Suhrawardi on Syllogisms

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Abstract

Shihāb al- Dīn al-Suhrawardī (d. 1194), in his most important book, *Hikmat al-Ishrāq*, claims that he has simplified the Aristotelian theory of syllogisms by reducing its many rules to a few by which the validity of all moods can be proven. This is done by reducing all negative and particular categorical propositions to universal affirmative propositions and introducing two meta-language rules, one for the second and the other for the third figure. This is an exposition of the non-modal part of his syllogism and an examination of his claim to simplifying the Aristotelian theory of syllogism.

key terms: Shihāb al- Dīn al-Suhrawardī, *Hikmat al-Ishrāq*, Aristotelian logic, theory of non-modal syllogism, conversion.

Introduction

A syllogism is a sequence of three propositions all in subject-predicate form. The first two propositions are called the premises and the third one the conclusion. Each of the subjects and predicates is called a *term* and each syllogism has three terms. The term common to the both premises is called the *middle term*. The term that is the subject of the conclusion is *the minor term* and the term that is the predicate of the conclusion is *the major term*. The premise containing the major term is called the major premise and the one containing the minor term the minor premise. In the traditional Islamic texts on logic, the minor premise comes before the major premise, which is the converse of their order in traditional European texts on logic.

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Syllogisms are classified according to the position of the middle term in the premises. Let us fix 'F' and 'H' as the subject and predicate of the conclusion, and 'G' as the middle term. This gives us the following possible permutations of the three terms. Each pattern of permutations is called a *figure*:

	1	2	3	4
Minor premise:	F, G	F, G	G, F	G,F
: Major premise:	G, H	H, G	G, H	H, G
Conclusion:	F, H	F, H	F, H	F, H

Given that each premise can be one of the four categorical propositions, i.e., universal affirmative, universal negative, particular affirmative, and particular negative proposition, each figure can have 16 patterns, not all of which are valid. Thus, we need some principles to distinguish the valid patterns (moods) from the invalid ones in each figure. There are different ways of doing that. The most important way which makes the theory of syllogism *looks like a deductive theory* is to give some rules, take one mood as axiom, and show the validity of certain other patterns by reducing them to that mood. This is the method the Peripatetic logicians used for the theory of the syllogism.

Suhrawardi, following Farabi and Ibn Sina, discards the fourth figure, as it is not intuitively plausible (4, p.34).

Suhrawardi's theory of the syllogism

Suhrawardī's main claim is that all propositions can be reduced to necessary universal affirmative propositions. So he writes:

> The mode of the relation of the predicate to the subject of a categorical proposition is either 'necessary', which is called 'wajeb' ['necessary'], or 'necessary not', which is called 'impossible', or 'not necessary' and 'not necessary not' which is called 'possible.' (4, p. 27, all translations from Corbin's edition are mine.)

Then by making modality a part of predicate and changing every existential proposition to universal one, he claims that the proposition if true becomes necessarily true with modality *De dicto.*

So his logic is fundamentally modal. But in this paper I am only interested in the non-modal part of his logic. I leave his modal claim, which is more complicated and controversial to another paper.

Suhrawardī's way of proving the validity of all moods of syllogisms is based on the following methods:

1. Reducing all negative categorical propositions to affirmatives by *obversion* of their predicates:

Thus,

Some A is not a B

becomes

Some A is a non-B.

Similarly,

No A is a B

becomes

Every A is a non-B.

2. Reducing all particular categorical propositions to universals by defining a new predicate like "D" for those individuals to which "some A" in "Some A is B" is referring (*ecthesis*).

Thus,

Some A is a B

changes to

Every D is a B.

So by the first principle,

Some A is not a B

becomes,

Every D is a non-B.

3. Two rules, one for the second and one for the third figure, as discussed below.

Now let us examine the three figures.

First figure

By applying the first and second method (obversion and ecthesis) to all moods of the first figure but Barbara, they are easily reduced to Barbara. For example, the syllogism:

Some animals are rational beings
No rational being is a stone
Some animals are not stones

is reduced to:

Every D is a rational being
Every rational being is a non-stone
Every D is a non-stone

The result is a Barbara syllogism, and the conclusion can easily be rewritten as:

Some animals are not stones

The other two moods in this figure are reduced in the same way to Barbara.

Now let us examine the other figures.

Second figure

According to the traditional texts, the valid moods of the second figure have the following four forms:

1-Every F is a G
No H is a G
No F is an H

2- No H is a G
Every F is a G
No H is an F

3- Some F is a G
No H is a G
Some F is not an H

4- Some H is not a G
Every F is a G
Some H is not an F

Now by using Suhrawardī's principles we get the following universal affirmative forms corresponding to each mood written above:

1'- Every F is a G
<u>Every H is a non-G</u>
Every F is a non-H

2'-Every H is a non-G
<u>Every F is a G</u>
Every H is a non-F

3'- Every D is a G
<u>Every H is a non-G</u>
Every D is a non-H

4'- Every D is a non-G
<u>Every F is a G</u>
Every D is a non-F

As it is clearly seen in this formalism, all these moods have only one form. We have two different subjects, to one of which a predicate is applied and to the other **the negated** or **obverse** of that predicate is applied. In order to prove the validity of this form, he introduces here a new principle at the meta-language level:

> If there are two [affirmative] universal (*mohitatan*) propositions with different subjects such that a predicate is applied to one of them, and it would be impossible to apply the same predicate to the other in all aspects or one aspect…then it is impossible to describe one of the subjects in terms of the other no matter which one of them would be taken as the subject or the predicate of the conclusion [of those two subjects] (4, P.36.)

By using this rule, the validity of the moods of this figure is secured, though not by reducing them to Barbara.

In the Aristotelian tradition, Suhrawardī's common form of this figure is written as

Every F is a G
Every H is a non-G
No F is an H

But to get this conclusion the second premise is converted to:

Every G is a non-H

Now from this and the first premise we get:

Every F is a non-H

This is a Barbara syllogism. Suhrawardī unwilling to use conversion applies his rule as follows:

> These two statements are two propositions that are impossible to predicate to the subject of one of them what is predicated to the subject of the other. Every two propositions that are impossible to predicate to the subject of one of them what is predicated to the subject of the other then their subjects are necessarily incompatible.
> Therefore the subjects of these two statements are necessarily incompatible. (4, P.37)

At the beginning of this meta-language Barbara syllogism he writes: "*Wa makhrajahoo men al-shikl al-awwal*" (3, p.39), meaning "And its derivation is from the first figure." He then writes his derivation mentioned above. The subtle point here is that Suhrawardi's rule is, in modern terms, a meta-language rule, while the conversion used by traditional logicians is a rule whose legitimacy is proven within the object language.

It should be mentioned in passing that J. Walbridge and H.Ziai's translation of this part (5, p.23) of *Hekmat al- Ishrāq* seems to be erroneous, as is the translation of the corresponding part in the third figure (5, p. 25).

It is worth mentioning that in this figure, like other figures, Suhrawardī extends his discussion to modal syllogisms as well. However, as I said before, I have set this part of his theory aside. He believes, as Ibn Sina did before him, that every proposition is modal whether its modality is enunciated or not. This is a position that even some contemporary modal logicians hold. But my purpose in this paper is to find out whether Suhravardī's claims concerning the non-modal part of his logic, the much simpler part, is tenable. This would prepare the ground for examining his more ambitious claim.

Third figure

Suhrawardī's treatment of the third figure is more interesting. He begins with two singular propositions:

> Zaid is a human being
> Zaid is an animal

Now he says from these two premises we can get:

> Some human beings are animals

and Some animals are human beings

Then he adds that if instead of a certain individual we have a term with a general meaning like "human being," we can generalize (of course, only in some cases) our premises to obtain:

> Every human being is an animal
> Every human being is a rational being

Here again, as in the case in the second figure, he introduces another meta-language principle:

> If a certain thing [here 'human being'] is described by two descriptions, then an individual [referred to by 'human being' here] described by one of them is to be described by the other.
> (4, pp.37-38)

He then concludes:

> Some animals are rational beings

The subtle part of his discussion on this figure is when one of the premises is a particular categorical one:

> Every human being is an animal
> Some human beings are writers

Here he says that since some human beings are included in every human being, it would be enough to select something described by both predicates.

The reader familiar with natural deduction rules realizes that Suhrawardi is implicitly using some rules like existential elimination, introduction rules, as well as the universal elimination rule. Also in his discussion of all moods, the non-emptiness of the subject terms is presupposed. To see how, the general pattern of his argument can be formulated as the following valid argument in modern predicate logic:

$$(x)(Fx\rightarrow Gx)$$
$$(x)(Fx\rightarrow Hx)$$
$$\underline{(\exists x)Fx}$$
$$(\exists x)(Gx\&Hx)$$

Again at the end of his discussion of this figure he formulates the application of his rule to the common pattern of this figure in the form of Barbara. He calls this "its derivation from the first figure:”

> These two statements are two propositions in which a certain thing is described by two predicates. In every two propositions in which a certain thing is described by two predicates, then something described <u>by one of the predicates</u> is described by the other.
> So these two statements are such that some of the individual described by one of the predicates is described by the other predicate. (4, p.39)

Suhrawardī here again uses a meta-language rule to prove the validity of the moods.

Of course any axiom, law or principle of any science can be used in or applied to a particular case and be formulated in the form of Barbara. Look at this argument:

The straight line L is passing through two points a and b
A straight line passing through two points is the shortest distance between them
Therefore, L is the shortest distance between a and b

Suhrawardī also formulates the application of each rule in the form of *modes ponens*.

Suhrawardī's discussion of the third syllogism involves some other new points too. So let us examine it more carefully.

Traditionally, the third figure has six moods as follows (respecting the Islamic logic texts, again I write the minor premise before the major and use modern symbolism):

1- $(x)(Gx\rightarrow Fx)$

$\underline{(x)(Gx \rightarrow Hx)}$
$(Ex)(Fx \& Hx)$

2-$(x)(Gx \rightarrow Fx)$
$\underline{(x)(Gx \rightarrow \neg Hx)}$
$(Ex)(Fx \& \neg Hx)$

3- $(Ex)(Gx \& Hx)$
$\underline{(x)(Gx \rightarrow Fx)}$
$(Ex)(Hx \& Fx)$

4- $(Ex)(Gx \& Hx)$
$\underline{(x)(Gx \rightarrow \neg Fx)}$
$(Ex)(Hx \& \neg Fx)$

5- $(x)(Gx \rightarrow Fx)$
$\underline{(Ex)(Gx \& Hx)}$
$(Ex)(Fx \& Hx)$

6- $(x)(Gx \rightarrow Fx)$
$\underline{(Ex)(Gx \& \neg Hx)}$
$(Ex)(Fx \& \neg Hx)$

Here, in mood (2), by making the negation a part of 'H' in the second premise (obversion), it is reduced to the first mood. Then, by applying Suhrawardī's second meta-language rule, the conclusion follows. As for the other four moods, it is enough to consider one of them. Let us consider mood (6).

By ecthesis and obversion, the second premise becomes:

$(x)(Dx \rightarrow \text{non-}Hx)$

Given that the objects referred to by 'Ex' of the second premise are included in '(x)' of the first premise we have:

$(x)(Dx \rightarrow Gx)$

From this and the first premise we have the following premises:

$(x)(Dx \rightarrow Fx)$
$(x)(Dx \rightarrow \text{non-}Hx)$

And again by applying the same rule we get:

$(Ex)(Fx \& \text{non-}Hx)$

And again by ecthesis it can be written as a universal affirmative proposition.

Two more moods

According to traditional logicians, from two negative propositions and two particulars, no valid syllogisms can be obtained. But Suhrawardī thinks differently. Let us see how.

Suppose we have:

No A is a B
No A is a C

By obversion he gets:

Every A is a non-B
Every A is a non-C

And by his rule:

Some non-B is a non-C

Of course this is a valid pattern but what he does not take into account is that he changes the minor and major terms of the premises. This is not a counter example to the traditional general rule for conversion. In that rule the major and minor remain and should remain intact but here 'A' and 'B' are changed to 'non-A' and 'non-B'.

Syllogism with particular premises

One of the general rules for a valid syllogism is that it should not have more than one particular premise. Here Suhrawardi writes:

> If some thing is described by one of the two predicates **or by both of them** and specified and made universal, then its case will be the same [as general form of the third figure.] (4, 38)

Here Suhrawardī seems to be saying that from two particular propositions such as:

Some G is a F
Some G is an H

One can derive,

Some F is an H

provided something can be **specified** having both F and H. But what is 'that something' supposed to be? Is it an individual like Zaid in his first example above? Or by ecthesis is it a predicate like 'D' with an extension containing those individuals referred to by 'Some G' in the two premises mentioned above? What about the following two premises:

Some numbers are evens
Some numbers are odds

There is no doubt that Suhrawardī's way of treating this pattern of syllogism as valid is a logical mistake. In fact Shahrazūrī in his commentary on *Hekmat al-Isrāq* does not comment on this part, and in a section preceding this pattern proves the invalidity of this syllogism through two examples (2, p.111). But Qutb-al-Dīn Shīrāzī, the other famous commentator of the book, seems to accept the validity of the pattern and gives as an example the following premises without giving the conclusion or commenting on it:

Some human beings are actually writers
Some human beings are actually laughers
(1, p.214)

It seems that the commentator, by adding 'actually' to each proposition, tries to make sure that there are actually some writers and some laughers and consequently some laughing writers! Of course if, as Suhrawardī says, one could find in such pattern an individual described by both predicates 'F' and 'H,' then one can conclude that 'Some F is a G.' However, by this kind of extra logical specification one can find true examples for any invalid syllogism. In fact this is one of the ways our traditional logicians use to prove the invalidities of some syllogisms.

Discussion

Let us compare the traditional logicians' treatment of syllogisms with that of Suhrawardī's. Traditional logicians, following Aristotle, take the moods of the first figure as self evident and reduce the moods of the second and the third figures by conversion, *ecthesis* and *reductio ad absurdum* to the moods of the first figure, mostly Barbara, to prove their validities.

Suhrawardī first reduces all premises of the syllogisms to universal affirmative categorical propositions by obversion and *ecthesis*. He also accepts Barbara as a self evident syllogism. As to the figures:

1. All moods of the first figure but Barbara are reduced to Barbara and their validities are secured by Barbara.
2. All the premises of the moods of the second figure will find, in modern symbolism, the following common pattern:

$$(x)(Fx \rightarrow Gx)$$
$$(x)(Hx \rightarrow \text{non-}Gx)$$

Then he applies a new rule at the meta-language level to get the conclusion:

$$(Ex)(Fx \& \text{non-}Hx)$$

In modern predicate logic, it is enough to add to the premises the non-emptiness of 'F' (or 'H') as:

$$(Ex)Fx \text{ (or } (Ex)Hx)$$

This shows the soundness of the Suhrawardī's rule. Of course, Suhrawardī presupposes the non-emptiness of the subject terms even for the negative categorical propositions.

Suhrawardī, at the end of his discussion of this figure, formulates the application of his rule to the common pattern of the figure, as I enunciated above, in the form of Barbara. Of course, as I explained, this is not the reduction of the common pattern to Barbara.

3. Suhrawardi's common form for the moods of the third figure is the following:

$$(x)(Gx \rightarrow Fx)$$
$$(x)(Gx \rightarrow Hx)$$

Here again he introduces another rule to get:

$$(Ex)(Fx \& Hx)$$

Again, if we add the existential proposition:

(Ex)Gx

to the premises, the legitimacy of the rule can be verified.

As I mentioned before, Suhrawardī's proof of this syllogism is very similar to quantifier rules of modern natural deduction.

Improvement on Suhrawardī's method

From what I have said at the end of the second figure above, it seems obvious that by applying a simple conversion to the single pattern of the second and the single pattern of the third figure, each pattern is reduced to Barbara. Thus, not only do we not need Suhrawardī's two new rules, but the theory becomes more elegant. Reducing all moods to Barbara in this way is more coherent and economical. I wonder why Suhrawardī did not choose this much simpler approach in doing syllogism.In my forthcoming paper on conversion I show that Suhrawardi had good reason to reject this rule.

Conclusion

From this exposition of Suhrawardī's treatment of the theory of syllogism, it seems that:

1. Suhrawardī's approach to the theory of syllogism and his method of proving validity of the moods are formally correct.
2. According to the textual evidence available to us, all materials used by Suhrawardī for forming his theory, including his meta-language rules, were known to Ibn Sina but scattered here and there in his writings and writings of his followers. Suhrawardī was the one who gathered them, combined bits and pieces together, and made a theory out of them. (Al-Qiyas, second part, 4th chapter).
3. Suhrawardī's version of syllogism can be more simplifieded by using simple conversion instead of adding two new rules to the traditional theory of syllogism.

It is worth mentioning that Suhrawardī calls his two rules *illuminationist rules*.

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