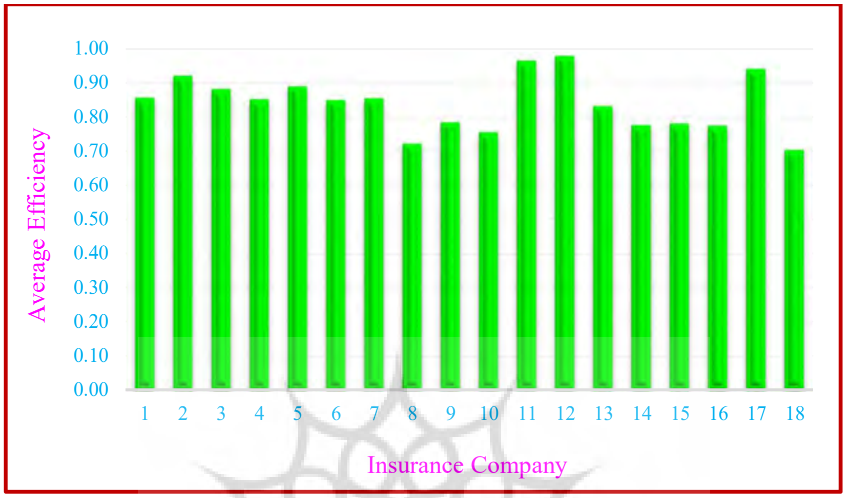

Fig. 2: The Average Efficiency for Stocks

According to the average of all results that are presented in Figure (2), Stock 12, Stock 11 and Stock 17 are the best stocks, respectively. As a result, stocks that are ranked superior can be qualified to be a candidate for portfolio and investing.

5 Conclusions

In this study, the fuzzy DEA approach for performance assessment and ranking of stocks in the presence of negative, imprecise and vague data was proposed. To reach this goal, the RDM model that was presented by Portela et al. [20], is used as the basic DEA model. Then, fuzzy RDM modeling in envelopment and multiplier forms based on possibility approach were proposed. Additionally, for solving and showing validation of the fuzzy DEA models, the proposed FRDM models are implemented for a real case study of Insurance industry of Tehran stock exchange. For the future studies, the fuzzy DEA models could be proposed based on other DEA models to deal with negative values such as variant of the radial measure models presented by Cheng et al. [3].

References

[1] Charnes, A. and Cooper, W.W., *Chance-constrained programming*, Management Science, 1959, **6**(1), P. 73-79. Doi: 10.1287/mnsc.6.1.73.

[2] Charnes, A., Cooper, W.W. and Rhodes, E., *Measuring the efficiency of decision making units*, European Journal of Operational Research, 1978, **2**(6), P. 429-444. Doi: 10.1016/0377-2217(78)90138-8.

[3] Cheng, G., Zervopoulos, P. and Qian. Z., *A variant of radial measure capable of dealing with negative inputs and outputs in data envelopment analysis*, European Journal of Operational Research, 2013, **225**(1), P. 100-105. Doi: 10.1016/j.ejor.2012.09.031.

[4] Emrouznejad, A. and Tavana, M., *Performance measurement with fuzzy data envelopment analysis*. Berlin and Heidelberg: Springer, 2014. Doi: 10.1007/978-3-642-41372-8.

[5] Emrouznejad, A., Anouze, A.L. and Thanassoulis, E., *A semi-oriented radial measure for measuring the efficiency of decision making units with negative data, using DEA*, European Journal of Operational Research, 2010, **200**(1), P. 297-304. DOI: 10.1016/j.ejor.2009.01.001.

[6] Esfandiar, M., Saremi, M., Jahangiri Nia, H., *Assessment of the efficiency of banks accepted in Tehran Stock Exchange using the data envelopment analysis technique*, Advances in Mathematical Finance and Applications, 2018, **3**(2), P. 1-11. Doi: 10.22034/amfa.2018.540815.

[7] Farrell, M., *The measurement of Productive Efficiency*, Journal of the Royal Statistics Society, 1957, **120**(3), P. 253-281. Doi: 10.2307/2343100.

[8] Guo, P., Tanaka, H. and Inuiguchi, M., *Self-organizing fuzzy aggregation models to rank the objects with multiple attributes*, IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, 2000, **30**(5), P. 573-580. Doi: 10.1109/3468.867864.

[9] Hatami-Marbini, A., Emrouznejad, A. and Tavana, M., *A taxonomy and review of the fuzzy data envelopment analysis literature: two decades in the making*, European Journal of Operational Research, 2011, **214**(3), P. 457-472. Doi: 10.1016/j.ejor.2011.02.001.

[10] Hossainzadeh, F., Jahanshahloo, G.R., Kodabakhshi, M. and Moradi, F., *A fuzzy chance constraint multi objective programming method in data envelopment analysis*, African Journal of Business Management, 2011, **5**(33), P. 12873-12881. Doi: 10.5897/ajbm11.1875.

[11] Izadikhah, M., *Improving the banks shareholder long term values by using data envelopment analysis model*, Advances in Mathematical Finance and Applications, 2018, **3**(2), P. 27-41. Doi: 10.22034/amfa.2018 .540829.

[12] Izadikhah, M., and Khoshroo, A., *Energy management in crop production using a novel fuzzy data envelopment analysis model*, RAIRO-Operations Research, 2018, **52**(2), P. 595-617. Doi: 10.1051/ro/2017082.

[13] Jafari, M., and Mousavi, M., *Performance analysis and rating of insurance companies using DEA in Iran capital market*, Advances in Mathematical Finance and Applications, 2017, **2**(3), P. 41-50. Doi: 10.22034/amfa. 2017.533100.

[14] Lertworasirikul, S., Fang, S.C., Joines, J.A. and Nuttle, H.L., *Fuzzy data envelopment analysis (DEA): a possibility approach*, Fuzzy sets and Systems, 2003, **139**(2), P. 379-394. Doi: 10.1016/S0165-0114(02)00484-0.

[15] Lertworasirikul, S., Fang, S.C., Nuttle, H.L. and Joines, J.A., *Fuzzy BCC model for data envelopment analysis*, Fuzzy Optimization and Decision Making, 2003, **2**(4), P. 337-358. Doi: 10.1023/b: fodm.0000003953 .39947. b4.

[16] Lertworasirikul, S., Fang, SC., Joines, J.A. and Nuttle, H.L., *Fuzzy data envelopment analysis: A credibility approach*, Fuzzy Sets based Heuristics for Optimization, Springer, Berlin, Heidelberg, 2003, P. 141-158. Doi: 10.1007/978-3-540-36461-0_10.

[17] Matin, R.K., Amin, G.R. and Emrouznejad, A., *A modified semi-oriented radial measure for target setting with negative data*, Measurement, 2014, **54**, P. 152-158. Doi: 10.1016/j.measurement.2014.04.018.

[18] Payan, A. and Shariff, M., *Scrutiny Malmquist productivity index on fuzzy data by credibility theory with an application to social security organizations*, Journal of Uncertain Systems, 2013, **7**(1), P. 36-49.

[19] Peykani, P., Mohammadi, E., Pishvaee, M.S., Rostamy-Malkhalifeh, M. and Jabbarzadeh, A., *A novel fuzzy data envelopment analysis based on robust possibilistic programming: possibility, necessity and credibility-based approaches*, RAIRO-Operations Research, 2018, **52**(4), P. 1445-1463. Doi: 10.1051/ro/2018019.

[20] Portela, M.S., Thanassoulis, E. and Simpson, G., *Negative data in DEA: A directional distance approach applied to bank branches*, Journal of the Operational Research Society, 2004, **55**(10), P. 1111-1121. DOI: 10.1057/palgrave.jors.2601768.

[21] Ruiz, J.L. and Sirvent, I., *Fuzzy cross-efficiency evaluation: a possibility approach*, Fuzzy Optimization and Decision Making, 2017, **16**(1), P. 111-126. Doi: 10.1007/s10700-016-9240-1.

[22] Saberi, M., Rostami, M. R., Hamidian, M., and Aghami, N., *Forecasting the profitability in the firms listed in Tehran Stock Exchange using data envelopment analysis and artificial neural network*, Advances in Mathematical Finance and Applications, 2016, **1**(2), P. 95-104. Doi: 10.22034/amfa.2016.527823.

[23] Sharp, J.A., Meng, W. and Liu, W., *A modified slacks-based measure model for data envelopment analysis with 'natural' negative outputs and inputs*, Journal of the Operational Research Society, 2007, **58**(12), P. 1672-1677. Doi: 10.1057/palgrave.jors.2602318.

[24] Zadeh, L.A., *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets and Systems, 1978, P. 3-28. Doi: 10.1016/0165-0114(78)90029-5.