

**TESTING FOR STOCHASTIC NON- LINEARITY  
IN THE RATIONAL EXPECTATIONS  
PERMANENT INCOME HYPOTHESIS**

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**ABSTRACT**

The Rational Expectations Permanent Income Hypothesis implies that consumption follows a martingale. However, most empirical tests have rejected the hypothesis. Those empirical tests are based on linear models. If the data generating process is non- linear, conventional tests may not assess some of the randomness properly. As a result, inference based on conventional tests of linear models can be misleading.

This paper tests for the presence of stochastic non- linearity in aggregate consumption of non- durable goods and services, using US and Canadian data. The two major tests applied are a test devised by Brock, 2 Dechert, and Scheinkman, and a test based on an Artificial Neural Network model. The results support the hypothesis that there is no non- linearity in the data. The forecast results, however, suggest that even though linearity is not rejected, the non- linear ANN model tends to outperform the linear ARIMA model over three different horizons.

Key words: Non- linear dynamic, Rational Expectations Permanent Income Hypothesis, BDS test, ANN test.

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## 1. INTRODUCTION

The Rational Expectations Permanent Income Hypothesis (REPIH) has been a subject of much research for the last two decades. Hall (1978) applied the Rational Expectations assumption to the Permanent Income Hypothesis and showed that under certain assumptions, consumption should follow a martingale or a random walk without drift. That is, the change in consumption should not be predictable. Hall (1978) derived the Euler equation assuming a linear marginal utility of consumption and a constant real interest rate, then tested its implied exclusion restrictions. Using post war aggregate data for consumer expenditures on non-durable goods and services in the United States, he found that no past variable other than stock prices was statistically significant in explaining the change in consumption. Some extensions of the Euler equation approach have also been proposed and implemented for testing the REPIH (for example, Ogaki, 1992).

There have been many subsequent tests in the literature evaluating the martingale hypothesis. For example, Flavin (1981) and Hahashi (1982) found that change in consumption is statistically significantly correlated with the lagged consumption and income. Campbell and Mankiw (1989) also found that income has a statistically significant effect on the change in consumption. As cited in Nelson (1987) and Speight (1990), most tests for REPIH have rejected the martingale process for the aggregate consumption series.

Researchers have provided two main explanations for the rejection of the martingale hypothesis for the aggregate consumption. The first is that some households are liquidity-constrained (Hall and Mishkin, 1982 and Zeldes, 1980). Consumption for liquidity-constrained households will depend on current income rather than permanent income, and would thus reject REPIH. Some households might behave as though liquidity constrained because of a combination of high discount rate and strong precautionary motive for saving. The argument is that some households follow a set of rule of thumb for their consumption behavior (Shefrin and Thaler, 1988), or because of a combination of high discount rate and precautionary saving households do not go heavily into debt (Deaton, 1991). The second reason advanced is that households may decide on consumption spending by rules of thumb, and let saving flows do the adjusting instead (Deaton 1991, Shefrin and Thaler, 1988); in such cases the random walk behaviour would be found mainly in saving rather than consumption behaviour.

The other set of analysis which are in favor of the REPIH deals with the way the aggregate consumption data is constructed. For instance, Christiano et al (1991) show that if households make their consumption decision at intervals of time finer than the data-sampling interval (normally quarterly) the empirical tests for the REPIH would be biased rejecting REPIH. Wilcok (1992) also argues that the sampling error which exists in the retail sales and personal consumption expenditure data distorts the autocorrelation of the observed series at lag one and higher. This implies that the tests of martingale hypothesis, using the sampling- error contaminated data will probably be biased rejecting the null of no autocorrelation in the first differences.

One reason for rejecting the REPIH could be that most of the tests have been carried out using linear models. If the data generating process for consumption behaviour is non- linear, these tests will be insufficient. Developments in the study of chaotic time series reveal that conventional tests of randomness may not be able to pick up some types of regularity in the series. Therefore, a test for randomness of the residuals obtained from a linear regression may classify such randomness wrongly as evidence of linear relationship. It is therefore useful to make sure that the data generating process is linear. If the data is found to be linear, then traditional time series models are appropriate. If non- linearity is present, then models with more flexible functional forms such as neural network models should be used (KuanandWhite, 1994).

While aggregate consumption behaviour has been analysed by many researchers, as far as we know this is the first study to test for stochastic non- linearity. The organisation of the paper is as follows. Section 2 briefly restates the Rational Expectations Permanent Income Hypothesis. Section 3 explains the theoretical background and procedures for the two tests of non- linearity. Section 4 presents the results of both tests. Section 5 compares the results of forecasting exercises with both linear and non- linear models, and Section 6 presents the conclusions.

## 2. THE RATIONAL EXPECTATIONS PERMANENT INCOME HYPOTHESIS

A consumer seeks to maximize the expected life- time utility function

$$E_t \left[ \sum_{t=0}^{T-1} (1+\theta)^{-t} U(C_t) \mid I_t \right]$$

Subject to the set of intertemporal budget constraints

$$A_{t+1} = (A_t + Y_t - C_t)(1+r), \quad Y_t \in I_t, A_t > 0.$$

Where  $E_t$  is the mathematical expectation conditional on all information in time  $t(I_t)$ ,  $\theta$  is the constant rate of subjective time preference,  $r$  is the constant real interest rate,  $U(C)$  is a one-period strictly concave utility function,  $C_t$  is consumption in time  $t$ ,  $Y_t$  is non-asset income, and  $A_t$  is non-human assets. Solving this inter-temporal optimization

Problem, we get the Euler equation as follows<sup>2</sup>.

$$E_t U'(C_{t+1}) = \gamma U'(C_t) \quad (1)$$

where  $\gamma = (1+\theta)/(1+r)$ . This is the standard equivalency between marginal rate of substitution and the relative prices of consumption this period and next. Introducing a disturbance term transforms equation (1) into the form of a regression equation:

$$U'(C_{t+1}) = \gamma U'(C_t) + \varepsilon_t \quad (2)$$

where  $\varepsilon_t$  is a random variable with the standard properties  $E_t(\varepsilon_t) = 0$  and  $\text{cov}[U'(C_t), \varepsilon_t] = 0$  Hall (1978) assumed a logarithmic utility function and so transformed the Euler equation into the linear equation

$$C_{t+1} = (1/\gamma) C_t + \varepsilon_t \quad (3)$$

If the real interest rate equals the rate of time preference, the coefficient is 1 and the auto regression in equation (3) is a random walk without drift, or a martingale.

The exclusion restriction implied by the Euler equation (3) is that no past variable other than lagged consumption should have a statistically

<sup>2</sup> For complete derivation of the Euler equation see Hall (1978), Blanchard and Fischer (1989) and Wirjanto (1991).

significant non- zero coefficient in a regression of current consumption on past variables. That is, no information in period tother than consumption itself can be helpful in explaining the consumption behaviour in period  $t + 1$ . The exclusion tests of Hall's REPIH are regressions to test whether  $E(C_{t+1}|C_t, I_t)$  is a function of  $I_t$  or not, where  $I_t$  is a vector of information in time  $t$  (other than  $C_t$ ).

In this paper we test to see if the log- linear model underlying Hall's Euler equation is appropriate. If it is not, empirical rejections of the REPIH could be due to an unrealized non- linearity in the aggregate time series data.

### 3. TESTS FOR NON- LINEARITY

#### 3.1 The BDS Test

The BDS test; named after Brock, Dechert, and Scheinkman (1988); is a test for randomness or irwhiteness\_ against the alternative general dependence in a series (Brock, Dechert, and Scheinkman, 1988). The BDS test can also be used to produce indirect evidence about non- linearity in data. If a linear structure in the data is extracted by an ARIMA process, then the BDS tet can be applied to the residuals to determine whether they are white noise. If the null of white noise is rejected, then there exists a general dependence in the residuals which may be due to the negelcted non- linearity in the estimation process. In this case, further investigation in needed to narrow down the alternative and determine the causes for the failure of the linear process.

The BDS test is based on the concept of *correlation integral* used by Grassberger and Procaccia (1983) in tests for chaos and the correlation integral of dimension  $m$  measures the spatial non-linearity. Correlation of a sample of  $T$  scattered points in  $m$ - dimensional space and picks up the fraction of  $m$ - dimensional pairs of points in that

sample  $(X^m_t, X^m_s)$  whose distance from each other is less than a fixed radius  $\epsilon$ . The correlation integral can be defined as follows:

$$C_{m,T}(\epsilon) = 2 \sum_{t < s} (||x^m_t - x^m_s|| < \epsilon) / (T_m(T_m - 1)), \quad (4)$$

where  $T$  is the size of the sample of points from the array  $x$ , and  $T_m = T - m + 1$ . In the context of this paper,  $X^m$  is a  $T \times m$  matrix whose vectors  $(X^m_0, X^m_1, X^m_2, \dots, X^m_t)$  refer to current and successively longer-

lagged values of a single time series variable  $X_t$ . Where the observations ( $X_t$ ) are identically and independently distributed, the true correlation integral for dimension  $m$  ( $C_{m,T}(\varepsilon)$ ) is related to the correlation integral for dimension 1 by the relation ( $C_{m,T}(\varepsilon) = [C_{1,T}(\varepsilon)]^m$ ), for all  $m$  and  $\varepsilon$ . Brock et al. (1988) showed that if the points  $X_t$  are iid, the standardized difference between  $C_{m,T}(\varepsilon)$  and  $[C_{1,T}(\varepsilon)]^m$  is asymptotically normally distributed. That is,

$$W_{m,T}(\varepsilon) = T^{1/2} [C_{m,T}(\varepsilon) - (C_{1,T}(\varepsilon))^m] / \sigma_{m,T}(\varepsilon) \sim N(0,1), \quad (5)$$

where  $T$  is the sample size,  $C_{m,T}(\varepsilon)$  is the correlation integral, and  $\sigma_{m,T}(\varepsilon) / T^{1/2}$  is the standard deviation of  $C_m$ .

The BDS test is able to pick up deviations from linearity in mean, and is also sensitive to series having autoregressive conditional heteroscedasticity (Lee et al, 1991). The BDS test performs best with 500+ observations, but Brock et al. (1992) point out that "even with 50 to 200 observations, BDS performs fairly well compared to the other tests."

### 3.2 The Artificial Neural Network Test

Artificial Neural Network (hereafter, ANN) models are non-linear input-output models with certain special features such as mass parallelism and non-linear processing of inputs which are also found in biological neural networks. By trying to mimic these basic features of biological neural networks, as well as processing inputs several times in separate stages, ANN models have succeeded in doing certain jobs very well, such as pattern recognition, optimisation, and forecasting (Kuan and White, 1994, Swanson and White, 1997, Moshiri and Cameron, 2000). In general form, the ANN output vector produced by a model or network with one output unit can be written as

$$y = F[\beta_0 + \sum_{j=1}^q G(x_j) \beta_j] = f(x, \theta), \quad (6)$$

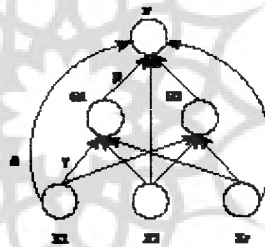
where  $y$  is the network's final output,  $F$  and  $G$  are (usually non-linear) transformation functions,  $X = [1, X_1, \dots, X_r]$  is a matrix of  $r$  input vectors (including the intercept term),  $\beta = [\beta_0, \beta_1, \dots, \beta_q]$  is a vector of weights for

the  $q$  intermediate transformations by the function  $G, \gamma = [\gamma_1, \dots, \gamma_q]$  is a matrix of  $q$  weight vectors, each vector relating the  $r$  input variables to one of the  $q$  intermediate totals, and  $\theta = (\gamma_1, \dots, \gamma_q, \beta_j)$  is a general term for the matrix containing both sets of weights.  $F$  and  $G$  can take any functional form, but the non-linear sigmoid function  $[y = 1/(1 + e^{-ax})]$  is a popular one, particularly for  $G$ .

The ANN model presented in (6) is a universal approximator. That is, given a sufficient number of intermediate units, it is capable of approximating any non-linear relationship [Rumelhart et al (1986), Kuan and White (1994)]. To use the ANN modelling approach to test for linearity, we first add an extra, direct connection between inputs  $x$  and output  $y$ , and then assume that the output transformation function ( $F$ ) is linear. With those two changes and after adding an error term ( $\varepsilon$ ), equation (6) becomes:

$$y = \beta_0 + x \delta + \sum_{j=1}^q G(x \gamma_j) \beta_j + \varepsilon \quad (7)$$

where  $(\delta)$  is a  $(1 \times r)$  vector of connection weights between inputs ( $x$ ) and output ( $y$ ), and other symbols are as in equation (6). Figure 1 shows such an augmented ANN model with  $r$  input units ( $x$ ), two intermediate units transformed by the function  $G$  (that is,  $q=2$ ), and one output unit ( $y$ ).



**Figure 1:** An augmented ANN model with  $r$  input units  $[x_1, \dots, x_r]$ , two intermediate units ( $G_1$  and  $G_2$ ), and one output unit ( $F$ ). ( $s$ ,  $\$s$ , and  $*s$  are connection weights for intermediate units, output units, and direct input-output units, respectively).

If the time series  $x$  is linear, the non- linear component of equation (7), i. e. the third term, would vanish, leaving just a linear regression model with the coefficients  $\beta_0$  and the vector  $\delta$ . Therefore, the neural network test for non- linearity uses equation 7, testing the null hypothesis  $\beta_0 = 0, j = 1, \dots, q$ . If we apply a linear filter, such as AR, to a time series, the residuals can be used to determine whether there is a non- linearity in the series. If the series is linear, the residuals obtained from a linear process should not be correlated with AR process and any function of the history. Thus, the null hypothesis can be expressed as  $E(e_t G_t) = 0$ , where  $e_t$  are the residuals from a linear regression of  $y$  on  $x$  ( $e_t = y_t - x_t \delta$ ) and  $G_t$  is the vector of  $q$  intermediate totals for date  $t$ . The test statistic  $Z_n$  for the ANN test can then be defined as:

$$Z_n = [T^{-1/2} \sum_{t=1}^T G_t e_t] \hat{W}^{-1} [T^{-1/2} \sum_{t=1}^T G_t e_t]$$

where  $\hat{W}$  is a consistent estimator of  $W = Var(T^{-1/2} \sum_{t=1}^T G_t e_t)$  (Lee et al, 1991). Under  $H_0$ ,  $Z_n$  has an asymptotic chi- square distribution with  $q$  degrees of freedom. To avoid the problem of collinearity between  $x$  and the components of  $G$ , the principal components of  $G$  which are not correlated with  $x$  can be used instead of  $G$ . In this case, there is an alternative test- statistic that is easier to calculate:

$$TR^2 \sim \chi^2(q), \quad (8)$$

where  $T$  is the number of observations and  $R^2$  is the squared multiple correlation coefficient from a standard linear regression of  $e$  on  $x$  and principal components of  $G$  which are not correlated with  $x$ . We use the simpler test statistic below.

### 3.3 Data

The BDS and ANN tests are carried out on quarterly US and Canadian data for personal consumption expenditures on non- durable goods and services, seasonally adjusted and in 1986 prices, for the period 1947: 1- 1996: 4. All data are from Statistics Canada's CANSIM databank, as maintained by CHASS at the University of Toronto.



#### 4. RESULTS

For both BDS and ANN tests we wish to test a transformation of the consumption series that could reasonably be expected to be iid. This means extracting the linear component of each consumption series and then testing the residuals for presence of a neglected the residuals will be the series  $x_t$  in (4) for calculating the non-linearity correlation integral in the BDS test, and  $e_t$  in (8) for the ANN test. To find the residuals with an ARMA or ARIMA model we need to establish the order of integration of the consumption series. Both ADF and Phillips- Perron unit root tests suggest that the consumption series is  $I(1)$ , so an ARIMA model is used<sup>3</sup>. To filter the data from serial dependency, the Box- Jenkins time series method was used. Results show that the following ARIMA models are appropriate for the original data set to make residuals of the model both stationary and without autocorrelation (t statistics are in brackets): Canada:

$$C_t = .006 + 0.136 C_{t-3} + 0.202 C_{t-4} + 0.154 C_{t-7} - 0.158 C_{t-8}$$

$$t \quad (4.8) \quad (1.95) \quad (2.88) \quad (2.24) \quad (-2.35)$$

R<sup>2</sup> = 0.12, Breusch- Godfrey LM test: F= 1.28, ARCH test: F = 0.17.

United States:

$$C_t = 0.007 + 0.191 C_{t-1} - 0.134 C_{t-8}$$

$$t \quad (8.82) \quad (2.68) \quad (-1.97)$$

R<sup>2</sup> = 0.05, Breusch- Godfrey LM test: F= 0.088, ARCH test: F = 2.15,

Where all C's are change in log of consumption series. Significant coefficients in the above models indicate that a change in current consumption is correlated with the past changes in consumption, and therefore, not random. In this respect our results agree with all of those

3 . ADF test statistics for stationarity of the levels of natural logarithms of Canadian and US consumption around a log- linear trend (with an intercept term) are -1.07 and -0.55, respectively, using AR orders of 11 for Canada and 1 for the US (the minimum AR order at which the longest lag coefficient is statistically significant). Phillips- Perron test statistics for the same assumptions are 0.097 and -0.720 respectively, with lag truncation at 4 quarters for both series. The MacKinnon critical value at 5 percent is -3.43 for both tests. Test results for stationarity of first differences around a mean are -1.77 and -11.94 for the ADF test for Canada and the US, respectively, with 12 and 0 lags, and and -12.30 for the Phillips- Perron test with lag truncation at 4 quarters. -15.58

previous tests that reject the exclusion restrictions of Hall<sup>TM</sup>'s REPIH. The ADF and Phillips- Perron test statistics for the residuals of the above regressions are all below -13.8, strongly rejecting the null of non-stationarity; this is weak evidence against those forms of non-linearity that would cause steady divergence from a linear relationship. To provide more direct evidence on the presence or absence of a non-linear component, we apply the BDS and ANN tests to the residuals of the Canadian and US models above.

#### 4.1 BDS Test Results

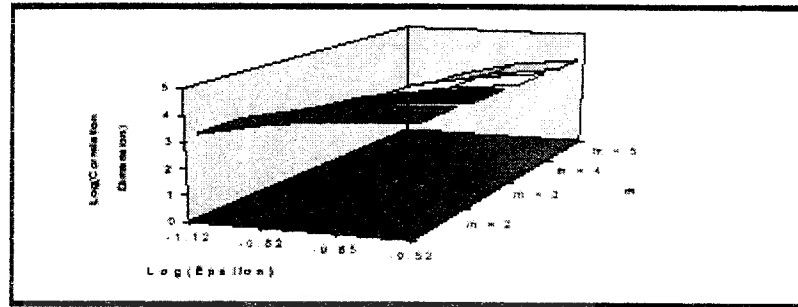
To carry out the BDS test, we let the series  $x$  of equation (4) be the residuals from each ARIMA model and calculate the correlation integral ( $C_{m,t}(\varepsilon)$ ) and the BDS statistics ( $W_{m,t}(\varepsilon)$ ) Table 1 shows the results of the BDS test for dimensions  $m$  from 2 to 5 and for sizes of threshold radius  $\varepsilon$  ranging from 0.5 to 2. The BDS test statistics are not large enough to reject the hypothesis of white noise residuals implying that the aggregate consumption series is linear.

**Table 1:** BDS statistics ( $W_{m,t}(\varepsilon)$  in equation 5) for Filtered Residuals of Linear Model, US and Canadian Consumption of Non-durable Goods and Services, 1946: 1- 1996: 4.

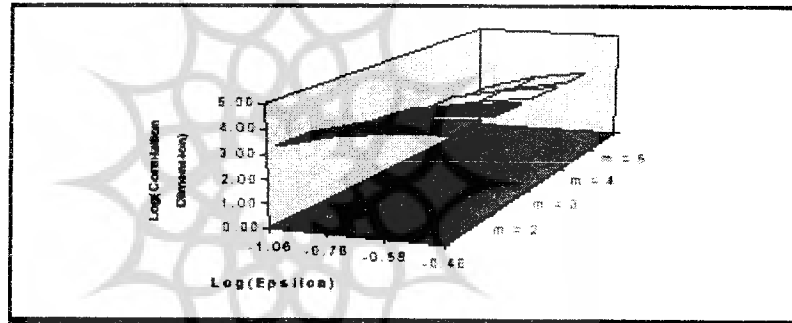
		$\varepsilon$							
		0.5		1		1.5		2	
$m^a$		CAN	US	CAN	US	CAN	US	CAN	US
2		0.03	0.02	0.15	0.08	0.19	0.91	0.16	0.06
3		0.02	0.007	0.18	0.08	0.26	0.18	0.24	0.192
4		0.01	0.01	0.22	0.10	0.37	0.31	0.38	0.35
5		0.009	0.004	0.21	0.10	0.48	0.34	0.51	0.48

a. Embedding dimensions.

The BDS test can also be presented visually. According to Grassberger and Procaccia (1983), if the correlation dimension ( $d_m$ ) continues to increase as the embedding dimension ( $m$ ) increases, the data is white noise. The correlation dimension ( $d_m$ ) is defined as the slope of the regression relationship of  $\ln(C_{m,T}(\epsilon))$  with  $\ln \epsilon$ . Whether this is the case for any dataset can be judged from a graph which plots  $\ln(C_{m,T}(\epsilon))$  against  $\ln(\epsilon)$  for various values of  $m$  and  $(\epsilon)$  as shown in Figure 2. The residuals in both models look like white noise.



a. Canada



b.US

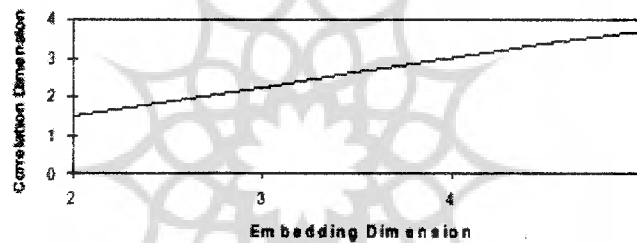
**Figure 2: Correlation Dimension for different dimensions.**

To see the relationship between  $(d_m)$  and  $m$  more clearly,  $\log(C_{m,T}(\epsilon))$  is regressed against  $\log(\epsilon)$ , and then the correlation dimensions of the data are estimated for various levels of the dimension  $m$ . The results are shown in Table 2.

**Table 2:** Correlation dimensions ( $\partial \ln C_{m,T}(\varepsilon) / \partial \ln \varepsilon$ ) of the US and Canadian aggregate consumption series.

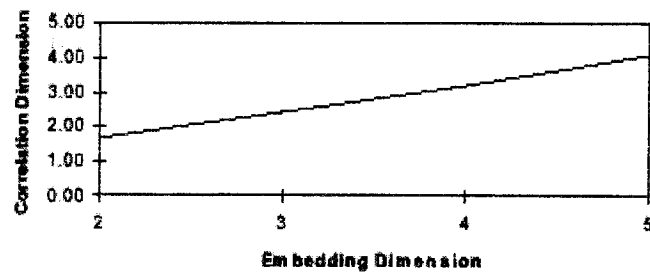
Embedding Dimension (m)	Correlation Dimension (dm)	
	Canada	US
2	1.54	1.53
3	2.31	2.3
4	3.05	3.03
5	3.76	3.8

The plot of  $d_m$  against m in Figure 3 shows that  $d_m$  is not stabilized. According to the test criterion, this means that the residuals of the linear filter of aggregate consumption series are iid and therefore there is no hidden non-linearity in the data set.



a. Canada

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b. US

Figure 3: Correlation Dimension vs Embedding Dimension

#### 4.2 ANN Test Results

To run an ANN test, the intermediate totals  $G(x\gamma_j)$  are calculated using random coefficients for  $\gamma$  and the lagged consumption series as the input vector  $x$ . Then we obtain those principal components of  $G(x\gamma_j)$  which are not correlated with the lagged values of the consumption series; we call this set  $G^*$ . The residuals of the ARIMA models obtained in section 3.4 are then regressed against the input vector and the principal components  $G^*$ , and the test statistic  $TR^2$  is calculated. The result of this test is shown in Table 3.

Table 3: The results of the ANN test for the US and Canadian aggregate consumption.

ANN Model	$TR^2$ Test Statistics*	Critical Values
Canada	8.22	9.35 (2.5%) 7.81 (5%)
US	1.34	5.99 (5%) 7.38 (2.5%)

\*The null hypothesis is that the underlying data generation model is linear.

The ANN test result in case of the US consumption series, as presented in Table 3, clearly supports the result of the BDS test: the hypothesis of linearity cannot be rejected at 5 percent significance level. Although the ANN test result in case of the Canadian consumption series is not consistent with the BDS test result at 5 percent significance level, the hypothesis of the linearity cannot be rejected at 2.5 percent significance level. Therefore, it seems to be quite appropriate to use linear models to estimate and forecast aggregate consumption series.

The BDS and the ANN tests can be considered as two complement tests for detecting non- linearity in a series. The BDS test tests a series for being white noise against a general dependence. In our application, we used the BDS test to test linearity in the series against a very general alternative of non- linearity. In the ANN test, the series is tested for linearity against an alternative of non- linearity in mean. Therefore, if the null of linearity in the BDS test is rejected, then one can use the ANN test to determine whether the process is in mean or not such as BARCH. If the null hypothesis of linear non-linear process in the BDS test is not rejected, then the ANN test results can be used as another evidence of linearity in the data.

While use of linear models is not rejected, there may be weak evidence in favour of non- linearity if non- linear models are still better for estimation and forecasting. To check on this possibility, in the next section we run a horse race between the two ARIMA models and an ANN model to forecast aggregate consumption.

## 5. FORECASTING AGGREGATE CONSUMPTION

The data set is divided into an estimation sample and an out- forecasting data set. The out- of- sample forecasting is done for of-sample three different horizons: one quarter ahead, four quarters ahead, and eight quarters ahead. Mean absolute percentage error (MAPE) is used as the performance criterion in comparing models. The ANN model used is a Back Propagation Network model using two intermediate stages; it is allowed 100,000 stochastic replications to learn the pattern of the data set . The results are summarized in the table 4.

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**Table 4:** Mean Absolute Percentage Error (MAPE) of Forecast using the ARIMA and ANN models for US and Canadian aggregate consumption series.

Model	1 - quarter	4 - quarter	8 - quarter
ARIMA- CAN	0.34	0.47	0.78
ANN- CAN	0.13	0.13	0.25
ARIMA - US	0.28	0.4	0.72
ANN - US	0.14	0.2	0.34

The results in Table 4 show the ANN model to outperform the ARIMA model. The results of Table 4 weaken the previous conclusion supporting linear models of consumption, and supports the conclusion of Swanson and White (1995) that even in the absence of explicit the adaptive ANN models appear to be promising for use non-linearity, in forecasting.

## 6. CONCLUSION

The Rational Expectations Permanent Income Hypothesis implies that aggregate consumption follows a martingale . Most empirical tests, based on a linear model of aggregate consumption behaviour, have rejected the hypothesis. In this study we have checked for the presence of any stochastic non- linearity in the aggregate consumption time series. Two different tests are applied to identify hidden a test developed by Brock, Dechert, and Scheinkman, non-linearities: and a test using an Artificial Neural Network approach. The results for US and Canadian aggregate consumption of non-durable goods and services from 1947 to 1996 do not reject the hypothesis of linearity. However, consumption behaviour is better forecast with the non-linear Neural Network model than with the linear ARIMA model, so allowing for some non- linearity also seems appropriate.

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